Konsultacije DJ 27.08.

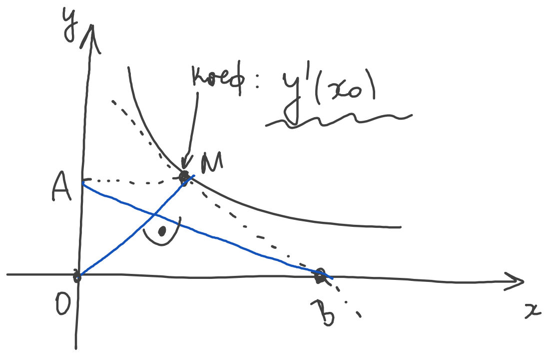
nedelja, 27. avgust 2023. 15:40

Tačka M krive Γ projektuje se na y-osu u tačku A, a tangenta krive Γ u tački M seče x-osu u tački B. Važi da je prava određena koordinatnim početkom i tačkom M ortogonalna na pravu AB, nezavisno od izbora tačke M.

a) Odrediti jednačinu krive Γ ako je poznato da prolazi kroz tačku (2,2).

b) Odrediti jednačinu tangente na krivu Γ u tački (2,2).

$\Gamma : \; y = y(x)$



$M(x_0, y(x_0))$

$A(0, y(x_0))$

$O(0,0)$

$$y = \underbrace{k}_{y'(x_0)} \cdot x + n$$

M ∈ тангенти $\Rightarrow y(x_0) = y'(x_0)\cdot x_0 + n$

$n = y(x_0) - y'(x_0)\cdot x_0$

B ∈ тант.

$y_B = 0$, $\overset{0}{y_B} = y'(x_0)\cdot x_B + y(x_0) - y'(x_0)\cdot x_0$

$$x_B = \frac{y'(x_0)\cdot x_0 - y(x_0)}{y'(x_0)}$$

B(, 0)

$$k_{AB} = \frac{y_B - y_A}{x_B - x_A}, \quad k_{OM} = \frac{y_O - y_M}{x_O - x_M}$$

$$AB \perp OM \iff \boxed{k_{AB}\cdot k_{OM} = -1}$$

→ диф. јна (хомогена $y' = f(\frac{y}{x})$)

(*) $X' = AX$, $A \in M_n(\mathbb{R})$

a) n непарно, $\det(A) \neq 0$ ∃ није период.

→ карακт. јна $\det(A - \lambda E) = 0$ реда n полином

n непарно ⇝ ∃ решење λ_0 које $\in \mathbb{R}$

v_0 сопст. вектор

$\boxed{x(t) = e^{\lambda_0 \cdot t} \cdot v_0}$ → ово решење се тражи

→ $\lambda_0 = 0$ → не може јер $\det(A) \neq 0$

б) $\det(A) = 0$ $\quad A = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ & & \ddots & \\ 0 & \cdots & & 0 \end{bmatrix}_{n \times n}$ → сва решења const → периодична

в) $A = [\,\cdots\,]$ → решити систем тешке и одредити конст.

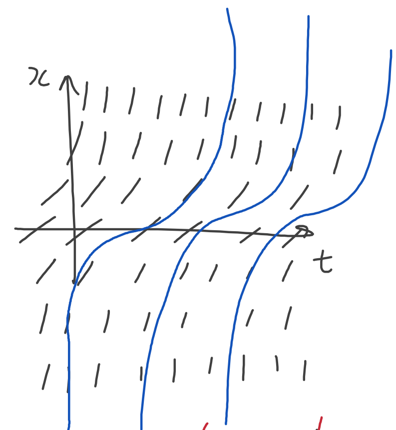
(*) $a \in \mathbb{R}$, $x' = a + x^2$ $\qquad x^2 + a = 0$

1° $a > 0$ $\quad x^2 + a > 0$

$x' > 0 \Rightarrow x \nearrow$

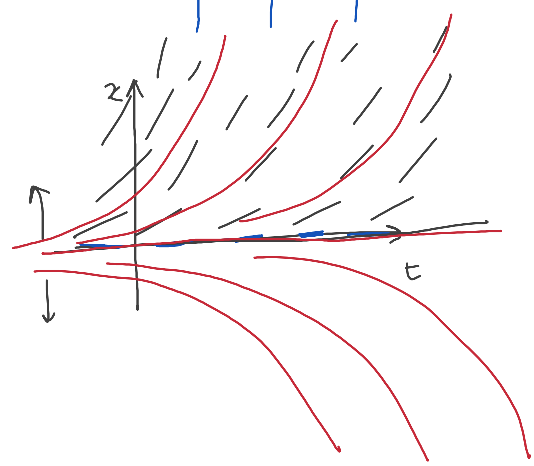
$x = 0: \ x' = a$

$x = \pm 1: \ x' = a + 1$



2° $a = 0$ $\quad x^2 + \cancel{a} = 0$

0 $\quad \underline{\underline{x = 0}}$



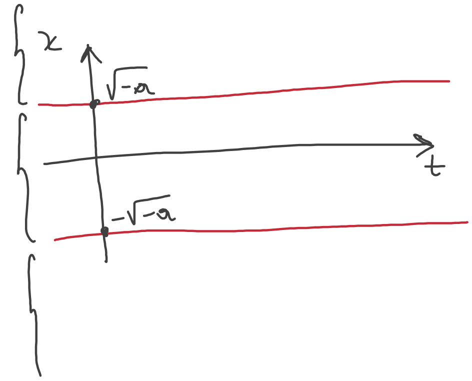
3° $a < 0$

$x^2 + a = 0$

$x^2 = -a$

$x = \pm\sqrt{-a}$

за вежбу ←



(*) г) хом. лин. диф. јнџ са КК мин. реда

$\boxed{x_1(t) = t}$

$x_2(t) = e^{-t}\sin t \rightsquigarrow e^{-t}\cos t$

$\boxed{-1 \pm i}$

$\lambda \rightsquigarrow e^{\lambda t},\ t\cdot e^{\lambda t}, \ldots$

$\alpha \pm i\beta \rightsquigarrow e^{\alpha t}\cos\beta t,\ e^{\alpha t}\sin\beta t$

$t\cdot e^{0t} = t \rightsquigarrow$ друго за $\lambda = 0$

$e^{0t} = 1$

$\boxed{\lambda = 0 \ \times 2}$

мин реда 4: $\lambda^2(\lambda^2 + 2\lambda + 2) = \lambda^4 + 2\lambda^3 + 2\lambda^2$

$(\lambda + 1 - i)(\lambda + 1 + i) = (\lambda+1)^2 - (i)^2 = \lambda^2 + 2\lambda + 2$

$x^{(4)} + 2x^{(3)} + 2x'' = 0$

в) $\left.\begin{array}{l} x_1(t) = t \\ x_2(t) = \ln t \end{array}\right\}$ $x''(t) + a(t)\cdot x'(t) + b(t)\cdot x(t) = 0$

(*) метода каракт. — метода 1. инт

дају исто решење

параметризовано — имплицитно задато

$(\cos t, \sin t)$ ✓ $x^2 + y^2 = 1$

(*) $a \in \mathbb{R}$

$$A = \begin{bmatrix} a+1 & a+1 & -a \\ a-2 & a+2 & -a \\ a-2 & a+1 & 1-a \end{bmatrix}$$

б) $a \neq 0$, $c \in \mathbb{R}^3$ $\exists_1$ $X' = AX$

$X'(1) = c$

дато са $X(t) = e^{A(t-1)}\cdot A^{-1}\cdot c$

$\left.\begin{array}{l} \lambda_1 = \lambda_2 = 1 \\ \lambda_3 = a \end{array}\right\}$ $\det A = a \neq 0 \Rightarrow \exists A^{-1}$

$X'(t) = (e^{A(t-1)})'\cdot A^{-1}\cdot c = A e^{A(t-1)}\cdot \cancel{X}\cdot A^{-1}\cdot c =$

$$= e^{A(t-1)} \cdot c$$

$$AX(t) = A \cdot e^{A(t-1)} \cdot A^{-1} \cdot c = e^{A(t-1)} \cdot c \quad \Big\} =$$

$$X'(1) = e^{A(1-1)} \cdot c = E \cdot c = c \quad \checkmark$$

$X'(1) = c$ $\qquad$ $X' = AX$ (1)

$X'(1) = AX(1) \Rightarrow X(1) = A^{-1}X'(1) = A^{-1} \cdot c$

$$\boxed{\begin{aligned} X' &= AX \\ X(1) &= A^{-1} \cdot c \end{aligned}} \to \text{има единствено решение}$$

(Пикар)

(*) $X' = e^t AX$

$X(t)$, $e^t = \tau$ ($t = \ln \tau$) $\rightsquigarrow X(\tau)$

$$\frac{dX}{d\tau} = \frac{dX}{dt} \cdot \underbrace{\frac{dt}{d\tau}}_{\frac{1}{\tau} = \frac{1}{e^t}} = \frac{dX}{dt} \cdot \frac{1}{e^t} \Rightarrow \underbrace{\frac{dX}{dt}}_{= e^t \cdot AX} = e^t \cdot \frac{dX}{d\tau}$$

$$\frac{dX}{d\tau} = AX(\tau)$$

а) -- $X(\tau) = ----$

$X(t) = (\tau \rightsquigarrow e^t)$