1 Automated Reasoning

The history of automated reasoning in Serbia is more than 20 years long. Most of research in this area focused on using decision procedures in automated theorem proving, but there are also results in automated theorem proving in geometry, in formal theorem proving, in uniform proof procedures (like tableaux and resolution methods), and in theorem proving in graph theory. In this section we briefly summarize some of the most interesting results achieved in this field in Serbia. In the following text, only publications by Serbian authors are cited (and not the other relevant publications). The home institutions are given for all the mentioned authors (either from Serbia or not).

Tableaux and Resolution Based Theorem Proving. In 1992, Krapež, Kapetanović, Ognjanović, and Petrović (Mathematical Institute, Belgrade), developed a parallel theorem prover `prover91`, based on the tableaux method [10].

Petar Hotomski (University of Novi Sad) and his collaborators have been developing an automated theorem prover based on ordered linear resolution and used it in practical problems like program verification and scheduling [1].

Automated Reasoning in Graph Theory. During 1980's, a team lead by Dragoš Cvetković (University of Belgrade) developed an interactive system GRAPH for the field of graph theory. The system consisted of several components, including an automated theorem prover. The theorem prover (working in first order logic) used libraries of definitions and lemmas, and supported both interactive theorem proving (in natural deduction) and automated theorem proving (based on the resolution method) [2]. The system had support for proving statements about concrete graphs and for checking conjectures on particular graphs (including random graphs), for using analogies, and for the interaction in natural language.

Automated Reasoning in Geometry. Predrag Janičić and Stevan Kordić (University of Belgrade) developed the first automated geometry theorem prover based on coherent logic [8]. The prover can generate human-readable, traditional proofs of conjectures in Euclidean geometry.

Since 1996, Predrag Janičić, with his collaborators, has been developing a geometry system GCLC[1] — a tool for describing and visualizing geometrical objects and for reasoning about them [3, 4]. There are three geometry theorem provers built in GCLC: a theorem prover based on the area method, implemented by Predrag Janičić and Pedro Quaresma (University of Coimbra) [9], and theorem provers based on the Gröbner bases method and on Wu's method, implemented by Goran Predović and Predrag Janičić. The built-in theorem provers can efficiently prove hundreds of complex geometry theorems. The tool

[1] `http://www.matf.bg.ac.rs/~janicic/gclc`

GCLC has thousands of users worldwide and is taught in a number of high-school and university courses.

Decision Procedures and Satisfiability Modulo Theory. Predrag Janičić, Ian Green (University of Edinburgh), and Alan Bundy (University of Edinburgh) developed a general framework GS that facilitates implementing, combining, augmenting decision procedures, and using them in theorem provers [7, 5]. Portions of this system were used in *Clam* and *LambdaCLam* proof planning systems. Along this line of research, Predrag Janičić and Alan Bundy developed an approach for automated synthesis of a class of decision procedure [6].

In early 2000's, a new generation of reasoning systems based decision procedures, with extensive use of propositional reasoning, emerged and led to a new area of *satisfiability modulo theory (*SMT*)*. Filip Marić (University of Belgrade) and Predrag Janičić implemented a SMT system ArgoLib based on one such approach (DPLL(T)), with a support for several decidable theories and their combinations [12]. The ArgoLib system successfully took part in the 2007 SMT competition and is used by several research groups in different countries. Motivated by the importance of propositional reasoning within a SMT solver, Filip Marić has been developing a new SAT solver ArgoSAT, with a flexible architecture. Both ArgoLib and ArgoSAT are under constant development and publicly available[2].

Aleksandar Jovanović and Žarko Mijajlović developed a new quantifier elimination method for the theory of algebraically closed fields and the theory of ordered real closed fields and implemented it with Filip Marić and Miroslav Marić [14].

Formal Theorem Proving. Because of importance of SAT solvers (not only within SMT solvers, but in many other applications), it is essential that they are correct. Filip Marić and Predrag Janičić have been working on verification of SAT solvers with a goal of producing solvers that are both efficient and fully trusted. This work includes proving correctness of the classical DPLL procedure [13] and verification of modern SAT solvers [11]. For the verification of modern SAT solvers three different paradigms were used: verification of abstract state transition systems describing SAT solvers, Hoare-style verification of an imperative implementation of a solver, and verification of a solver defined as a set of recursive functions within higher order logic (these descriptions are the most detailed ones considered so far in the context of solvers' verification). In order to achieve the highest level of trust, all these verification tasks were performed within the Isabelle proof assistant. This verification project is accompanied by the SAT solver ArgoSAT and a trusted, automatically generated solver implemented in a functional language.

[2] `http://argo.matf.bg.ac.rs`

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