

хоризонтална
и коса: развијемо $f(x)$ кад $x \rightarrow +\infty$, или $\frac{1}{x} \rightarrow 0$

$$f(x) = \frac{1}{\log(1 + \frac{1}{x})} - x = \frac{1}{\frac{1}{x} - \frac{1}{2x^2} + \frac{1}{3x^3} + o(\frac{1}{x^3})} - x$$

$$= \frac{1}{\frac{1}{x} (1 - \frac{1}{2x} + \frac{1}{3x^2} + o(\frac{1}{x^2}))} - x$$

$$= x \cdot \left[\left(1 - \frac{1}{2x} + \frac{1}{3x^2} + o(\frac{1}{x^2})\right)^{-1} - 1 \right]$$

$$= x \cdot \left[1 - \left(-\frac{1}{2x} + \frac{1}{3x^2} + o(\frac{1}{x^2})\right) + \left(-\frac{1}{2x} + \frac{1}{3x^2} + o(\frac{1}{x^2})\right)^2 + o(\frac{1}{x^2}) - 1 \right]$$

$$= x \left(\frac{1}{2x} - \frac{1}{3x^2} + \frac{1}{4x^2} + o(\frac{1}{x^2}) \right)$$

$$= \frac{1}{2} - \frac{1}{12x} + o(\frac{1}{x}), \quad x \rightarrow +\infty$$

$\Rightarrow y = \frac{1}{2}$ је хоризонтална а. кад $x \rightarrow +\infty$

(Горе и кад $x \rightarrow -\infty$, због симетрије)

5° монотоност и локални екстремуми:

$$f(x) = \frac{1}{\log(1 + \frac{1}{x})} - x \quad (\text{јер } x > 0, \text{ па } |x| = x)$$

$$f'(x) = \frac{-\frac{1}{1 + \frac{1}{x}} \cdot \frac{-1}{x^2}}{\log^2(1 + \frac{1}{x})} - 1 = \frac{\frac{1}{x^2 + x} - \log^2(1 + \frac{1}{x})}{\log^2(1 + \frac{1}{x})}$$

$$= \frac{1 - (x^2 + x) \log^2(1 + \frac{1}{x})}{(x^2 + x) \log^2(1 + \frac{1}{x})} = \frac{(1 - \sqrt{x^2 + x} \log(1 + \frac{1}{x})) (1 + \sqrt{x^2 + x} \log(1 + \frac{1}{x}))}{(x^2 + x) \log^2(1 + \frac{1}{x})}$$

$$= \frac{\left(\frac{1}{\sqrt{x^2 + x}} - \log(1 + \frac{1}{x})\right) \cdot \sqrt{x^2 + x} (1 + \sqrt{x^2 + x} \log(1 + \frac{1}{x}))}{(x^2 + x) \log^2(1 + \frac{1}{x})}$$

$$\psi(x) = \frac{1}{\sqrt{x^2 + x}} - \log(1 + \frac{1}{x})$$

> 0

$$\boxed{\text{sgn } f' = \text{sgn } \psi}$$