$\boxed{16}$ $$p(x;\lambda,\mu,\theta)=\theta\frac{e^{-\lambda}\lambda^x}{x!}+(1-\theta)\frac{e^{-\mu}\mu^x}{x!},\quad x\in\mathbb{N}_0$$

$$L(x;\lambda,\mu,\theta)=\prod_{k=1}^{n}p(x_k;\lambda,\mu,\theta)$$

$$\ln L(x;\lambda,\mu,\theta)=\sum_{k=1}^{n}\ln\left(\theta\frac{e^{-\lambda}\lambda^{x_k}}{x_k!}+(1-\theta)\frac{e^{-\mu}\mu^{x_k}}{x_k!}\right)$$

За конкретан узорак могуће је добити оцене параметара. Прва два нумеричка метода су сувише компликована па се примењује ЕМ алгоритам.

$(X_1,Y_1),\ldots,(X_n,Y_n)$ независни и једнако расподељени парови случајних величина, тако да

$$P\{Y_i=y\}=\theta^y(1-\theta)^{1-y},\quad y\in\{0,1\}$$

$$P\{X_i=x\mid Y=0\}=\frac{e^{-\mu}\mu^x}{x!},\quad x=0,1,2,\ldots$$

$$P\{X_i=x\mid Y=1\}=\frac{e^{-\lambda}\lambda^x}{x!},\quad x=0,1,2,\ldots$$

$$L_c(x,y;\lambda,\mu,\theta)=\prod_{k=1}^{n}P\{X=x_k,Y=y_k\}$$

$$P\{X=x,Y=y\}=\left((1-\theta)\frac{e^{-\mu}\mu^x}{x!}\right)^{1-y}\left(\theta\frac{e^{-\lambda}\lambda^x}{x!}\right)^y$$

$$\ln L_c(x,y;\lambda,\mu,\theta)=\sum_{k=1}^{n}\ln\left[\left((1-\theta)\frac{e^{-\mu}\mu^{x_k}}{x_k!}\right)^{1-y_k}\left(\theta\frac{e^{-\lambda}\lambda^{x_k}}{x_k!}\right)^{y_k}\right]$$

$$=\sum_{k=1}^{n}y_k[\ln\theta-\lambda+x_k\ln\lambda-\ln x_k!]+\sum_{k=1}^{n}(1-y_k)[-\mu+\ln(1-\theta)+x_k\ln\mu-\ln x_k!]$$

$$=\sum_{k=1}^{n}y_k[\ln\theta-\lambda+x_k\ln\lambda]+\sum_{k=1}^{n}(1-y_k)[\ln(1-\theta)-\mu+x_k\ln\mu]-\sum_{k=1}^{n}\ln x_k!$$

Е корак: $$\ln L^{(m)}(\lambda,\mu,\theta)=E(\ln L_c(X,Y;\lambda,\mu,\theta)\mid X=x;\hat\lambda^{(m)},\hat\mu^{(m)},\hat\theta^{(m)})$$
$$=(\ln\theta-\lambda+x\ln\lambda)\sum_{k=1}^{n}E(Y_k\mid X_k=x;\hat\lambda^{(m)},\hat\mu^{(m)},\hat\theta^{(m)})$$
$$+(\ln(1-\theta)-\mu+x\ln\mu)(n-\sum E(Y_k\mid X_k=x;\hat\lambda^{(m)},\hat\mu^{(m)},\hat\theta^{(m)}))$$
$$-\sum\ln x!$$

$$\hat y_k^{(m)}=E(Y_k\mid X_k=x;\hat\lambda^{(k)},\hat\mu^{(k)},\hat\theta^{(k)})=P\{Y_k=1\mid X_k=x\}=\frac{P\{Y_k=1,X_k=x\}}{P\{X_k=x\}}$$
$$=\frac{\hat\theta^{(m)}e^{-\hat\lambda^{(m)}}(\hat\lambda^{(m)})^x}{\hat\theta^{(m)}e^{-\hat\lambda^{(m)}}(\hat\lambda^{(m)})^x+(1-\hat\theta^{(m)})e^{-\hat\mu^{(m)}}(\hat\mu^{(m)})^x}$$

$$\Rightarrow\ln L^{(m)}(\lambda,\mu,\theta)=(\ln\theta-\lambda+x\ln\lambda)\sum\hat y_k^{(m)}+(\ln(1-\theta)-\mu+x\ln\mu)(n-\sum\hat y_k^{(m)})-\sum\ln x!$$

М корак:

$$\ln'_\theta L^{(m)}(\lambda,\mu,\theta)=\frac{\sum\hat y_k^{(m)}}{\theta}-\frac{n-\sum\hat y_k^{(m)}}{1-\theta}=0$$

$$\ln'_\lambda L^{(m)}(\lambda,\mu,\theta)=-\sum\hat y_k^{(m)}+\frac{\sum x_k\hat y_k^{(m)}}{\lambda}=0$$

$$\ln'_\mu L^{(m)}(\lambda,\mu,\theta)=-n+\sum\hat y_k^{(m)}+\frac{\sum(1-\hat y_k^{(m)})x_i}{\mu}=0$$

$$\Rightarrow\hat\theta^{(m+1)}=\frac{1}{n}\sum^{n}\hat y_k^{(m)},\quad\hat\lambda^{(m+1)}=\frac{\sum x_k\hat y_k^{(m)}}{\sum\hat y_k^{(m)}},\quad\hat\mu^{(m+1)}=\frac{\sum(1-\hat y_k^{(m)})x_k}{\sum(1-\hat y_k^{(m)})}$$