PISMENI ISPIT IZ MNM B (SEPTEMBAR 2, 2020.)

1 $mx^2-2mx+4m=3x^2+6x+8$, $m\neq 3$

$(m-3)x^2-2(m+3)x+4(m-2)=0$, $m\neq 3$ $a=m-3\neq 0,\ b=-2(m+3),\ c=4(m-2)$

KVADRATNA JEDNAČINA IMA POZITIVNA REŠENJA AKO JE

$D\geq 0$, $x_1\cdot x_2>0$ i $\frac{x_1+x_2}{2}>0$

tj. $D\geq 0$, $\frac{c}{a}>0$ i $-\frac{b}{2a}>0$

$D=b^2-4ac=4(m+3)^2-4\cdot(m-3)\cdot 4(m-2)$

$=4m^2+24m+36-16(m^2-5m+6)=4m^2+24m-36-16m^2+80m-96$

$=-12m^2+104m-60=-4(3m^2-26m+15)$

$D\geq 0 \iff 3m^2-26m+15<0 \iff m\in\left(\frac{13-\sqrt{124}}{3},\ \frac{13+\sqrt{124}}{3}\right)$

$m_1=\frac{26\pm\sqrt{26^2-4\cdot 3\cdot 15}}{6}=\frac{13\pm\sqrt{13^2-3\cdot 15}}{3}=\frac{13\pm\sqrt{124}}{3}$

$x_1+x_2>0 \iff \frac{b}{a}<0 \iff \frac{-2(m+3)}{m-3}<0 \iff \frac{m+3}{m-3}>0 \iff m\in(-\infty,-3)\cup(3,+\infty)$

$x_1\cdot x_2>0 \iff \frac{c}{a}>0 \iff \frac{4(m-2)}{m-3}>0 \iff m\in(-\infty,2)\cup(3,+\infty)$

$0<\frac{13-\sqrt{124}}{3}<1$ $\boxed{m\in\left(3,\ \frac{13+2\sqrt{31}}{3}\right)}$

2 $x^{2\log^3 x-\log\sqrt{x^3}}=10$, $x>0$

$2\log^3 x-\log\sqrt{x^3}=\log_x 10$, VAŽI JER JE $x\neq 1$

$2\log^3 x-\frac{3}{2}\log x=\frac{1}{\log x}$, $\log x=t$

$2t^3-\frac{3}{2}t=\frac{1}{t}\ /\cdot t$ $(x\neq 1 \Rightarrow t\neq 0)$

$2t^4-\frac{3}{2}t^2=1\ /\cdot 2 \iff 4t^4-3t^2-2=0 \iff u_{1,2}=\frac{3\pm\sqrt{9+32}}{8}=\frac{3\pm\sqrt{41}}{8}$

$t^2=u\geq 0$

$u=\frac{3+\sqrt{41}}{8}\geq 0$ $\left(\frac{3-\sqrt{41}}{8}<0\right)$

$t^2=\frac{3+\sqrt{41}}{8} \Rightarrow t=\pm\sqrt{\frac{3+\sqrt{41}}{8}} \Rightarrow \log x=\pm\sqrt{\frac{3+\sqrt{41}}{8}} \Rightarrow x=10^{\pm\sqrt{\frac{3+\sqrt{41}}{8}}}$

$\boxed{x\in\left\{10^{\sqrt{\frac{3+\sqrt{41}}{8}}},\ 10^{-\sqrt{\frac{3+\sqrt{41}}{8}}}\right\}}$

3. $\sin(x-\sqrt{x^2})=-1$

$$x-\sqrt{x^2}=x-|x|=\begin{cases}0 & , x\geq 0\\ 2x & , x<0\end{cases}$$

$$(x\geq 0 \wedge \sin 0=-1) \vee (x<0 \wedge \sin 2x=-1)$$

$$(x\geq 0 \wedge x\in\emptyset) \vee (x<0 \wedge 2x=-\frac{\pi}{2}+2k\pi, k\in\mathbb{Z})$$

$$x<0 \wedge x=-\frac{\pi}{4}+k\pi, k\in\mathbb{Z}$$

$$x=-\frac{\pi}{4}+k\pi, k\in\{0,-1,-2,\dots\}$$

ili $x\in\{-\frac{\pi}{4}-k\pi \mid k\in\mathbb{N}\cup\{0\}\}$ ili $\boxed{x\in\{\frac{3\pi}{4}-k\pi \mid k\in\mathbb{N}\}}$

4. $P(x)=x^{2n}+3x^{2n-1}+1, n\geq 2$

$$P(x)=Q(x)\cdot(x^3+3x^2-x-3)+(ax^2+bx+c)$$

$$x^3+3x^2-x-3=x^2(x+3)-(x+3)=(x^2-1)(x+3)=(x-1)(x+1)(x+3)$$

$$P(x)=Q(x)\cdot(x-1)(x+1)(x+3)+ax^2+bx+c$$

$$\left.\begin{array}{l}P(1)=Q(1)\cdot 0\cdot 2\cdot 4+a+b+c\\ P(1)=1^{2n}+3\cdot 1^{2n-1}+1=1+3+1=5\end{array}\right\} a+b+c=5$$

$$\left.\begin{array}{l}P(-1)=Q(-1)\cdot(-2)\cdot 0\cdot(2)+a-b+c\\ P(-1)=(-1)^{2n}+3\cdot(-1)^{2n-1}+1=1+3\cdot(-1)+1=-1\end{array}\right\} a-b+c=-1$$

$$\left.\begin{array}{l}P(-3)=Q(-3)\cdot(-4)\cdot(0)+9a-3b+c\\ P(-3)=(-3)^{2n}+3\cdot(-3)^{2n-1}+1=3^{2n}-3^{2n}+1=1\end{array}\right\} 9a-3b+c=1$$

$$\begin{array}{l}a+b+c=5\\ a-b+c=-1\\ 9a-3b+c=1\end{array} \quad - \qquad 2b=6 \Rightarrow b=3$$

$$\begin{array}{l}a+c=2\\ 9a+c=10\end{array} \quad - \qquad 8a=8,\ a=1,\ c=1$$

$$\boxed{R(x)=x^2+3x+1}$$

5. $\sqrt{x+2\sqrt{x-1}}+2\sqrt{x-2\sqrt{x-1}}>x-4$

$x+2\sqrt{x-1}=x-1+2\sqrt{x-1}+1=(\sqrt{x-1}+1)^2$

$x-2\sqrt{x-1}=x-1-2\sqrt{x-1}+1=(\sqrt{x-1}-1)^2$

$\sqrt{(\sqrt{x-1}+1)^2}+2\sqrt{(\sqrt{x-1}-1)^2}>x-4$

$|\sqrt{x-1}+1|+2|\sqrt{x-1}-1|>x-4$

$\sqrt{x-1}=t\geq 0\quad,\quad x=t^2+1$

$|t+1|+2|t-1|>t^2-3$

$t+1+2|t-1|>t^2-3,$

$(t\geq 1\ \wedge\ t+1+2(t-1)>t^2-3)\ \vee\ (t\in[0,1)\ \wedge\ t+1-2(t-1)>t^2-3)$

$(t\geq 1\ \wedge\ t^2-3t-2<0)\ \vee\ (t\in[0,1)\ \wedge\ t^2+t-6<0)$

$\left(t\geq 1\ \wedge\ t\in\left(\frac{3-\sqrt{17}}{2},\frac{3+\sqrt{17}}{2}\right)\right)\ \vee\ (t\in[0,1)\ \wedge\ t\in(-3,2))$

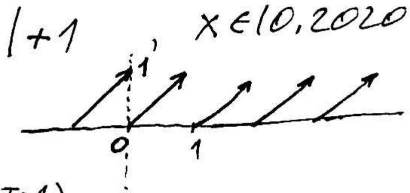
$t\in\left[1,\frac{3+\sqrt{17}}{2}\right)\qquad\vee\qquad t\in[0,1]$

$t\in\left[0,\frac{3+\sqrt{17}}{2}\right)$

$0\leq\sqrt{x-1}<\frac{3+\sqrt{17}}{2}\iff 1\leq x<1+\left(\frac{3+\sqrt{17}}{2}\right)^2=1+\frac{9+6\sqrt{17}+17}{4}=\frac{30+6\sqrt{17}}{4}$

$\boxed{x\in\left[1,\frac{15+3\sqrt{17}}{2}\right)}$

6. $2\arcsin(x-[x])=4|\cos(2\pi x+\frac{\pi}{2})|+1\quad,\quad x\in(0,2020)$

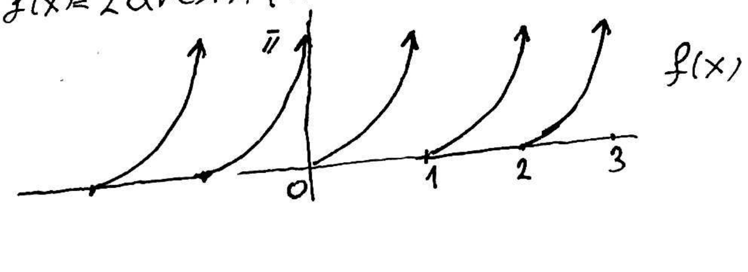


$x-[x]$ JE PERIODIČNA F-JA $(T=1)$

$f(x)=2\arcsin(x-[x])$ JE PERIOD. F-JA $(T=1)$

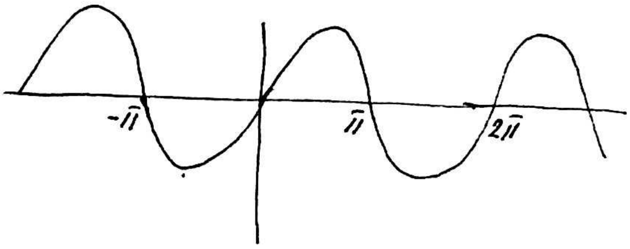
PRVO ĆEMO JE SKICIRATI NA $[0,1)$, A ONDA ISKORISTITI PERIODIČNOST

$x\in[0,1)$, $f(x)=2\arcsin(x-[x])=2\arcsin x$



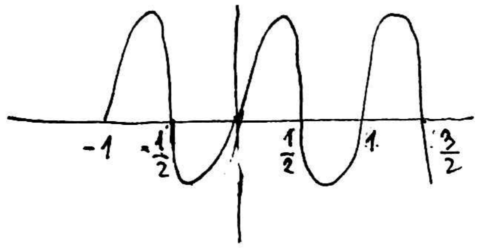
$g(x)=4|\cos(2\pi x+\frac{\pi}{2})|+1 = 4|\sin(2\pi x)|+1$

$g_1(x)=\sin x$



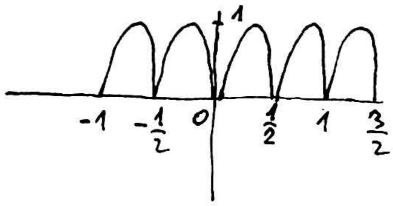
g_1

$g_2(x)=\sin(2\pi x)$
$=g_1(2\pi x)$



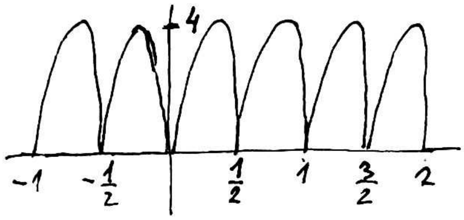
g_2

$g_3(x)=|\sin 2\pi x|=|g_2(x)|$

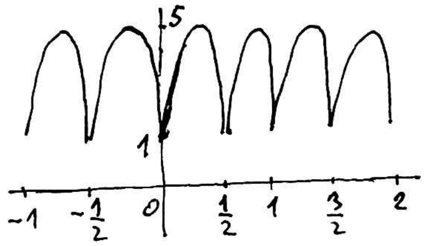


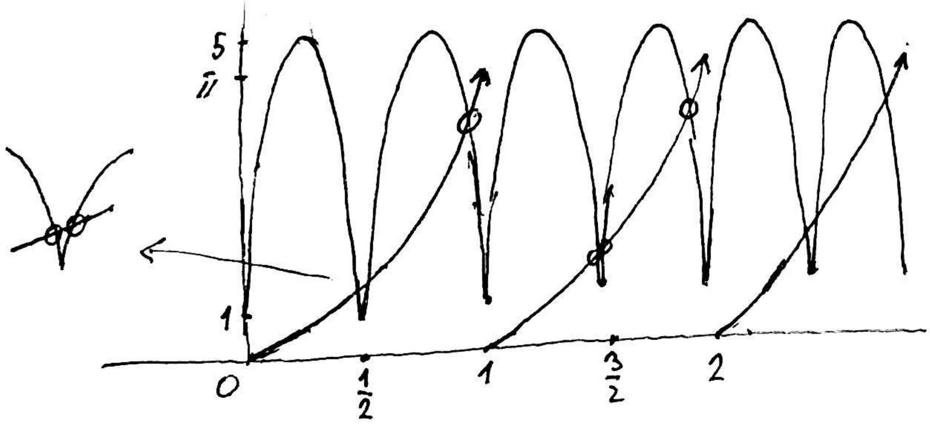
g_3

$g_4(x)=4|\sin(2\pi x)|=4g_3(x)$



$g(x)=4|\sin(2\pi x)|+1=g_4(x)+1$





$f(\frac{1}{2})=2\arcsin(\frac{1}{2})=2\cdot\frac{\pi}{6}=\frac{\pi}{3}>1$

POSTOJE 3 REŠENJA KOJA PRIPADAJU $(0,1)$

POSTOJE $3\cdot 2020=\boxed{6060}$ REŠENJA KOJA PRIPADAJU $(0,2020)$