Одредимо кој је знака фја φ.

$$\varphi(x)=\frac{1}{\sqrt{x^2+x}}-\log\left(1+\frac{1}{x}\right)$$

$$\varphi'(x)=\frac{-\frac{1}{2}\cdot\frac{1}{\sqrt{x^2+x}}(2x+1)}{x^2+x}-\frac{1}{1+\frac{1}{x}}\cdot\frac{-1}{x^2}$$

$$=\frac{1-\frac{2x+1}{2\sqrt{x^2+x}}}{x^2+x}\ ,\qquad \frac{2x+1}{2\sqrt{x^2+x}}\overset{?}{>}1 \ ✓$$

$$2x+1>2\sqrt{x^2+x}\ /^2$$

$$4x^2+4x+1>4x^2+4x$$

$$1>0\ ✓$$

$\Rightarrow \varphi'(x)<0$ за $\forall x\in(0,+\infty)$

$$\left.\begin{array}{l}\Rightarrow \varphi\downarrow \text{ на } (0,+\infty)\\ \lim\limits_{x\to+\infty}\varphi(x)=0\end{array}\right\}\Rightarrow \varphi(x)>0\ \ \forall x\in(0,+\infty)$$

$\Rightarrow f'(x)>0\ \ \forall x\in(0,+\infty)$

$\Rightarrow f\uparrow$ на $(0,+\infty)$

и нема локалних екстремума

6° $$f''(x)=\left(\frac{1}{(x^2+x)\log^2\left(1+\frac{1}{x}\right)}-1\right)'$$

$$=\frac{-(2x+1)\log^{2}\left(1+\frac{1}{x}\right)-(x^2+x)\,2\log\left(1+\frac{1}{x}\right)\frac{1}{1+\frac{1}{x}}\cdot\frac{-1}{x^2}}{(x^2+x)^2\log^{4}\left(1+\frac{1}{x}\right)}$$

$$=\frac{-(2x+1)\log\left(1+\frac{1}{x}\right)+2}{(x^2+x)^2\log^3\left(1+\frac{1}{x}\right)}=\frac{(2x+1)\left(\frac{2}{2x+1}-\log\left(1+\frac{1}{x}\right)\right)}{(x^2+x)^2\log^3\left(1+\frac{1}{x}\right)}$$

$$\psi(x)=\frac{2}{2x+1}-\log\left(1+\frac{1}{x}\right)\ ,\quad \operatorname{sgn}\psi=\operatorname{sgn}f''$$

$$\psi'(x)=\frac{-4}{(2x+1)^2}-\frac{1}{1+\frac{1}{x}}\cdot\frac{-1}{x^2}=\frac{-4x^2-4x+4x^2+4x+1}{(2x+1)^2\cdot(x^2+x)}=\frac{1}{(2x+1)^2(x^2+x)}>0$$

$\Rightarrow \psi\uparrow$ на $(0,+\infty)$, $\lim\limits_{x\to+\infty}\psi(x)=0 \Rightarrow \psi(x)<0$ за $\forall x\in(0,+\infty)$