Teaching-Scientific Council
Faculty of Mathematics
University of Belgrade

By the decision of the Teaching-Scientific Council of the Faculty of Mathematics at the University of Belgrade, adopted at the meeting held on September 27, 2024, we were appointed as members of the jury for the evaluation of the doctoral dissertation *Statistical tests based on Laplace and Hankel transforms, and their application in change point detection* by doctoral candidate Žikica Lukić. After reviewing the submitted manuscript, we hereby present the following:

REPORT

1 Biography

Žikica Lukić was born on October 25, 1996, in Užice. He enrolled in undergraduate studies at the Faculty of Mathematics (Mathematics program, Statistics, Actuarial and Financial Mathematics module) in 2015 and graduated in 2019 with an average grade of 9.47. He completed his Master's studies in the same module in 2020 with an average grade of 10.00, defending his Master's thesis titled "Characterization of the Levy Distribution and a New Goodness-of-Fit Test Based on It," under the mentorship of Professor Dr. Bojana Milošević. The thesis was awarded the Mathematics Institute of the Serbian Academy of Sciences and Arts prize for the best Master's thesis.

He enrolled in doctoral studies at the Faculty of Mathematics, University of Belgrade (Mathematics program) in 2020 and has since passed all exams with a grade of 10.

From 2017 to 2021, he held the position of research associate at the Petnica Science Center, while in 2019, he completed an internship at Raiffeisen Bank.

In the 2020/2021 academic year, he was employed at the Faculty of Mathematics as a teaching associate for the scientific field of Probability and Statistics. From 2020 to 2022, he worked at TeleTrader. After that, he continued his career at Parexel, where he holds the position of biostatistician.

Papers in SCI-indexed journals

1. Lukić, Ž., Milošević B. Change-point analysis for matrix data: the empirical Hankel transform approach. Statistical Papers, 2024,1-26. doi: 10.1007/s00362-024-01596-4 accepted for publication (IF2023=1.2, M22),

2. Lukić, Ž. A Laplace transform–based test for the equality of positive semidefinite matrix distributions. Filomat, 2024, accepted for publication, (IF2021=0.988, M22)

3. Lukić, Ž., Milošević B. A novel two-sample test within the space of symmetric positive definite matrix distributions and its application in finance. Annals of the Institute of Statistical Mathematics, 2024, 1-24. doi: 10.1007/s10463-024-00902-z, (IF2021=1.180, M23)

4. Lukić, Ž., Milošević, B. Characterization-based approach for construction of goodness-of-fit test for Lévy distribution. Statistics, 2023, 1087-1116. (IF2021=2.346, M21).

Conferences

- Lukić, Ž., Milošević, B. On some change-point tests based on the Laplace transform, September 2024, Statistical Modeling with Applications, Belgrade, Serbia, M34, presenting author
- Lukić, Ž. On a two-sample test for equality of matrix distributions based on Laplace transforms, June 2024, 15th Serbian Mathematical Congress, Belgrade, Serbia, M34, presenting author
- Lukić, Ž, Milošević, B. On Recent Developments in Change Point Analysis using Integral Transforms, December 2023, The thirteenth symposium Mathematics and Applications, Belgrade, Serbia, M64, presenting author
- Lukić, Ž., Milošević, B. On the novel two-sample tests and their application for change point analysis, CMStatistics 2023, December 17-19 2022, Berlin, Germany, M34, coauthor
- Lukić, Ž., Milošević, B. On a Novel Two-Sample Test for Matrix Distributions and Application in Finance, September 2023, 23rd European Young Statisticians Meeting (EYSM), Ljubljana, Slovenia, M32, presenting author, invited talk
- Lukić, Ž., Milošević, B. Change point analysis – the empirical Hankel transform approach GOFCP August 2023, Kruger Park, South Africa, M34, coauthor
- Lukić, Ž., Milošević, B. Testing for the equality of matrix distributions in the space of positive semi-definite random matrices, April 2023, The Spring HiTEC meeting, Larnaca, Cyprus, M34, coauthor
- Lukić, Ž., Milošević, B. On equality of matrix distributions in the space of positive semi – definite random matrices, December 2022, The twelfth symposium Mathematics and Applications, Belgrade, Serbia, M64, presenting author
- Lukić, Ž. On some new characterizations of stable distributions, October 2022, The Congress of Young Mathematicians KMMNS2, Novi Sad, Serbia, M64, presenting author
- Lukić, Ž., Milošević, B. On goodness-of-fit tests for the Lévy distribution, December 2021, The eleventh symposium Mathematics and Applications, Belgrade, Serbia, M64, presenting author
- Lukić, Ž., Milošević, B., Obradović, M. On the fiducial distribution of the reliability parameter of a two-component system with independent exponential distributions, September 2021, XLVIII International Symposium on Operational Research SYM-OP-IS, Banja Koviljača, Serbia, M34, presenting author

2 The Subject of the Dissertation

The subject of the doctoral dissertation is the development of statistical tests based on Laplace and Hankel transforms and their application in change point detection. Special focus is on the matrix-valued data for which the literature is very sparse. On the other hand the need for the development of methodology for such data is growing with the advance of technology.

The dissertation is divided into two main parts. In the first part, two new two-sample tests for matrix data are introduced and their properties are studied in detail. The second part of the dissertation focuses on data segmentation, more specifically change-point detection. Two novel

classes of univariate tests for offline data segmentation are developed and their theoretical properties are studied. These tests are applied to datasets from meteorology and macroeconomics, showcasing their practical use in real-world.

3 Overview of the Dissertation

The dissertation is written on $x + 97$ pages, and its structure is as follows:

In the Preface, the author provides a brief historical account of aspects of the development of "matrix methods" in mathematical statistics. Although the matrix methods discussed are limited to the areas that appear in the dissertation, the account given here is extensive since the substantial and wide-ranging material is covered in less than two full pages, from which it can be seen that the author read widely so as to be able to distill such wide-ranging material into a relatively brief preface.

In Chapter 1, the author provides a detailed introduction to the statistical and mathematical concepts that are necessary to read the dissertation. The outcome is that this introductory chapter results in a largely self-contained dissertation, thereby enabling its readers to understand

the technical details without having to consult external sources at each stage of the development of the material. This all makes for a highly readable document.

In Chapter 2, the author develops and treats the "two-sample problem" of testing that two statistical matrix populations have the same distribution. In particular, two novel tests are proposed: one utilizing Hankel transform and one utilizing Laplace transform. For both tests, the author derived asymptotic and small sample properties as well. The novel tests are applied to multiple real-data cases, primarily stemming from the field of finance. These tests are the first of their kind for two-sample tests for positive-definite symmetric matrix distributions and are based on Laplace and Hankel transforms.

In Chapter 3, the author provides an introduction to change-point statistical inference and then he also develops some astute modifications of classical integral transform two-sample test statistics for the detection of change-point within one-dimensional data.

The results of Chapter 4 are truly unparalleled. In this chapter, novel and unique change-point test statistics for matrix data are introduced, and their limiting and small-sample properties are studied. The test is then applied to financial data, demonstrating the practical applicability of this approach.

4 Conclusion

The results presented in this dissertation represent a significant contribution to the development of matrix statistics. For the first time, two-sample tests for matrix distributions are considered. Additionally, a methodology for detecting changes in sequences of random variables of different types, including matrix data, is proposed.

The candidate has demonstrated excellent knowledge of the field, the ability to conduct independent research, and the skill to derive complex mathematical conclusions and present them appropriately.

Žikica Lukić has published three co-authored papers and one solo-authored paper in SCI-indexed journals and has presented his work at many scientific conferences. Taking all this into account, and since all formal requirements have been met, we propose that the thesis *Statistical tests based on Laplace and Hankel transforms, and their application in change point detection* be accepted as a doctoral dissertation and the jury for its public defense be assigned.

Belgrade, November 15 2024

Jury:

Marko Obradović
Assistant Professor
University of Belgrade
Faculty of Mathematics

Marija Cuparić
Assistant Professor
University of Belgrade
Faculty of Mathematics

Donald Richards,
Distinguished Professor Emeritus
Penn State University