Metoda konjugovanih gradijenata

$X^{(k+1)} = X^{(k)} - \lambda_k p^{(k)},$

$r^k = AX^{(k)} - b,$

$p^{(k+1)} = r^{(k+1)} - \mu_k p^{(k)}, \quad p^0 = r^0$

$\lambda_k = \dfrac{(r^{(k)}, p^{(k)})}{(Ap^{(k)}, p^{(k)})}$

$\mu_k = \dfrac{(Ap^{(k)}, r^{(k+1)})}{(Ap^{(k)}, p^{(k)})}$

Kriterijum zaustavljanja: $||r|| < \varepsilon$.
Zadatak:
Metodom konjugovanih pravaca naći minimum funkcionala $F = \frac{1}{2}(Ax, x) - (b, x)$ sa tačnošću 10^{-3}, ako je

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}, \quad b = [1,\ 0,\ 1]^T$$

1. iteracija:

redosled: $x - r - p - \lambda - x^1$

$X^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$r^0 = AX^0 - b = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$

$p^0 = r^0 = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$

$\lambda_0 = \frac{(r^{(0)}, p^{(0)})}{(Ap^{(0)}, p^{(0)})} = 2/3$

$X^1 = X^0 - \lambda_0 p^0 = \begin{bmatrix} 2/3 \\ 0 \\ 2/3 \end{bmatrix}$

2. iteracija:

redosled: $r - \mu - p - \lambda - x^2$

$r^1 = AX^1 - b = \begin{bmatrix} 1/3 \\ -4/3 \\ -1/3 \end{bmatrix}$

$\mu_0 = \frac{(Ap^{(0)}, r^{(1)})}{(Ap^{(0)}, p^{(0)})} = -1$

$p^1 = r1 - \mu_0 p^0 = \begin{bmatrix} -2/3 \\ -4/3 \\ -4/3 \end{bmatrix}$

$\lambda_1 = \frac{(r^{(1)}, p^{(1)})}{(Ap^{(1)}, p^{(1)})} = 2.25$

$X^2 = X^1 - \lambda_1 p^1 = \begin{bmatrix} 2.1666 \\ 3 \\ 3.6666 \end{bmatrix}$

3. iteracija:

redosled: $r - \mu - p - \lambda - x^3$

$r^2 = AX^2 - b = \begin{bmatrix} 1/3 \\ 1/6 \\ -1/3 \end{bmatrix}$

$\mu_1 = \frac{(Ap^{(1)}, r^{(2)})}{(Ap^{(1)}, p^{(1)})} = -0.125$

$p^2 = r^2 - \mu_1 p^1 = \begin{bmatrix} 1/4 \\ 0 \\ -1/2 \end{bmatrix}$

$\lambda_2 = \frac{(r^{(2)}, p^{(2)})}{(Ap^{(2)}, p^{(2)})} = 2/3$

$X^3 = X^2 - \lambda_2 p^2 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$

$X^* = X^3$ Iterativni niz dobijen ovom metodom iskonvergira u najviše n (u ovom zadatku $n = 3$) iteracija.