Hermiteov interpolacioni polinom

1. Broj podataka: $\sum_{i=0}^{m} n_i$
2. Polinom stepena $n = \sum_{i=0}^{m} n_i - 1$
3. Podeljene razlike reda $0, 1, ...n$

$$
\begin{aligned}
P_n(x) = f[x_0] + f[x_0, x_0](x - x_0) + f[x_0, x_0, x_0](x - x_0)^2 + ... + f[\overbrace{x_0, x_0, ...x_0}^{n_0}](x - x_0)^{n_0 - 1} \\
+ f[x_0, x_0, ...x_0, x_1](x - x_0)^{n_0} + f[x_0, x_0, ...x_0, x_1, x_1](x - x_0)^{n_0}(x - x_1)... \\
... + f[x_0, ...x_0, ..., x_m, ..., x_m](x - x_0)^{n_0}(x - x_1)^{n_1}...(x - x_m)^{n_m - 2} \\
+ f[\underbrace{x_0, ...x_0}_{n_0}, ..., \underbrace{x_m, ..., x_m}_{n_m}](x - x_0)^{n_0}(x - x_1)^{n_1}...(x - x_m)^{n_m - 1}.
\end{aligned}
$$

Formiranje polinoma u Matlabu

$$
\begin{aligned}
P_n(x) &= f[x_0] + (x - x_0)P_{n-1}(x) \\
P_{n-1}(x) &= f[x_0, x_0] + f[x_0, x_0, x_0](x - x_0) + ... + f[x_0, x_0, ...x_0](x - x_0)^{n_0 - 2} \\
&+ f[x_0, x_0, ...x_0, x_1](x - x_0)^{n_0 - 1} + f[x_0, x_0, ...x_0, x_1, x_1](x - x_0)^{n_0 - 1}(x - x_1)... \\
&... + f[x_0, ...x_0, ..., x_m, ..., x_m](x - x_0)^{n_0 - 1}(x - x_1)^{n_1}...(x - x_m)^{n_m - 2} \\
&+ f[x_0, ...x_0, ..., x_m, ..., x_m](x - x_0)^{n_0 - 1}(x - x_1)^{n_1}...(x - x_m)^{n_m - 1} \\
P_{n-1}(x) &= f[x_0, x_0] + (x - x_0)P_{n-2}(x) \\
P_{n-2}(x) &= f[x_0, x_0, x_0] + ... + f[x_0, x_0, ...x_0](x - x_0)^{n_0 - 3} \\
&+ f[x_0, x_0, ...x_0, x_1](x - x_0)^{n_0 - 2} + f[x_0, x_0, ...x_0, x_1, x_1](x - x_0)^{n_0 - 2}(x - x_1)... \\
&... + f[x_0, ...x_0, ..., x_m, ..., x_m](x - x_0)^{n_0 - 2}(x - x_1)^{n_1}...(x - x_m)^{n_m - 2} \\
&+ f[x_0, ...x_0, ..., x_m, ..., x_m](x - x_0)^{n_0 - 2}(x - x_1)^{n_1}...(x - x_m)^{n_m - 1} \\
&.... \\
P_1(x) &= f[\underbrace{x_0, ...x_0}_{n_0}, ..., \underbrace{x_m, ..., x_m}_{n_m - 1}] + (x - x_m)P_0(x) \\
P_0(x) &= f[\underbrace{x_0, ...x_0}_{n_0}, ..., \underbrace{x_m, ..., x_m}_{n_m}]
\end{aligned}
$$

Najpre formiramo $P_0(x)$, pa $P_1(x)$ i t.d. sve do $P_n(x)$. Slično kao u sledćem primeru:
$P_4(x) = e + dx + cx^2 + bx^3 + ax^4$
$P_4(x) = e + x(d + cx + bx^2 + ax^3) = e + xP_3(x)$
$P_3(x) = d + x(c + bx + ax^2) = d + xP_2(x)$
$P_2(x) = c + x(b + ax) = c + xP_1(x)$
$P_1(x) = b + ax$
$P_0(x) = a$
Krenemo od $P_0(x)$ i formiramo unazad: $P_1(x) = b + xP_0(x)$,$P_2(x) = c + xP_1(x)$,$P_3(x) = d + xP_2(x)$,$P_4(x) = e + xP_3(x)$.