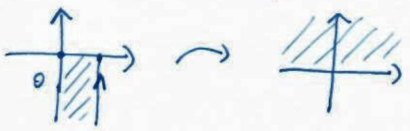
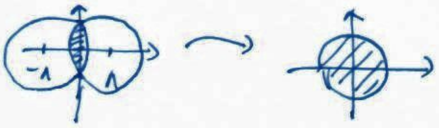
① Одредити бар 1 „1-1" холоморфно пресликавање $w=f(z)$ којим се:

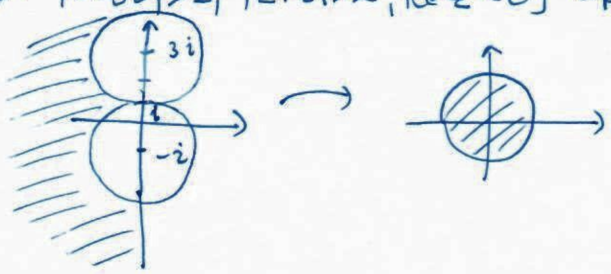
а) $\{z\in\mathbb{C}: 0<\operatorname{Re} z<1, \operatorname{Im} z<0\}$ пресликава на $\{w\in\mathbb{C}: \operatorname{Im} w>0\}$.



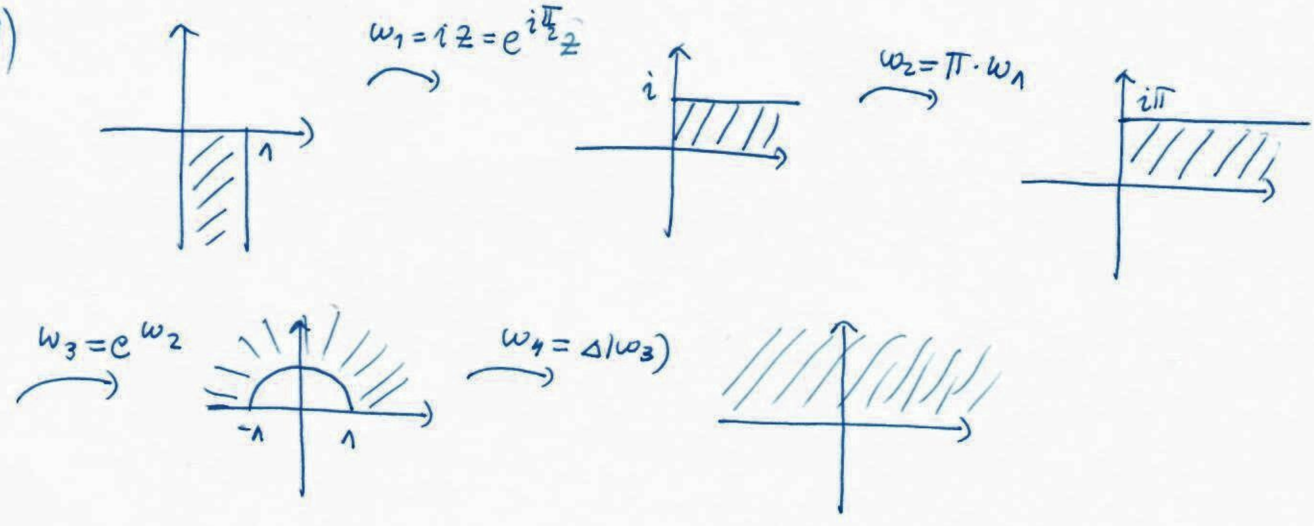
б) $\{z\in\mathbb{C}: |z-1|<\sqrt{2}, |z+1|<\sqrt{2}\}$ пресликава на $\{w\in\mathbb{C}: |w|<1\}$



в) $\{z\in\mathbb{C}: |z-3i|>2, |z+i|>2, \operatorname{Re} z<0\}$ пресликава на $\{w\in\mathbb{C}: |w|<1\}$

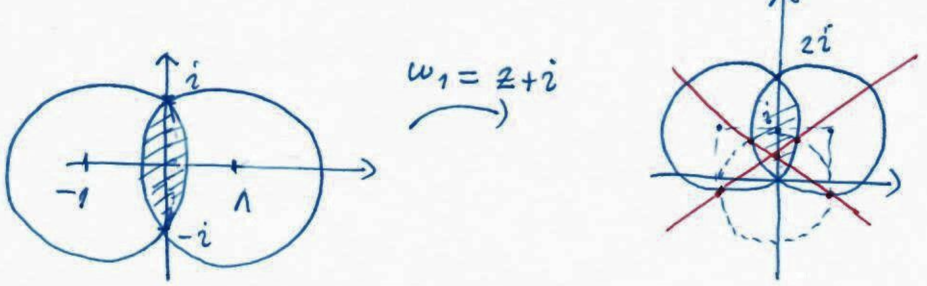


а)



$\Rightarrow \quad w=f(z)=\Delta\left(e^{i\pi z}\right)$

б)



слика у црвене праве
сада у односу на
$w_2=\frac{1}{\overline{w_1}}$

$2i \mapsto \frac{1}{-2i}=\frac{1}{2}i$ ← пресек правих
↓
пресек кругова

! Потребно нам је да се кружнице пресликају у праве $\Rightarrow$ кружнице треба да прођу кроз центар инверзије
да бисмо добили полураван у неком моменту (после лако долазимо до ⊜)

① Одредити бар 1 „1-1" холоморфно пресликавање $w=f(z)$ којим се :

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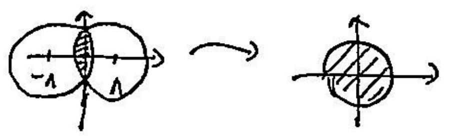
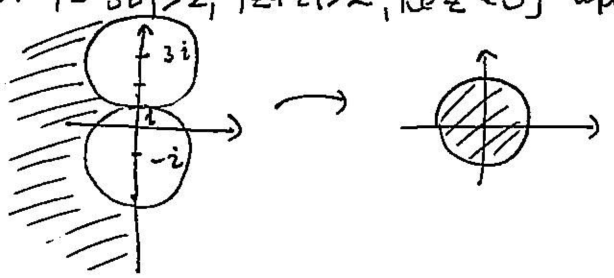
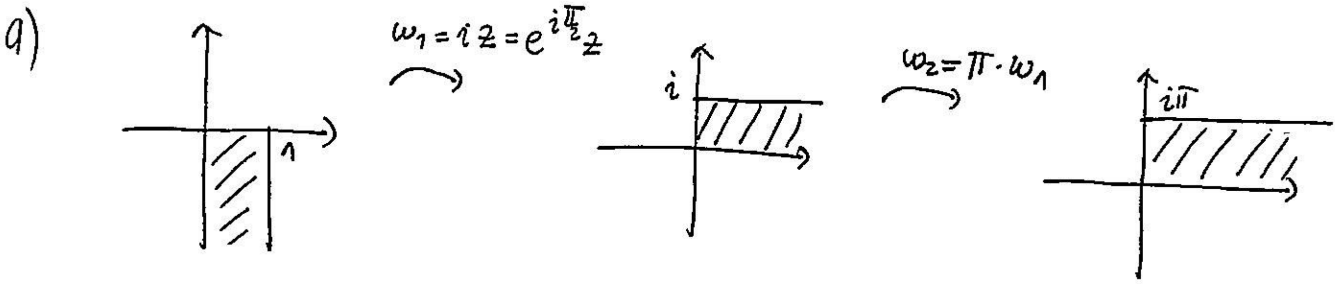
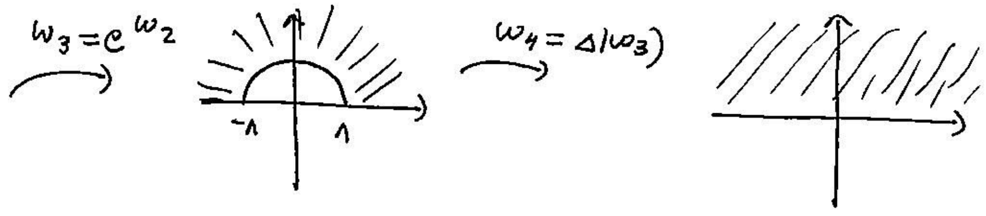


в) $\{z\in\mathbb{C}: |z-3i|>2, |z+i|>2, \operatorname{Re} z<0\}$ пресликава на $\{w\in\mathbb{C}: |w|<1\}$



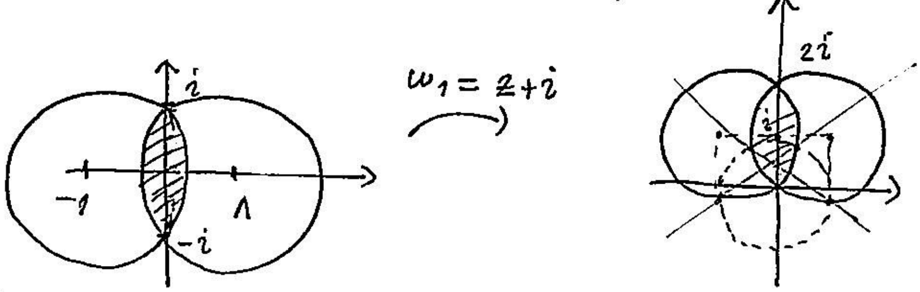
а)





$$\Rightarrow \quad w=f(z)=\Delta\left(e^{i\pi z}\right)$$

б)



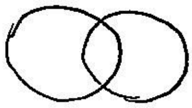
слике у црвене праве сада у односу на

$w_2 = \frac{1}{\bar{w}_1}$

$2i \mapsto \frac{1}{-2i} = \frac{1}{2}i$ ← пресек правих

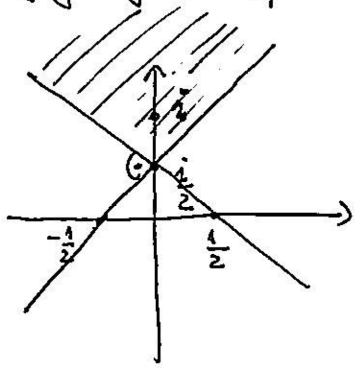
↓ пресек кругова

! Потребно нам је да се кружнице пресликају у праве ⇒ кружнице треба да прођу кроз центар инверзије да бисмо добили полураван у неком моменту (после лако долазимо до ⊜)



* пошто инверзија чува углове између кривих, а кружнице су нормалне, то су добијене праве нормалне и још су симетричне у односу на y осу (јер су кругови били симетрични, а y оса се слика на себе инверзијом)

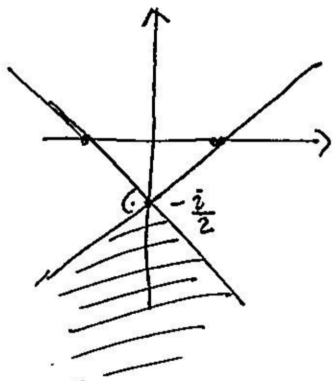
$$w_2 = \frac{1}{\overline{w_1}}$$



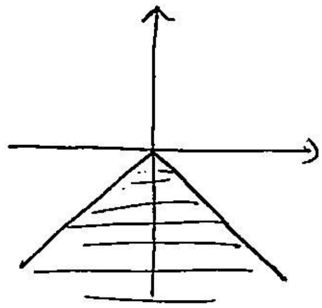
$$i \mapsto \frac{1}{-i} = i$$

$\Rightarrow$ слика је осенчени део

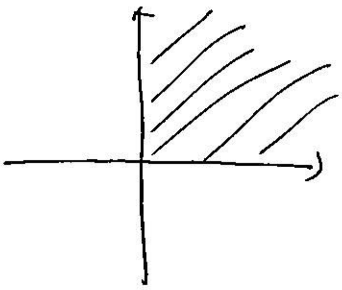
$$w_3 = \overline{w_2}$$



$$w_4 = w_3 + \frac{i}{2}$$



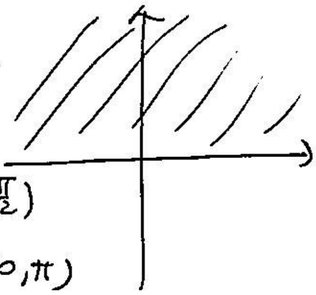
$$w_5 = w_4 \cdot e^{i\frac{3\pi}{4}}$$



$$w_6 = w_5^{\,2}$$

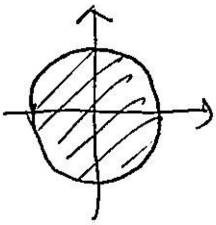
$z = re^{i\theta}$, $\theta \in (0, \frac{\pi}{2})$

$z^2 = r^2 e^{2i\theta}$, $2\theta \in (0, \pi)$



$$w_7 = \frac{w_6 - i}{w_6 + i}$$

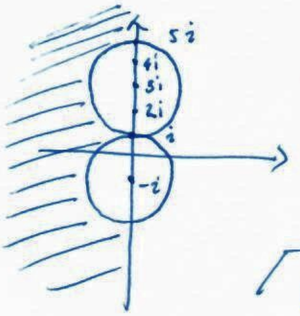
(нпр.)



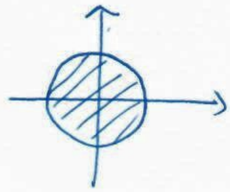
$$w = f(z) = \frac{w_5^{\,2} - i}{w_5^{\,2} + i} = \frac{w_4^{\,2} e^{i\frac{3\pi}{2}} - i}{w_4^{\,2} e^{i\frac{3\pi}{2}} + i} = \frac{\left(w_3 + \frac{i}{2}\right)^2 e^{i\frac{3\pi}{2}} - i}{\left(w_3 + \frac{i}{2}\right)^2 e^{i\frac{3\pi}{2}} + i}$$

$$= \frac{\left(\frac{1}{z+i} + \frac{i}{2}\right)^2 e^{i\frac{3\pi}{2}} - i}{\left(\frac{1}{z+i} + \frac{i}{2}\right) e^{i\frac{3\pi}{2}} + i} = \frac{\left(\frac{1}{z+i} + \frac{i}{2}\right)^2 \cdot (-i) - i}{\left(\frac{1}{z+i} + \frac{i}{2}\right)^2 \cdot (-i) + i} = \frac{\left(\frac{1}{z+i} + \frac{i}{2}\right)^2 + 1}{\left(\frac{1}{z+i} + \frac{i}{2}\right)^2 - 1}$$

в)

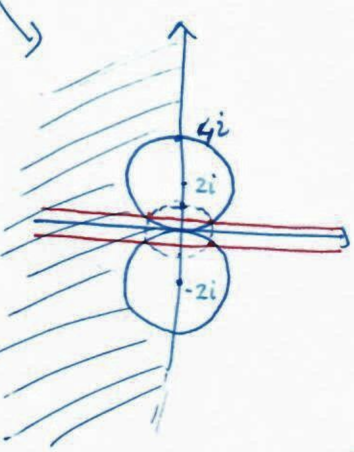


$w = f(z)$?

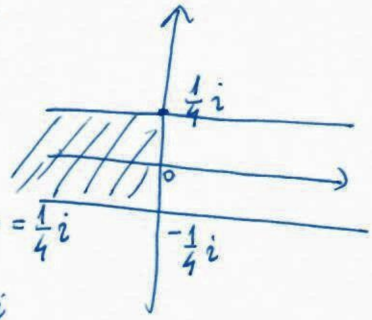


Опет желимо да се кругови сликају на праве!
транслирамо најпре све за $-i$

$w_1 = z - i$



$w_2 = \frac{1}{\overline{w_1}}$

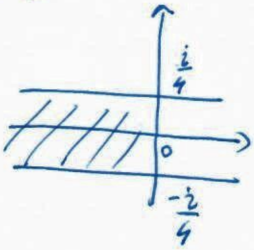


$4i \mapsto \frac{1}{-4i} = \frac{1}{4}i$

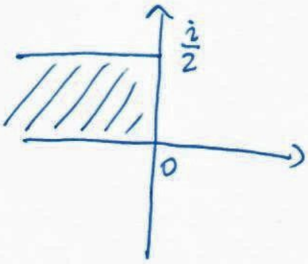
$-4i \mapsto -\frac{1}{4}i$

$-1 \mapsto -1$

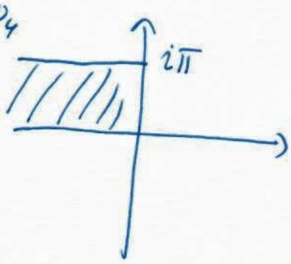
$w_3 = \overline{w_2}$



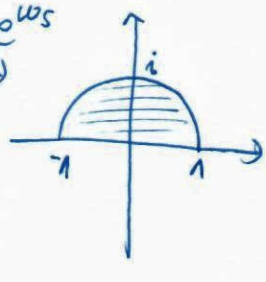
$w_4 = w_3 + \frac{i}{4}$



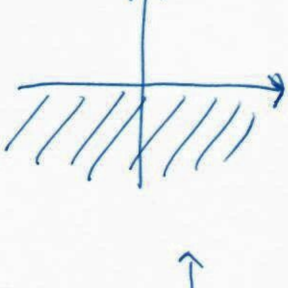
$w_5 = 2\pi \cdot w_4$



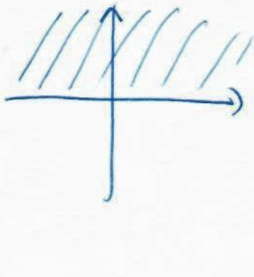
$w_6 = e^{w_5}$



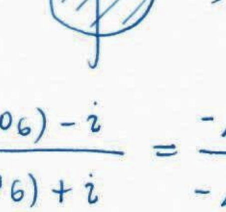
$w_7 = \Delta(w_6)$



$w_8 = -w_7$



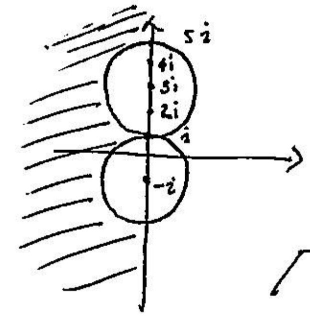
$w_9 = \frac{w_8 - i}{w_8 + i}$



$$w = f(z) = \frac{-\Delta(w_6) - i}{-\Delta(w_6) + i} = \frac{-\Delta(e^{w_5}) - i}{-\Delta(e^{w_5}) + i} = \frac{-\Delta\left(e^{2\pi\left(\frac{1}{z-i} + \frac{i}{4}\right)}\right) - i}{-\Delta\left(e^{2\pi\left(\frac{1}{z-i} + \frac{i}{4}\right)}\right) + i}$$

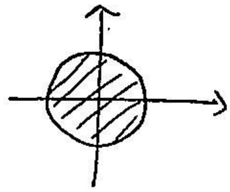
$$f(z) = \frac{\Delta\left(e^{2\pi\left(\frac{1}{z-i} + \frac{i}{4}\right)}\right) + i}{\Delta\left(e^{2\pi\left(\frac{1}{z-i} + \frac{i}{4}\right)}\right) - i}$$

в)



$w = f(z)$

?



Опет желимо да се кругови сликају на праве! транслирано најпре све за $-i$

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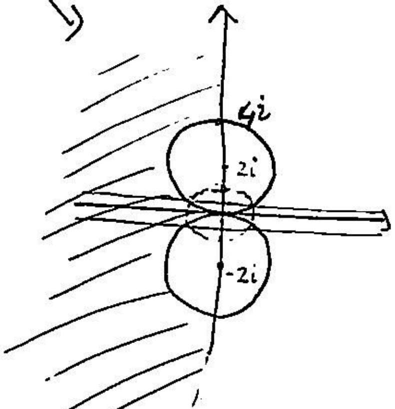
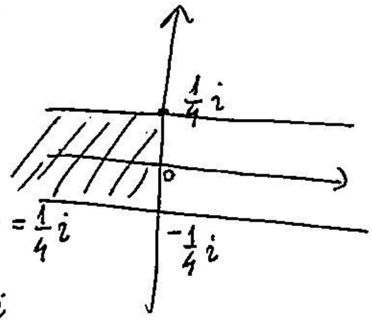


$w_2 = \frac{1}{\overline{w_1}}$



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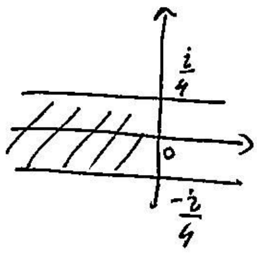
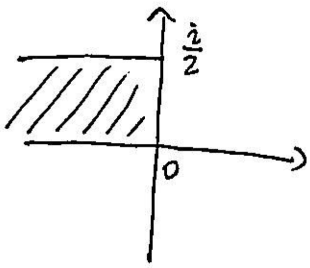
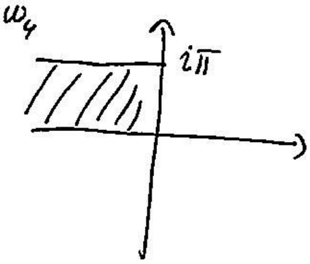


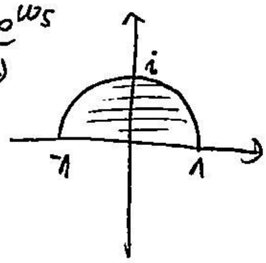
$w_4 = w_3 + \frac{i}{4}$



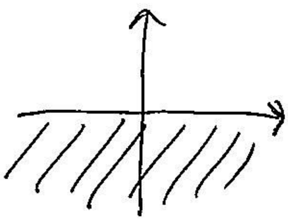
$w_5 = 2\pi \cdot w_4$



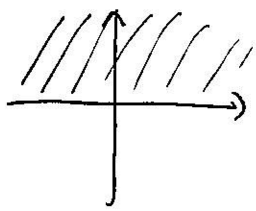
$w_6 = e^{w_5}$



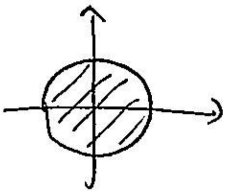
$w_7 = \Delta(w_6)$



$w_8 = -w_7$



$w_9 = \frac{w_8 - i}{w_8 + i}$



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$$f(z) = \frac{\Delta\left(e^{2\pi\left(\frac{1}{z-i} + \frac{i}{4}\right)}\right) + i}{\Delta\left(e^{2\pi\left(\frac{1}{z-i} + \frac{i}{4}\right)}\right) - i}$$