$F = f \circ g$, $f(g(x,y)) = f(u(x,y), v(x,y)) = F(x,y)$

$g(x,y) = (u(x,y), v(x,y))$ (ima $u+iv$)

$F_x = f_u \cdot u_x + f_v \cdot v_x$

$F_{xx} = (f_u)_x u_x + f_u (u_x)_x + (f_v)_x v_x + f_v \cdot v_{xx}$

$F_{xx} = (f_{uu} \cdot u_x + f_{uv} \cdot v_x) u_x + f_u u_{xx} + f_{vu} u_x + f_{vv} v_x) v_x + f_v \cdot v_{xx}$

$F_{yy} = (f_{uu} \cdot u_y + f_{uv} \cdot v_y) u_y + f_u \cdot u_{yy} + (f_{vu} u_y + f_{vv} v_y) v_y + f_v \cdot v_{yy}$

$\Delta F = f_{uu}(u_x^2 + u_y^2) + f_{uv}(u_x v_x + u_y v_y) + f_u \Delta u + f_v \Delta v + f_{vu}(v_x^2 + v_y^2) + f_{vv}(u_x v_x + u_y v_y)$

$$\boxed{\Delta F = f_{uu}(u_x^2 + u_y^2) + f_{vv}(v_x^2 + v_y^2) + f_u \Delta u + f_v \Delta v + 2 f_{uv}(u_x v_x + u_y v_y)}$$

$f = (U, V)$, $f = U + iV$ $f_u = U_u + iV_u$, $f_v = U_v + iV_v$

$$\boxed{\Delta f = \Delta U + i \Delta V}$$

Ako je $\Delta g = \Delta u + i \Delta v = 0$, onda je $\Delta F = f_{uu}(u_x^2 + u_y^2) + f_{vv}(v_x^2 + v_y^2) + 2 f_{uv}(u_x v_x + u_y v_y)$

- Ako je g analitička, onda je $u_x = v_y$, $u_y = -v_x$ $\Rightarrow \Delta F = f_{uu}(u_x^2 + v_x^2) + f_{vv}(v_x^2 + u_x^2) + 2 f_{uv}(u_x(-u_y) + u_y u_x)$

$\Rightarrow \Delta F = (f_{uu} + f_{vv})(u_x^2 + v_x^2) = 0$ za f harmonijsku ?