Rungeova ocena greške

$$I = I_h + M_1 h^k$$

$$I = I_H + M_2 H^k$$

pretpostavka $M_1 = M_2 = M$

$$I_h - I_H = M(H^k - h^k)$$

$$M = \frac{I_h - I_H}{H^k - h^k}$$

$$R_h \cong M \cdot h^k = \frac{I_h - I_H}{\left(\frac{H}{h}\right)^k - 1}$$

Za $H = 2h$ je

$$R_h \cong \frac{I_h - I_{2h}}{2^k - 1}$$

Greška računa kvadraturnih formula

$$S_n(f) = \sum_i c_i f(x_i)$$

Ako su greške u funkciji ne veće od ε, greška računa nije veća od

$$R_R \leq \sum_i |c_i| \cdot \varepsilon = \varepsilon \cdot \sum_i |c_i|$$

Pošto je svaka kvadraturna formula tačna za $f(x) \equiv 1$

$$\int_a^b 1 \cdot dx = (b-a) = \sum_i c_i \cdot 1 = \sum_i c_i$$

Ako su u formuli svi $c_i \geq 0$, tada je

$$\sum_i |c_i| = \sum_i c_i = (b-a)$$

$$R_R \leq (b-a) \cdot \varepsilon$$

Ležandrovi $[-1, 1]$ $\quad p(x) \equiv 1$

$$(n+1) L_{n+1}(x) = (2n+1) x L_n(x) - n L_{n-1}(x)$$

$$(1-x^2) L_n''(x) - 2x L_n'(x) + n(n+1) L_n(x) = 0$$

$$L_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}\left[(x^2-1)^n\right]$$

Čebiševljevi $[-1, 1]$ $\quad p(x) = \frac{1}{\sqrt{1-x^2}}$

$$T_{n+1}(x) = 2x T_n(x) - T_{n-1}(x)$$

$$(1-x^2) T_n''(x) - x \cdot T_n'(x) + n^2 T_n(x) = 0$$

$$T_n(x) = \cos(n \arccos x) \qquad -1 \leq x \leq 1$$

Lagerovi $[0, \infty)$ $\quad p(x) = e^{-x}$

$$(n+1) L_{n+1}(x) = (2n+1-x) L_n(x) - n L_{n-1}(x)$$

$$x L_n''(x) + (1-x) L_n'(x) + n L_n(x) = 0$$

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n}\left[e^{-x} x^n\right]$$

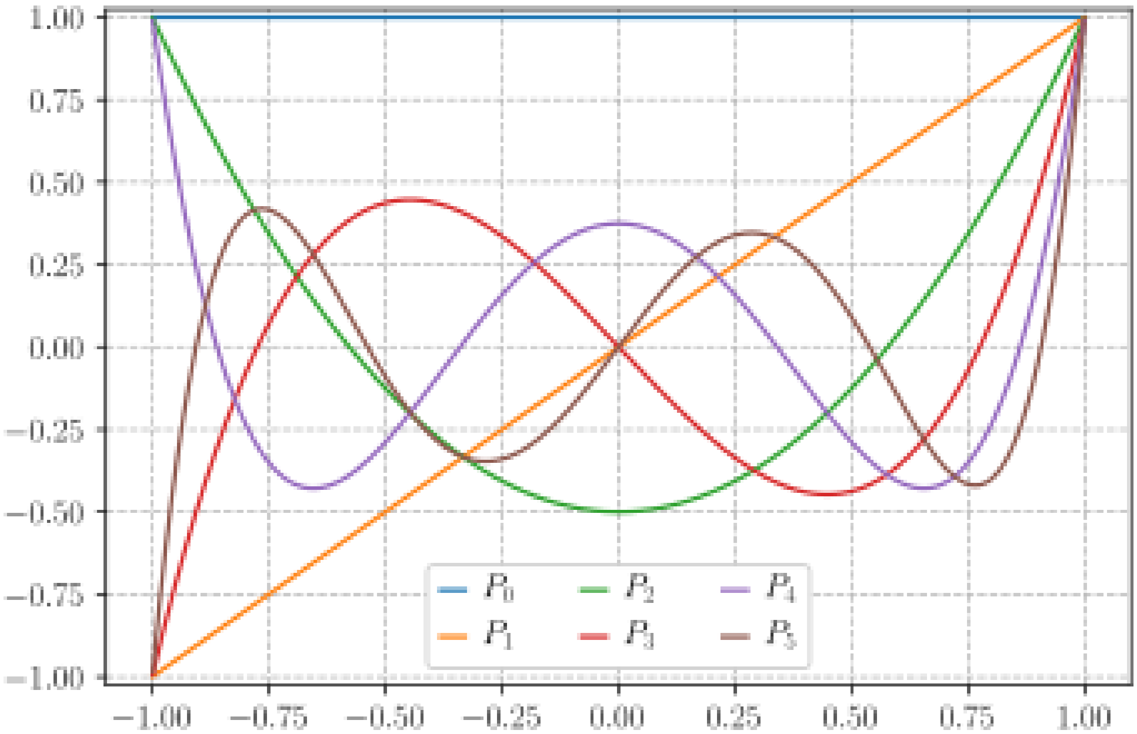
Hermitovi $(-\infty, \infty)$ $\quad p(x) = e^{-x^2}$

$$H_{n+1}(x) = 2x H_n(x) - 2n H_{n-1}(x)$$

$$H_n''(x) - 2x H_n'(x) + 2n H_n(x) = 0$$

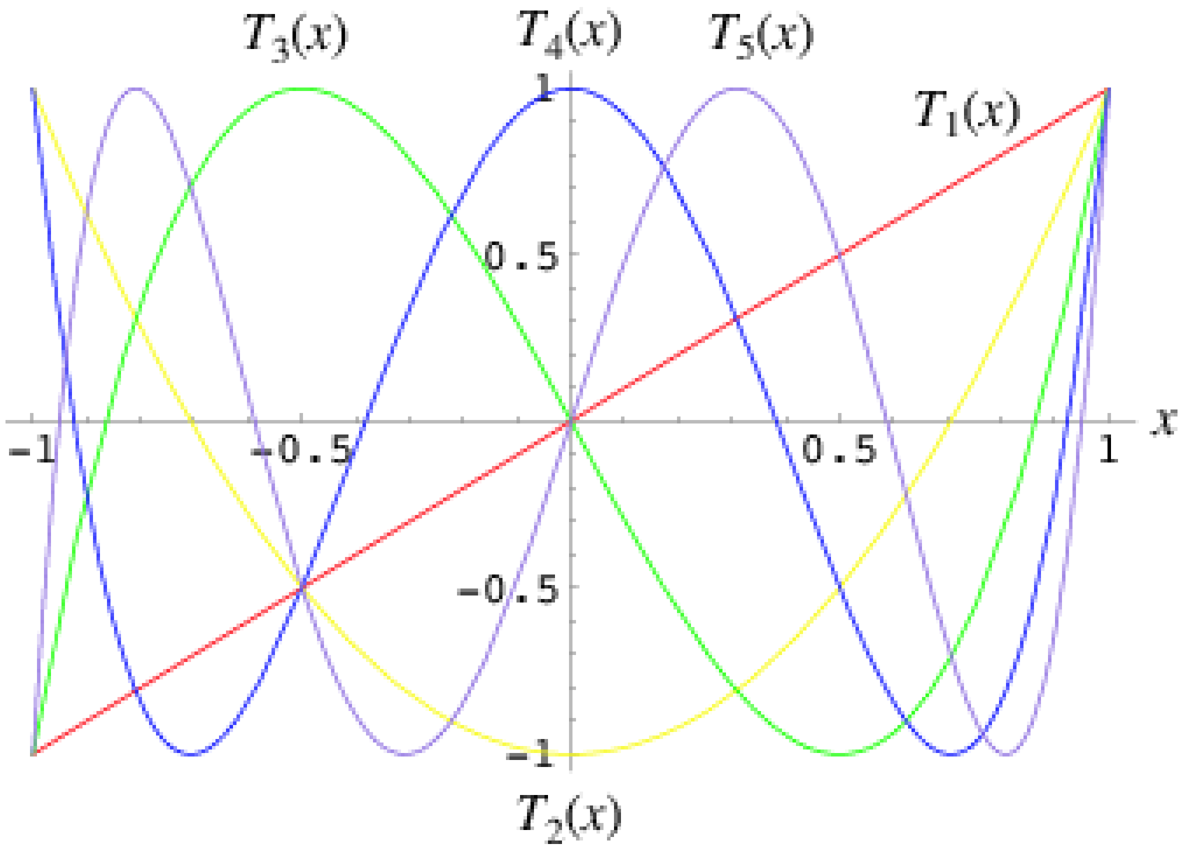
$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}\left[e^{-\frac{x^2}{2}}\right]$$

Ležandrovi polinomi [-1, 1] p(x) = 1



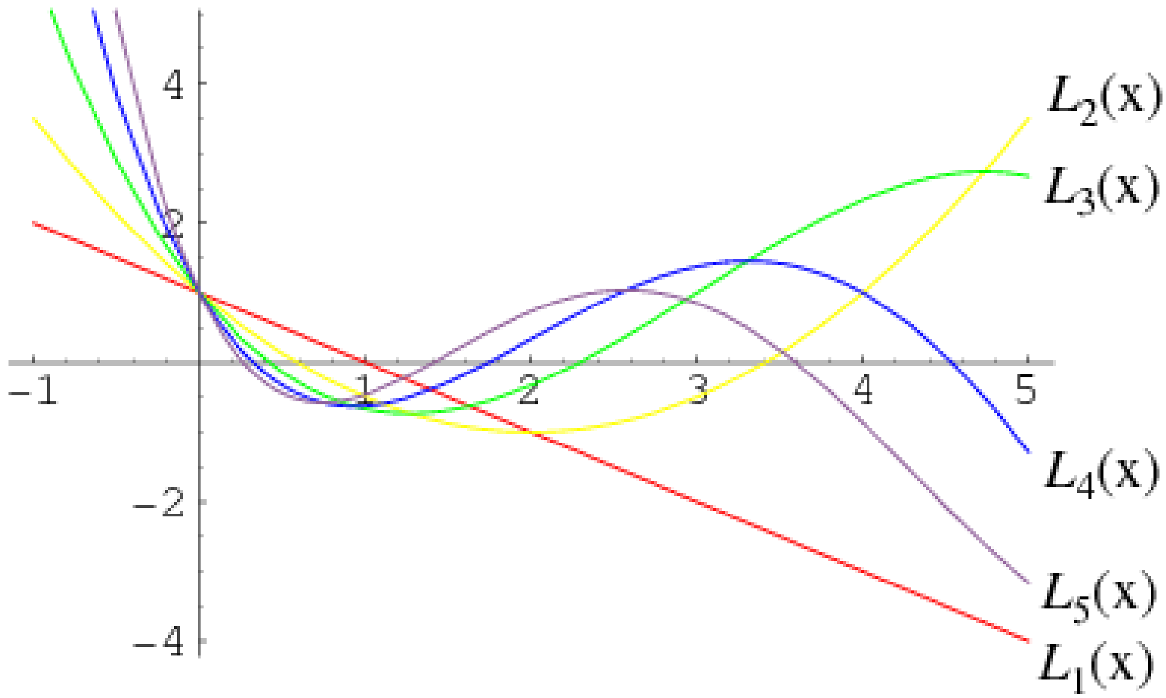
n	$P_n(x)$
0	1
1	x
2	$\frac{1}{2}\left(3x^2-1\right)$
3	$\frac{1}{2}\left(5x^3-3x\right)$
4	$\frac{1}{8}\left(35x^4-30x^2+3\right)$
5	$\frac{1}{8}\left(63x^5-70x^3+15x\right)$
6	$\frac{1}{16}\left(231x^6-315x^4+105x^2-5\right)$
7	$\frac{1}{16}\left(429x^7-693x^5+315x^3-35x\right)$
8	$\frac{1}{128}\left(6435x^8-12012x^6+6930x^4-1260x^2+35\right)$
9	$\frac{1}{128}\left(12155x^9-25740x^7+18018x^5-4620x^3+315x\right)$
10	$\frac{1}{256}\left(46189x^{10}-109395x^8+90090x^6-30030x^4+3465x^2-63\right)$

Čebiševljevi polinomi (prve vrste) [-1, 1] p(x) = 1/sqrt(1-x^2)



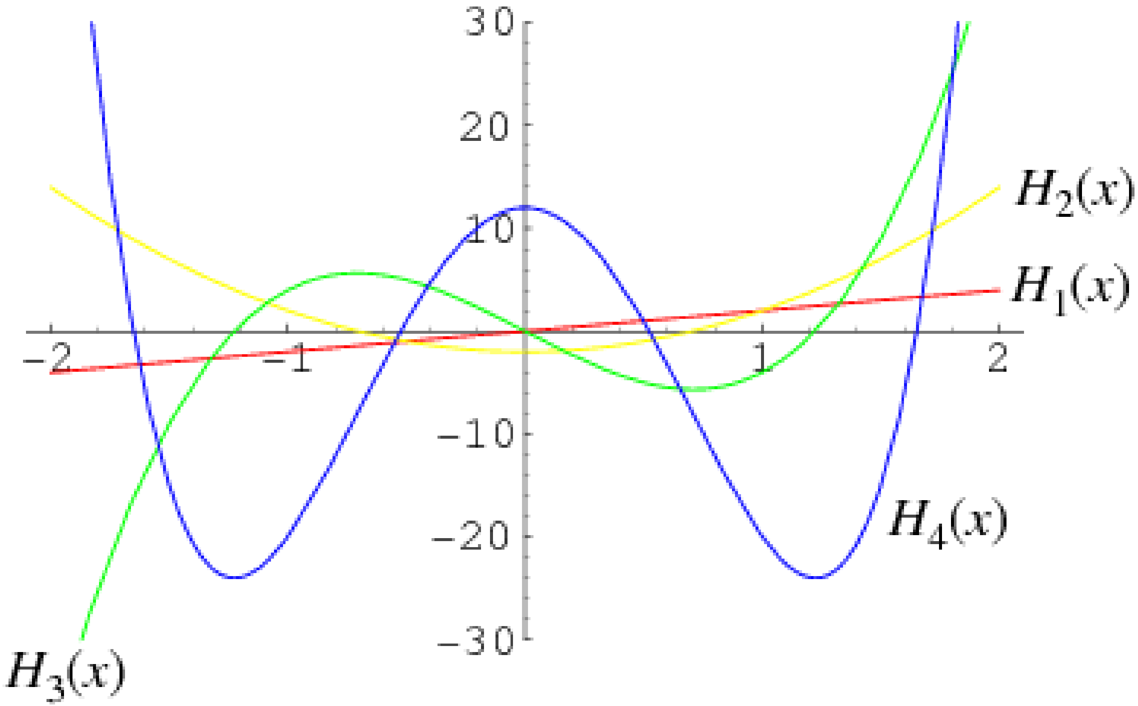
$$T_0(x) = 1$$
$$T_1(x) = x$$
$$T_2(x) = 2x^2 - 1$$
$$T_3(x) = 4x^3 - 3x$$
$$T_4(x) = 8x^4 - 8x^2 + 1$$
$$T_5(x) = 16x^5 - 20x^3 + 5x$$
$$T_6(x) = 32x^6 - 48x^4 + 18x^2 - 1$$
$$T_7(x) = 64x^7 - 112x^5 + 56x^3 - 7x$$
$$T_8(x) = 128x^8 - 256x^6 + 160x^4 - 32x^2 + 1$$
$$T_9(x) = 256x^9 - 576x^7 + 432x^5 - 120x^3 + 9x$$
$$T_{10}(x) = 512x^{10} - 1280x^8 + 1120x^6 - 400x^4 + 50x^2 - 1$$
$$T_{11}(x) = 1024x^{11} - 2816x^9 + 2816x^7 - 1232x^5 + 220x^3 - 11x$$

Lagerovi polinomi [0, +inf) p(x) = exp(-x)



n	$L_n(x)$
0	1
1	$-x+1$
2	$\frac{1}{2}(x^2-4x+2)$
3	$\frac{1}{6}(-x^3+9x^2-18x+6)$
4	$\frac{1}{24}(x^4-16x^3+72x^2-96x+24)$
5	$\frac{1}{120}(-x^5+25x^4-200x^3+600x^2-600x+120)$
6	$\frac{1}{720}(x^6-36x^5+450x^4-2400x^3+5400x^2-4320x+720)$
n	$\frac{1}{n!}((-x)^n+n^2(-x)^{n-1}+\ldots+n(n!)(-x)+n!)$

Hermitovi polinomi (-inf, +inf) p(x) = exp(-x^2)



$H_0(x) = 1$
$H_1(x) = 2x$
$H_2(x) = 4x^2 - 2$
$H_3(x) = 8x^3 - 12x$
$H_4(x) = 16x^4 - 48x^2 + 12$
$H_5(x) = 32x^5 - 160x^3 + 120x$
$H_6(x) = 64x^6 - 480x^4 + 720x^2 - 120$
$H_7(x) = 128x^7 - 1344x^5 + 3360x^3 - 1680x$
$H_8(x) = 256x^8 - 3584x^6 + 13\,440x^4 - 13\,440x^2 + 1680$
$H_9(x) = 512x^9 - 9216x^7 + 48\,384x^5 - 80\,640x^3 + 30\,240x$
$H_{10}(x) = 1024x^{10} - 23\,040x^8 + 161\,280x^6 - 403\,200x^4 + 302\,400x^2 - 30\,240.$