APPROXIMATE ALGEBRAIC STRUCTURES AND ARITHMETIC COMBINATORICS

Schedule and Abstracts



Mathematical Institute of the Serbian Academy of Sciences and Arts September $3-5,\,2025$

Schedule

Time	Wednesday	Thursday	Friday
9.30-9.45	Registration		
9.45-10	MI SANU Welcome		
10–11	Mrazović	Jamneshan	Pach
11–12	Beker	Ranđelović	Bucić
12-13.45	Lunch	Lunch	Lunch
13.45-14.45	González-Sánchez	Milićević	Milojević
14.45–15.45	Candela	Open problems	Zahirović
		and discussion	

Abstracts

The Erdős-Moser sum-free set problem via improved bounds for k-configurations

Adrian Beker University of Zagreb

In a recent remarkable breakthrough, Kelley and Meka significantly improved the bounds for sets of integers lacking three-term arithmetic progressions. Soon thereafter, Filmus, Hatami, Hosseini and Kelman extended their methods to sets lacking so-called binary systems of linear forms. Central to their approach is a new sparse graph counting lemma which is tailored to density increment arguments in additive number theory. In this talk, I will discuss how one can combine improvements in certain quantitative aspects of their graph counting lemma with further Kelley–Meka-style arguments to get new bounds for sets lacking k-configurations, i.e. collections of k integers together with their pairwise arithmetic means. As a consequence, this gives an alternative proof of a bound for the Erdős–Moser sum-free set problem of best known shape.

ON GRAHAM'S REARRANGEMENT CONJECTURE

Matija Bucić

University of Vienna and Princeton University

A well known question in combinatorial group theory, going back to a conjecture of Graham from 1971, asks if given a subset S of some group it is possible to order S as s_1, s_2, \ldots, s_t so that the partial sums s_1, \ldots, s_j are all distinct for each j < t. We discuss recent progress on this question based on a synergy between ideas from additive combinatorics and graph theory. Based on a joint work with Benjamin Bedert, Alp Müyesser, Noah Kravitz, and Richard Montgomery.

A SPECTRAL APPROACH TO HIGHER-ORDER FOURIER ANALYSIS - PART II

Pablo Candela

Instituto de Ciencias Matemáticas (ICMAT)

We propose a general framework in higher-order Fourier analysis, based on the spectral decomposition of matrices that represent functions on abelian groups. This framework offers new theoretical insights on higher-order Fourier analysis, and also practical potential via algorithms for the calculation of higher-order decompositions of functions.

In this second part of this presentation, we shall focus on explaining how nilspace theory comes into the picture to shed light on the structure of the dominant components of functions produced by our algorithms. In particular, we shall discuss the notion of a nilspace character (which extends naturally the better known concept of a nilcharacter) and outline our proof that, in the quadratic setting, the dominant eigenvectors produced by our spectral approach are essentially 2-step nilspace characters which are pairwise quasiorthogonal. We will also discuss various related results and theoretical consequences, including new higher-order versions of classical results such as Parseval's identity.

Joint work with Diego González-Sánchez and Balázs Szegedy.

A SPECTRAL APPROACH TO HIGHER-ORDER FOURIER ANALYSIS - PART I

DIEGO GONZÁLEZ-SÁNCHEZ

HUN-REN Alfréd Rényi Institute of Mathematics

We propose a general framework in higher-order Fourier analysis, based on the spectral decomposition of matrices that represent functions on abelian groups. This framework offers new theoretical insights on higher-order Fourier analysis, and also practical potential via algorithms for the calculation of higher-order decompositions of functions.

In this first part of this presentation, we will present the elementary theory of our approach. We will show how to create matrices associated with functions defined on finite abelian groups, and discuss some of their elementary properties. Along the way, we will present some elementary notions of higher-order characters on finite abelian groups. We will then explain how such characters naturally emerge in our spectral approach as isolated eigenvectors of matrices associated with functions.

Joint work with Pablo Candela and Balázs Szegedy.

QUADRATIC MAPS BETWEEN NON-ABELIAN GROUPS

Asgar Jamneshan

University of Bonn

Gowers and Hatami initiated the inverse theory for the uniformity norms U^k of matrix-valued functions on non-abelian groups, proving a 1% inverse theorem for the U^2 -norm and linking it to stability questions for almost representations. In this talk, I will present recent joint work with Andreas Thom that takes a step toward an inverse theory for higher-order uniformity norms of matrix-valued functions on arbitrary groups. Specifically, we investigate the 99% regime for the U^k -norm on perfect groups of bounded commutator width.

Our analysis leads to a classification of Leibman's quadratic maps between non-abelian groups. The main result is a complete description of these maps via an explicit universal construction. This classification yields several applications: a full classification of quadratic maps on arbitrary abelian groups; a proof that perfect groups admit no nontrivial polynomial maps of degree greater than two; and stability results for approximate polynomial maps.

TBA

Luka Milićević

Mathematical Institute of the Serbian Academy of Sciences and Arts

TBA

ZARANKIEWICZ'S PROBLEM AND ITS APPLICATIONS TO INCIDENCE GEOMETRY

Aleksa Milojević ETH Zurich

Zarankiewicz's problem is one of the central problems in extremal graph theory, and it asks for the maximum number of edges in a bipartite graph with vertex classes of size n, which does not contain the complete bipartite graph $K_{s,s}$. In recent years, an interesting twist on this problem attracted the interest of many researchers: given a family of graphs \mathcal{F} with certain additional structure (e.g. certain incidence graphs), what is the maximum number of edges in a graph of \mathcal{F} with n vertices, which does not contain $K_{s,s}$? In this talk, I will show how the study of such problems through the lens of extremal graph theory can lead to new incidence bounds, applicable to the setting of finite fields, going beyond the reach of space-partitioning techniques.

This talk is based on joint work with Zach Hunter, Benny Sudakov and Istvan Tomon.

TRANSVERSALS IN QUASIRANDOM LATIN SQUARES

Rudi Mrazović

University of Zagreb

A transversal in a $n \times n$ latin square is a set of entries not repeating any row, column, or symbol. A famous conjecture of Brualdi, Ryser, and Stein predicts that every latin square has at least one transversal provided n is odd. We will discuss an approach motivated by the circle method from the analytic number theory which enables us to count transversals in latin squares which are quasirandom in an appropriate sense.

THE ALON-JAEGER-TARSI CONJECTURE VIA GROUP RING IDENTITIES

PÉTER PÁL PACH

HUN-REN Alfréd Rényi Institute of Mathematics and BME

The Alon-Jaeger-Tarsi conjecture states that for any finite field \mathbb{F} of size at least 4 and any nonsingular matrix A over \mathbb{F} there exists a vector x such that neither x nor Ax has a 0 component. In this talk we discuss the proof of this result for $|\mathbb{F}| > 79$ and further applications of our method about coset covers and additive bases. Joint work with János Nagy and István Tomon.

TBA

ŽARKO RANDELOVIĆ

Mathematical Institute of the Serbian Academy of Sciences and Arts

TBA

ALGEBRAIC GRAPH THEORY (OR THE OTHER WAY AROUND?)

Samir Zahirović

University of Novi Sad, Faculty of Sciences

Algebraic graph theory seeks algebraic properties within graphs. In this talk, however, we will not focus on identifying groups within graphs. Instead, we will discuss various graphs constructed from groups and other algebraic structures. The Cayley graph is the most well-known example, but there are many others as well, such as the commuting graph, the power graph, the enhanced power graph, the difference graph, the intersection subgroup graph, and more. We will examine how effectively these graphs reflect the underlying algebraic structure, explore their combinatorial properties, and consider how they relate to one another.