Одредити локалне екстреме функције више промењљивих:

- $z = \frac{xy}{2} + \frac{2}{x} - \frac{1}{y}$. одређивање стацио. тачака

$x \neq 0$, $y \neq 0$

$z'_x = \frac{y}{2} - \frac{2}{x^2}$

$z'_y = \frac{x}{2} + \frac{1}{y^2}$

$\frac{y}{2} - \frac{2}{x^2} = 0$

$\frac{x}{2} + \frac{1}{y^2} = 0$

$y = \frac{4}{x^2}$

$\frac{x}{2} + \frac{x^4}{16} = 0 \quad / \cdot 16$

$x(8 + x^3) = 0$

$x = -2$

$M(-2, 1)$

$z''_{xx} = \frac{4}{x^3}$ $\quad z''_{xx}(M) = -\frac{1}{2}$

$z''_{xy} = \frac{1}{2}$

$z''_{yy} = -\frac{2}{y^3}$ $\quad z''_{yy}(M) = -2$

$A_1 = -\frac{1}{2} < 0$

$A_2 = \begin{vmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -2 \end{vmatrix} = 1 - \frac{1}{4} > 0$

$z_{max}(M) = -3$

- $f(x,y) = x^2 + y^2 - 2\ln x - 18\ln y$ $\quad x>0,\ y>0$

$f'_x = 2x - \frac{2}{x}$ $\qquad 2x - \frac{2}{x} = 0$

$f'_y = 2y - \frac{18}{y}$ $\qquad 2y - \frac{18}{y} = 0$

$x^2 = 1 \Rightarrow x = \pm 1$

$y^2 = 9 \Rightarrow y = \pm 3$

$M(1,3)$

$f''_{xx} = 2 + \frac{2}{x^2}$ $\qquad f''_{xx}(M) = 4$

$f''_{xy} = 0$

$f''_{yy} = 2 + \frac{18}{y^2}$ $\qquad f''_{yy}(M) = 4$

$A_1 = 4 > 0$

$A_2 = \begin{vmatrix} 4 & 0 \\ 0 & 4 \end{vmatrix} = 16$

$f_{min}(M) = 10 - 18\ln 3$

- $f(x, y) = \frac{x}{y} + \frac{1}{x} + y$

- $f(x, y) = 3x^2 - 2x\sqrt{y} + y - 8x + 8.$

- $f(x,y)=(x^2-2y^2)e^{y-x}$

$$f'_x = 2x e^{y-x} + (x^2-2y^2)e^{y-x}(-1) = e^{y-x}(2x-x^2+2y^2)$$

$$f'_y = -4y e^{y-x} + (x^2-2y^2)e^{y-x}\cdot 1 = e^{y-x}(-4y+x^2-2y^2)$$

$$\cancel{e^{y-x}}(2x-x^2+2y^2)=0$$

$$\cancel{e^{y-x}}(-4y+x^2-2y^2)=0 \Rightarrow -4y+4y^2-2y^2=0$$

$$+ \quad 2x-4y=0 \Rightarrow x=2y$$

$$2y(y-2)=0$$

$$y=0 \lor y=2$$

$M_1(0,0)$

$M_2(4|2)$

$$f''_{xx} = e^{y-x}(-1)(2x-x^2+2y^2)+e^{y-x}(2-2x)$$

$$f''_{xy} = e^{y-x}\cdot 1\,(2x-x^2+2y^2)+e^{y-x}\cdot 4y$$

$$f''_{yy} = e^{y-x}\cdot 1\cdot(-4y+x^2-2y^2)+e^{y-x}(-4-4y)$$

$f''_{xx}(M_1)=2$ $\quad f''_{xx}(M_2)=-6e^{-2}$

$f''_{xy}(M_1)=0$ $\quad f''_{xy}(M_2)=8e^{-2}$

$f''_{yy}(M_1)=-4$ $\quad f''_{yy}(M_2)=-12e^{-2}$

• $f(x,y) = y - 2x + \underbrace{\ln\sqrt{x^2+y^2}}_{\frac{1}{2}\ln(x^2+y^2)} + 3\,\mathrm{arctg}\,\frac{x}{y}$.

$x^2+y^2>0,\ y\neq 0$

$$f'_x = -2+\frac{1}{2}\,\frac{2x}{x^2+y^2} + 3\,\frac{1}{1+\frac{x^2}{y^2}}\cdot\frac{1}{y} = -2+\frac{x+3y}{x^2+y^2}$$

$$f'_y = 1+\frac{1}{2}\,\frac{2y}{x^2+y^2}+3\,\frac{1}{1+\frac{x^2}{y^2}}\left(-\frac{x}{y^2}\right) = 1+\frac{y-3x}{x^2+y^2}$$

$$-2+\frac{x+3y}{x^2+y^2}=0$$

$$1+\frac{y-3x}{x^2+y^2}=0$$

$$x+3y = 2(x^2+y^2)$$

$$x^2+y^2 = 3x-y$$

$$x+3y=6x-2y \Rightarrow y=x$$

$$x^2+x^2=3x-x$$

$$2x(x-1)=0$$

$$\cancel{x=0} \vee x=1$$

$M(1,1)$

$$f''_{xx} = \frac{x^2+y^2-(x+3y)\,2x}{(x^2+y^2)^2} \qquad f''_{xx}(M) = -\frac{6}{4}=-\frac{3}{2}$$

$$f''_{xy} = \frac{3(x^2+y^2)-(x+3y)\,2y}{(x^2+y^2)^2} \qquad f''_{xy}(M)=-\frac{1}{2}$$

$$f''_{yy} = \frac{x^2+y^2-(y-3x)\,2y}{(x^2+y^2)^2} \qquad f''_{yy}(M)=\frac{3}{2}$$

Одредити локалне екстреме функције које су задате имплицитно $z = f(x, y)$:

- $z^2 + x^2 + 2y^2 - 2xy + 4y = 0$

• $z^3 - z^2x + x^2 + y^2 + 4y + 4 = 0,\ z \neq 0$

$( )'_x : \ 3z^2 \cdot z'_x - 2z \cdot z'_x \cdot x - z^2 + 2x = 0$

$( )'_y : \ 3z^2 \cdot z'_y - 2z z'_y x + 2y + 4 = 0$

одређивање стацио. тачака

$z'_x(M) = 0 \quad z'_y(M) = 0$

$-z^2 + 2x = 0$

$2y + 4 = 0$

$y = -2$

$x = \frac{z^2}{2}$

$z^3 - \frac{z^4}{2} + \frac{z^4}{4} + 4 - 8 + 4 = 0$

$z^3\left(1 - \frac{z}{4}\right) = 0$

$z = 4 \quad M(8, -2)$

$$\left.\begin{array}{l}
( )''_{xx} : \ 6z \cdot z'_x \cdot z'_x + 3z^2 \cdot z''_{xx} - 2z'_x \cdot z'_x \cdot x - 2z \cdot z''_{xx} \cdot x - 2z z'_x - 2z \cdot z'_x + 2 = 0 \\
( )''_{xy} : \ 6z \cdot z'_y z'_x + 3z^2 \cdot z''_{xy} - 2z'_y z'_x x - 2z \cdot z''_{xy} x - 2z z'_y = 0 \\
( )''_{yy} : \ 6z z'_y \cdot z'_y + 3z^2 \cdot z''_{yy} - 2z'_y \cdot z'_y x - 2z \cdot z''_{yy} x + 2 = 0
\end{array}\right\} \overset{M}{\Rightarrow}
\begin{array}{l}
48 z''_{xx}(M) - 64 z''_{xx}(M) + 2 = 0 \\
48 z''_{xy}(M) - 64 z''_{xy}(M) = 0 \\
48 z''_{yy}(M) - 64 z''_{yy}(M) + 2 = 0
\end{array}$$

- $2x + 6y - 3x^2 - 2xy + y^2 - 2z^2 + 13 = 0$, $z < 0$
- $z^3 - xyz - x^2 - 3y^2 - 1 = 0$.