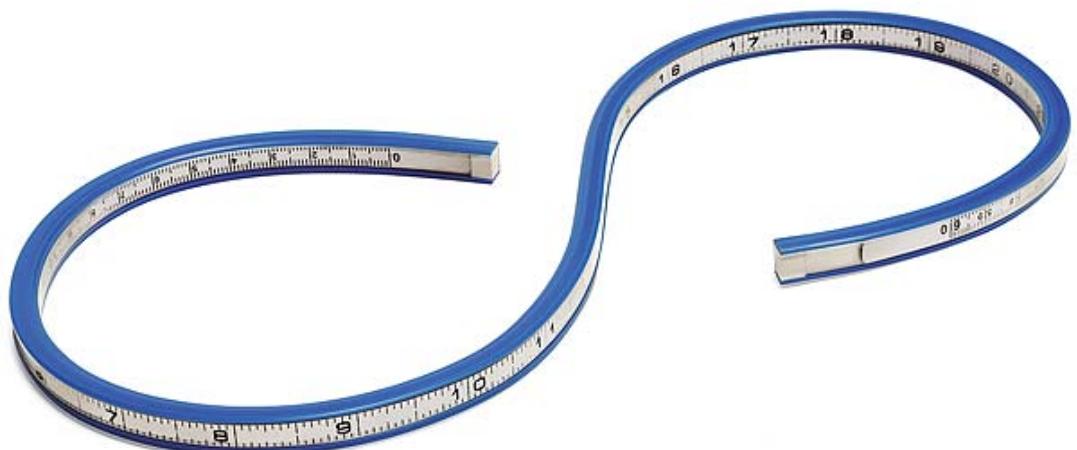
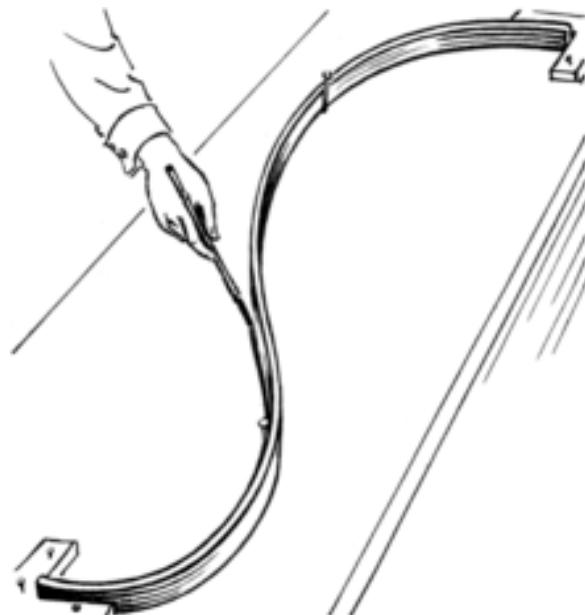
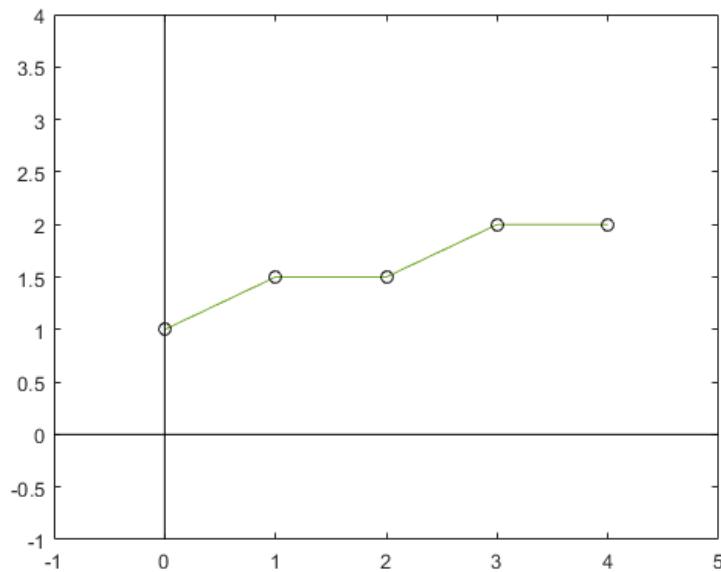


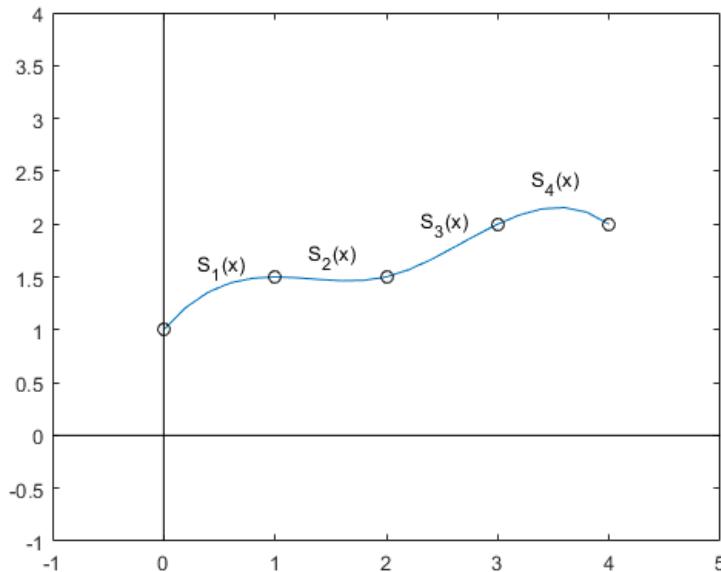
Splajn



Linearni splajn



Kubni splajn



$$S_i(x) = A_i x^3 + B_i x^2 + C_i x + D_i \quad \text{ukupno } 4n \text{ neodređenih koeficijenata}$$

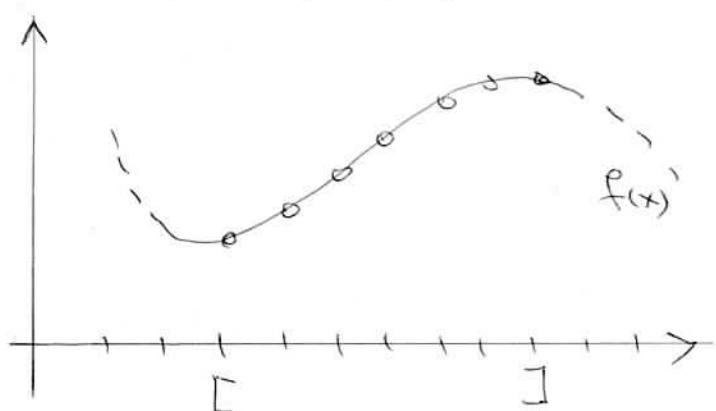
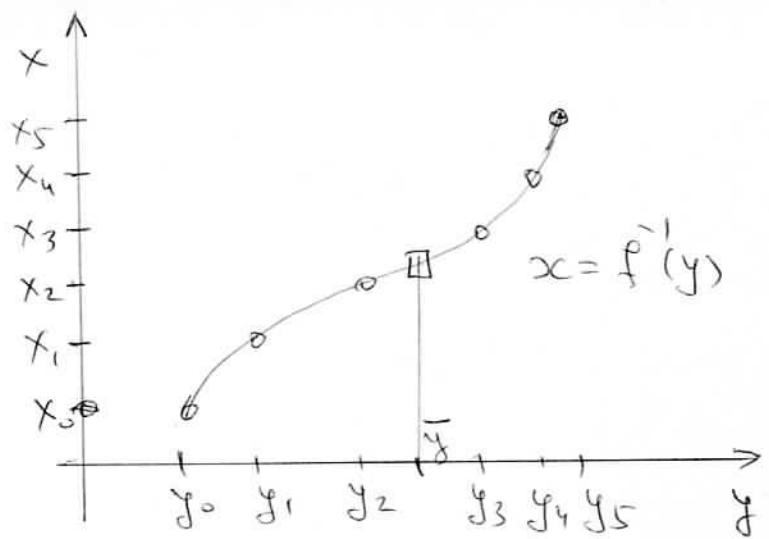
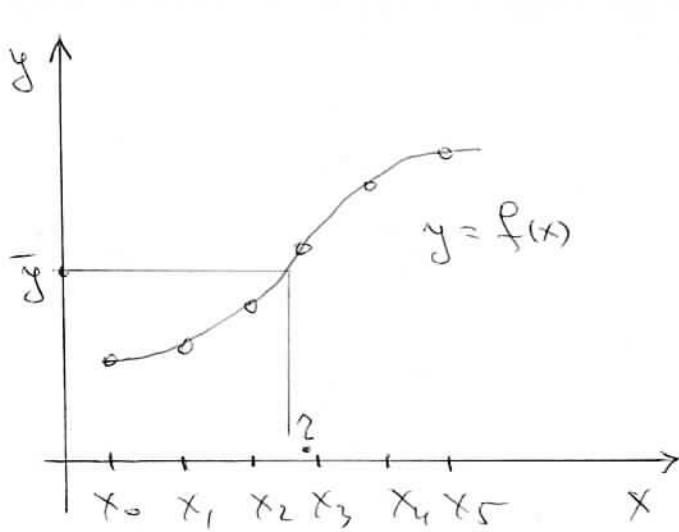
Uslovi:

neprekidnost u svim tačkama: $2n$ uslova (svaki u dva kraja)

neprekidnost prvog izvoda u unutrašnjim tačkama: $n-1$ uslov

neprekidnost drugog izvoda u unutrašnjim tačkama: $n-1$ uslov

Ukupno $4n-2$ uslova, $4n$ koeficijenata - potrebno još 2 uslova. Obično da je drugi izvod u krajevima =0



$$y = f(x) \quad x = f^{-1}(y)$$

I Извршавани подаци у којима је монотон (монаштвја!)
Задаја се подаци за $x = f^{-1}(y) \Rightarrow$ решете $x = f^{-1}(\bar{y})$
 $\approx L_n(\bar{y})$

II $f(x) = \bar{y}$ је решите $L_n(x) = \bar{y} \Leftrightarrow L_n(y) - \bar{y} = 0$

a) Нуле посматрајте $L_n(x) - \bar{y}$

b) Испољијте тумерашко метода за нуле фукције
Нпр. I Капитол

$$\bar{y} = L_n(x_0 + gh) = f_0 + g \Delta f_0 + \frac{g(g-1)}{2!} \Delta^2 f_0 + \dots + \frac{g(g-1)\dots(g-n+1)}{n!} \Delta^n f_0$$

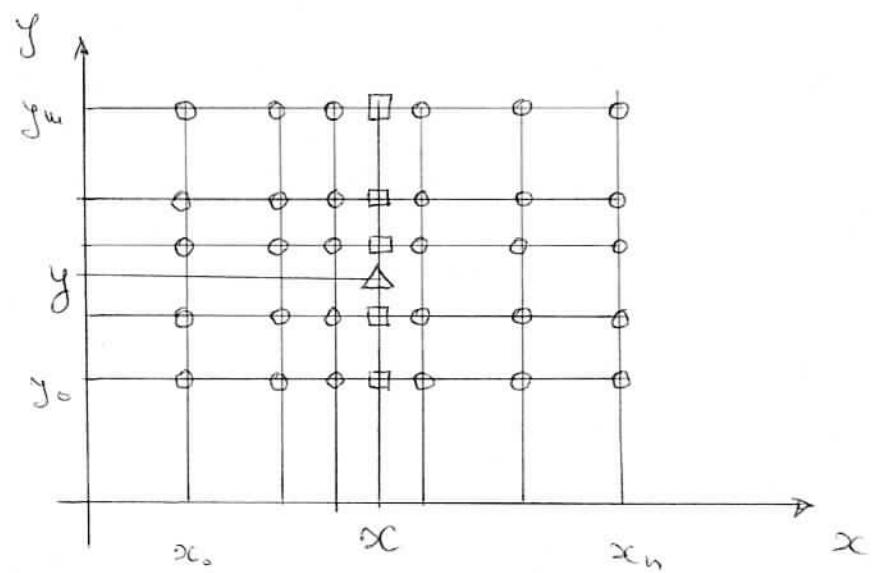
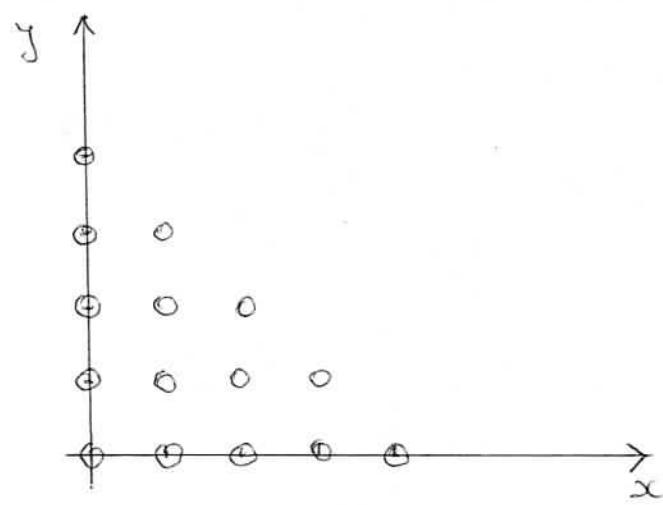
$$g = \frac{1}{\Delta f_0} \left[\bar{y} - f_0 - \frac{g(g-1)}{2!} \Delta^2 f_0 - \dots - \frac{g(g-1)\dots(g-n+1)}{n!} \Delta^n f_0 \right]$$

$$g = g(g)$$

$$g_{n+1} = g(g_n), \quad n=0, 1, 2, \dots \quad g_0 \text{ засновано на}$$

Aко $g_n \rightarrow \bar{g}$ и то је \bar{g} решете $\bar{g} = g(\bar{g})$

$$x = x_0 + \bar{g} h$$



I Найменше у. об.

$$L_n(x) = y_0 + g \Delta y_0 + \frac{g(g-1)}{2!} \Delta^2 y_0 + \frac{g(g-1)(g-2)}{3!} \Delta^3 y_0 + \dots$$

$$L_n'(x) = \frac{1}{h} \left[\Delta y_0 + \frac{2g-1}{2} \Delta^2 y_0 + \frac{3g^2 - 6g + 2}{6} \Delta^3 y_0 + \frac{4g^3 - 18g^2 + 22g - 6}{24} \Delta^4 y_0 + \dots \right]$$

$$L_n''(x) = \frac{1}{h^2} \left[\Delta^2 y_0 + (g-1) \Delta^3 y_0 + \frac{6g^2 - 18g + 11}{12} \Delta^4 y_0 + \dots \right]$$

$$L_n'(x_0) = \frac{1}{h} \left[\Delta y_0 - \frac{1}{2} \Delta^2 y_0 + \frac{1}{3} \Delta^3 y_0 - \frac{1}{4} \Delta^4 y_0 + \dots \right]$$

$$L_n''(x_0) = \frac{1}{h^2} \left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12} \Delta^4 y_0 - \frac{5}{6} \Delta^5 y_0 + \dots \right]$$

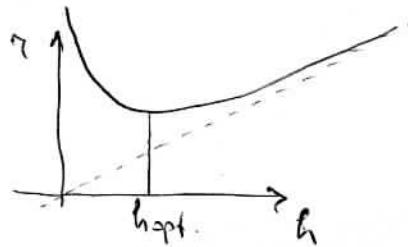
$$f'(x_0) \approx \frac{f(x_1) - f(x_0)}{h}$$

$$x_1 = x_0 + h \quad f(x_1) = f(x_0 + h) = f(x_0) + h f'(x_0) + \frac{h^2}{2} f''(\xi)$$

$$\begin{aligned}\frac{f(x_1) - f(x_0)}{h} &= \frac{1}{h} \left[f(x_0) + h f'(x_0) + \frac{h^2}{2} f''(\xi) - f(x_0) \right] \\ &= f'(x_0) + \frac{h}{2} f''(\xi)\end{aligned}$$

$$f'(x_0) = \frac{f(x_1) - f(x_0)}{h} - \frac{h}{2} f''(\xi) \Rightarrow R_M \leq \frac{h}{2} M_2$$

$$R_R \leq \frac{2\varepsilon}{h}$$



$$R \leq R_1 + R_2 = r(h) = \frac{h}{2} M_2 + \frac{2\varepsilon}{h} \rightarrow \min_{\frac{h}{2}}$$

$$r'(h) = \frac{M_2}{2} - \frac{2\varepsilon}{h^2} = 0 \Rightarrow h_{opt} = \sqrt{\frac{4\varepsilon}{M_2}}$$

$$f''(x_0) \approx \frac{f(x_0-h) - 2f(x_0) + f(x_0+h)}{h^2}$$

$$\begin{aligned}\frac{1}{h^2} [f(x_0-h) - 2f(x_0) + f(x_0+h)] &= \frac{1}{h^2} \left[f(x_0) - h f'(x_0) + \frac{h^2}{2} f''(x_0) - \frac{h^3}{6} f'''(x_0) + \frac{h^4}{24} f''''(\xi_1) \right. \\ &\quad \left. - 2f(x_0) + f(x_0) + h f'(x_0) + \frac{h^2}{2} f''(x_0) + \frac{h^3}{6} f'''(x_0) + \frac{h^4}{24} f''''(\xi_2) \right] \\ &= f''(x_0) + \frac{h^2}{24} (f''''(\xi_1) + f''''(\xi_2))\end{aligned}$$

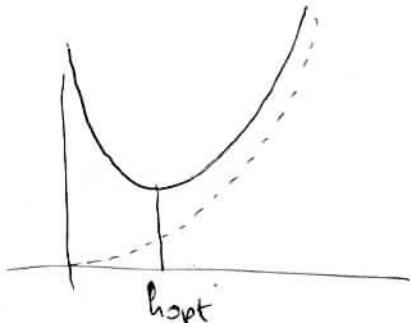
$$(d_1 f(\xi_1) + d_2 f(\xi_2) = (d_1 + d_2) f(\xi_3))$$

$$d_1, d_2 > 0$$

$$= f''(x_0) + \frac{h^2}{12} f''''(\xi)$$

$$f''(x_0) = \frac{f(x_0-h) - 2f(x_0) + f(x_0+h)}{h^2} - \frac{h^2}{12} f''''(\xi) \Rightarrow R_M \leq \frac{h^2}{12} M_4$$

$$R_R \leq \frac{4\varepsilon}{h^2}$$



$$R \leq R_1 + R_2 = r(h) = \frac{h^2}{12} M_4 + \frac{4\varepsilon}{h^2} \rightarrow \min_{\frac{h}{2}}$$

$$r'(h) = \frac{h}{6} M_4 - \frac{8\varepsilon}{h^3} = 0 \Rightarrow h_{opt} = \sqrt[4]{\frac{48\varepsilon}{M_4}}$$