• $f(x)=\begin{cases} x\, e^{\operatorname{arctg}\frac{1}{x}}, & x\neq 0 \\ 0, & x=0 \end{cases}$

1° $D_f=\mathbb{R}$

2° НУЛЕ: $x=0$

ЗНАК: $- \; - \; \underset{0}{|} \; + \; +$

3° ПАР/НЕП - ништа..

4° АСИМПТОТЕ:

• вертикалне нема :) јер је домен $\mathbb{R}$

• $\lim\limits_{\substack{x\to+\infty \\ (-\infty)}} f(x)=\lim\limits_{x\to+\infty} x\, e^{\operatorname{arctg}\frac{1}{x}}=\underset{(-\infty)}{+\infty}$ => нема хоризонталне!

:) желимо да развијемо $\operatorname{arctg} x$ у нули:

$g(x)=\operatorname{arctg} x = g(0)+\frac{g'(0)}{1!}\cdot x+\frac{g''(0)}{2!}x^2+\dots \quad, x\to 0$

$\underbrace{g'(x)}_{h(x)}=\frac{1}{1+x^2}=(1+x^2)^{-1}=1-x^2+x^4+o(x^4), \; x\to 0$

$\equiv h(0)+\frac{h'(0)}{1!}x+\frac{h''(0)}{2!}x^2+\frac{h'''(0)}{3!}x^3+\frac{h^{IV}(0)}{4!}x^4+o(x^4), \; x\to 0$

$\Rightarrow \underset{\underset{g'(0)}{\|}}{h(0)=1}, \; \underset{\underset{g''(0)}{\|}}{h'(0)=0}, \; \underset{\underset{g'''(0)}{\|}}{h''(0)=-2}, \; \underset{\underset{g^{IV}(0)}{\|}}{h'''(0)=0}, \; h^{IV}(0)=4! \; \dots$

а знамо $h^{(n)}(x)=g^{(n+1)}(x)$:

Још нам треба $g(0)=\operatorname{arctg} 0=0$

$\Rightarrow g(x)=\operatorname{arctg} x = 0+\frac{1}{1}x+0+\frac{-2}{3!}x^3+0+o(x^4) \quad, x\to 0$

$$\boxed{\operatorname{arctg} x = x-\frac{1}{3}x^3+o(x^4), \; x\to 0}$$

Применимо то:

$f(x)=x\cdot e^{\frac{1}{x}-\frac{1}{3x^3}+o\left(\frac{1}{x^4}\right)} \quad, x\to\pm\infty$

$=x\cdot\left(1+\left(\frac{1}{x}-\frac{1}{3x^3}+o\left(\frac{1}{x^4}\right)\right)+\frac{1}{2}\left(\frac{1}{x}+\frac{1}{3x^3}+o\left(\frac{1}{x^4}\right)\right)^2+o\left(\frac{1}{x^2}\right)\right)$

$=x\cdot\left(1+\frac{1}{x}+\frac{1}{2}\cdot\frac{1}{x^2}+o\left(\frac{1}{x^2}\right)\right)$

$=\underbrace{x+1}+\frac{1}{2}\cdot\frac{1}{x}+o\left(\frac{1}{x}\right), \quad x\to\pm\infty$

$\Rightarrow \boxed{y=x+1 \text{ к.а. } x\to\pm\infty}$

у $+\infty$ Γ_f изнад, у $-\infty$ Γ_f испод ње

5° $f'(x) = e^{\operatorname{arctg}\frac{1}{x}} + x\cdot e^{\operatorname{arctg}\frac{1}{x}}\cdot \frac{1}{1+\frac{1}{x^2}}\cdot \frac{-1}{x^2}$

$x \neq 0$

$$= e^{\operatorname{arctg}\frac{1}{x}}\left(1-\frac{x}{x^2+1}\right) =$$

$$= e^{\operatorname{arctg}\frac{1}{x}}\,\frac{x^2-x+1}{x^2+1} > 0, \quad \forall x \neq 0$$

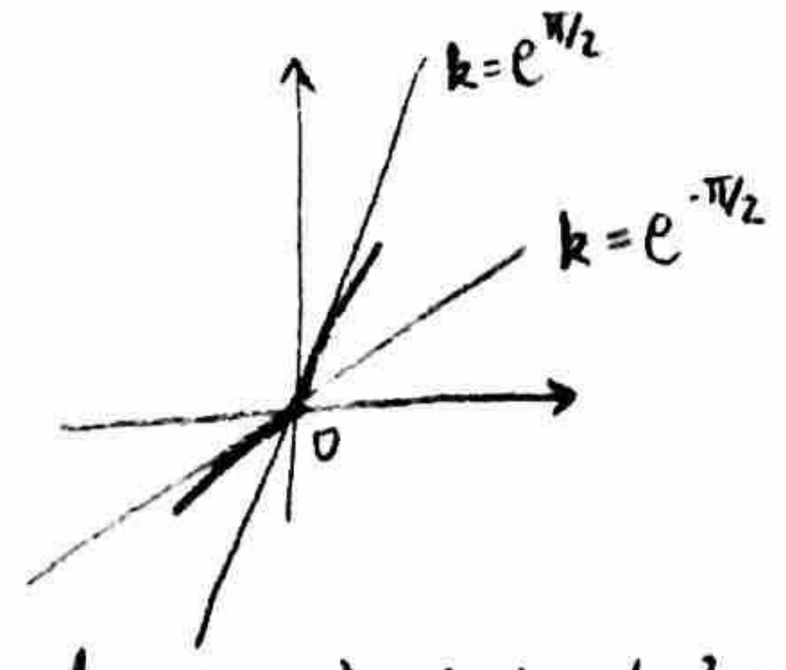
$+ \quad 0 \quad +$

$\nearrow \quad \nearrow$

Шта се дешава у нули?

$$\lim_{x\to 0+} f'(x) = \lim_{x\to 0+} \underbrace{e^{\operatorname{arctg}\frac{1}{x}}}_{\to \pi/2} \underbrace{\frac{x^2-x+1}{x^2+1}}_{\to 1} = e^{\pi/2}$$

$$\lim_{x\to 0-} f'(x) = e^{-\pi/2}$$



6° $f''(x) = e^{\sqcup}\cdot\frac{1}{1+\frac{1}{x^2}}\cdot\frac{-1}{x^2}\cdot\frac{x^2-x+1}{x^2+1} + e^{\operatorname{arctg}\frac{1}{x}}\cdot\frac{(2x-1)(x^2+1)-(x^2-x+1)\cdot 2x}{(x^2+1)^2}$ $= 2x^3+2x-x^2-1-2x^3+2x^2-2x$

$x \neq 0$

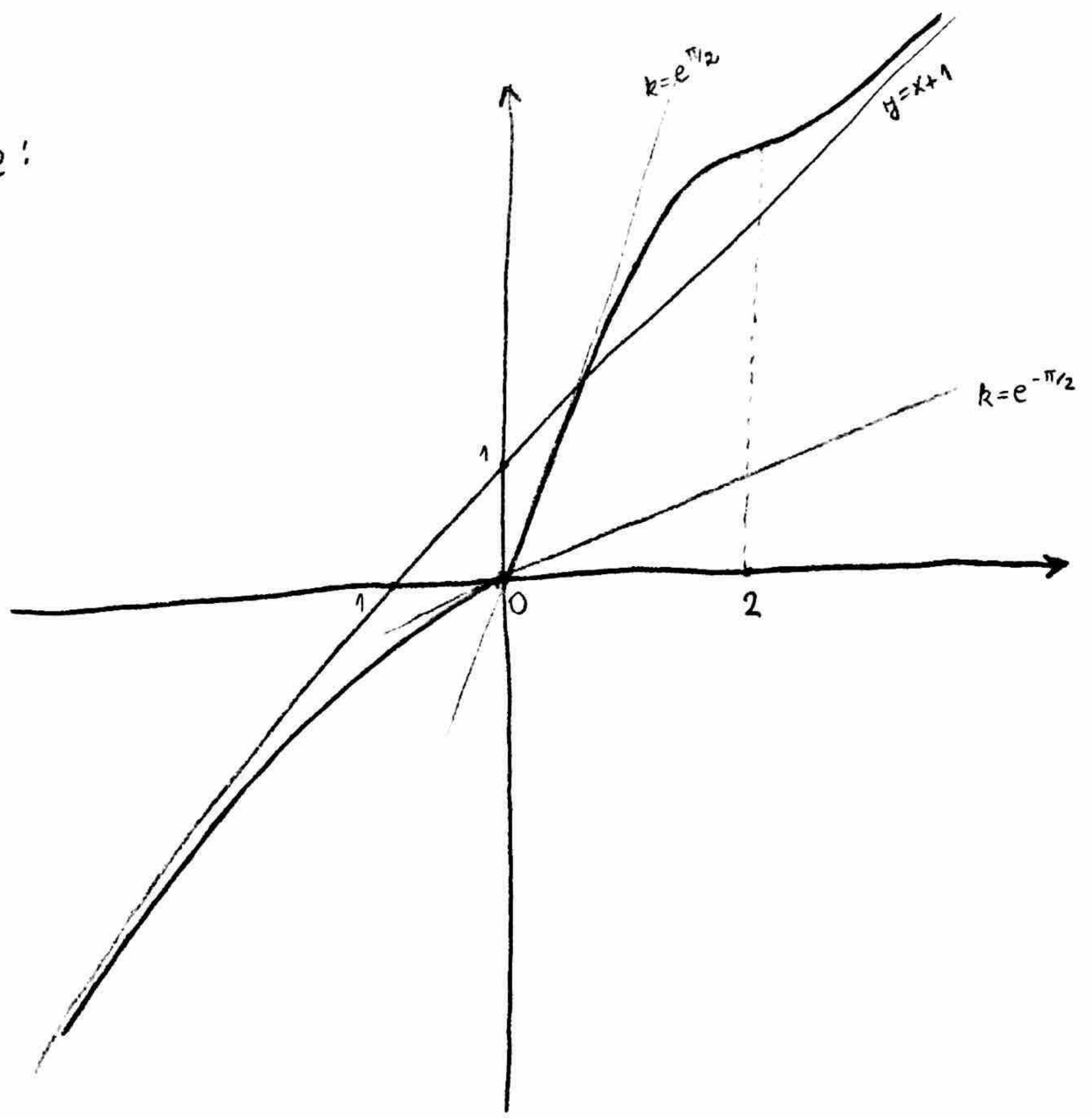
$$= \frac{e^{\operatorname{arctg}\frac{1}{x}}}{(x^2+1)^2}\cdot(-x^2+x-1+x^2-1) = \frac{e^{\operatorname{arctg}\frac{1}{x}}}{(x^2+1)^2}\cdot(x-2), \quad x\neq 0$$

f'': $-$ (0) $-$ (2) $+$

f: $\frown$ $\frown$ $\smile$

2 - превојна тачка

7° Γ_f:



I Испитати фју $f(x)=\sqrt[3]{(x-2)^2|x-1|}$

☺ abs нам само каже да раздвојимо случајеве

1° $D_f=\mathbb{R}$

2° пар / неп / период – није ништа

3° нуле, знак:

$x=2,\ x=1$

$+\ +\ +\ +\ +\ \overset{}{_1}\ \ +\ +\ +\ \overset{}{_2}$ – не урадиј! просто свуда је >0 јер $(\ )^2$ и abs

$f(x)\geq 0,\ \forall x$

$>0\quad \forall x\neq 2,1$

4° АСИМПТОТИКА: $x\to+\infty$: $f(x)=\sqrt[3]{(x-2)^2(x-1)}=(x-2)^{2/3}\cdot(x-1)^{1/3}=$

$$=x^{2/3}\cdot\left(1-\frac{2}{x}\right)^{2/3}\left(1-\frac{1}{x}\right)^{1/3}\cdot x^{1/3}=$$

$$=x\cdot\left(1+\frac{2}{3}\cdot\frac{-2}{x}+\binom{2/3}{2}\frac{4}{x^2}+o\left(\frac{1}{x^2}\right)\right)\left(1+\frac{1}{3}\cdot\frac{-1}{x}+\binom{1/3}{2}\cdot\frac{1}{x^2}+o\left(\frac{1}{x^2}\right)\right)$$

$$=x\left(1-\frac{4/3}{x}+\frac{-4}{9}\cdot\frac{1}{x^2}+o\left(\frac{1}{x^2}\right)\right)\left(1-\frac{1}{3x}+\frac{-1}{9}\cdot\frac{1}{x^2}+o\left(\frac{1}{x^2}\right)\right)$$

$$=x\cdot\left(1-\frac{5}{3}\cdot\frac{1}{x}+\frac{1}{x^2}\cdot\left(\frac{-4}{9}-\frac{1}{9}+\frac{4}{9}\right)+o\left(\frac{1}{x^2}\right)\right)$$

$$=x-\frac{5}{3}-\frac{1}{9}\cdot\frac{1}{x}+o\left(\frac{1}{x}\right)$$

$\boxed{\text{К.А. } x\to+\infty\quad y=x-5/3}$ ← f је испод АСИМПТОТЕ

$x\to-\infty$: $f(x)=\sqrt[3]{(x-2)^2\cdot(1-x)}=-(x-2)^{2/3}(x-1)^{1/3}=$ - претходно

$$=-x+\frac{5}{3}+\frac{1}{9}\cdot\frac{1}{x}+o\left(\frac{1}{x}\right)$$

$\boxed{\text{К.А. } x\to-\infty\quad y=-x+5/3}$ ↑ f је испод АСИМПТОТЕ

5° $f'(x)=?$ прво: $f(x)=\begin{cases}(x-2)^{2/3}(x-1)^{1/3}, & x\geq 1\\ -(x-2)^{2/3}(x-1)^{1/3}, & x<1\end{cases}$ – НА ПОЧЕТАК

$x\geq 1$: $f'(x)=\frac{2}{3}\cdot\frac{1}{(x-2)^{1/3}}(x-1)^{1/3}+\frac{1}{3}\cdot\frac{(x-2)^{2/3}}{(x-1)^{2/3}}=\frac{1}{3\cdot(x-2)^{1/3}(x-1)^{2/3}}\cdot((x-1)\cdot 2+(x-2))$

$\boxed{f'(x)=\frac{3x-4}{3\cdot\sqrt[3]{(x-2)(x-1)^2}}}$ ← $x>1,\ x\neq 2$

Не може тако у 2 и 1

$x<1$: $f(x)=-$претх $\Rightarrow$ $f'(x)=-$претх

$\boxed{f'(x)=\frac{4-3x}{3\sqrt[3]{(x-2)(x-1)^2}},\quad x<1}$

		1		4/3		2	
f'	−		+		−	×	+
f	↘		↗		↘		↗

$\boxed{1,2\ \min\quad 4/3\ \max}$

да ли је диф. у 1 и 2 ?

то ћемо видети рачунањем лимеса (ако су коначни, да, ако нису, не)

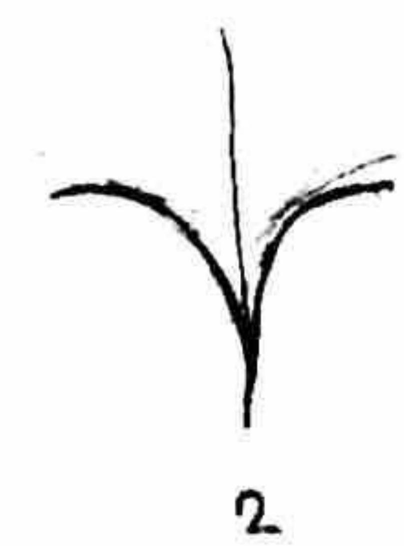
1, 2 - шта је ту? <0

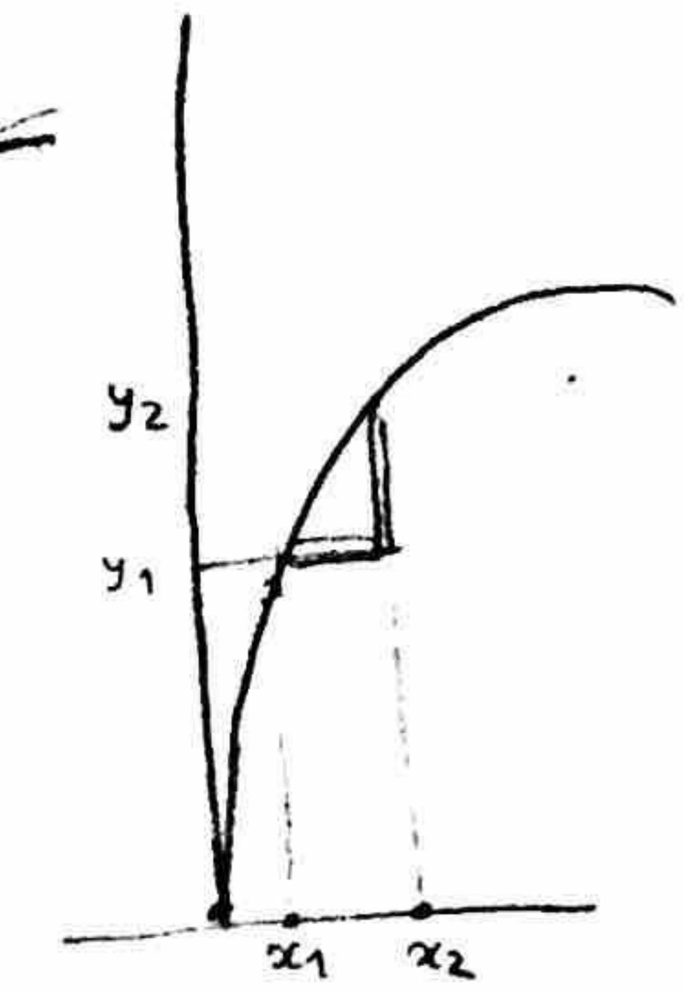
$$\lim_{x\to 1+} f'(x) = \lim_{x\to 1+} \frac{3x-4}{3\sqrt[3]{(x-2)(x-1)^2}} = +\infty$$

$$\lim_{x\to 1-} f'(x) = \lim_{x\to 1-} \frac{4-3x}{3\sqrt[3]{(x-2)(x-1)^2}} = -\infty$$

$$\lim_{x\to 2+} f'(x) = \lim_{x\to 2+} \frac{3x-4}{3\sqrt[3]{(x-2)(x-1)^2}} = +\infty$$

$$\lim_{x\to 2-} f'(x) = \lim_{x\to 2-} \frac{3x-4}{3\sqrt[3]{(x-2)(x-1)^2}} = -\infty$$





$$\frac{y_1-y_2}{x_1-x_2}$$

$\Rightarrow$ Није диференцијабилна у 1 и 2,
имамо шпицеве,
и вертикално се „пењу"

$$f'_+(2) = \lim_{h\to 0+} \frac{f(2+h)-f(2)}{h} = +\infty$$

$\Rightarrow f(2+h) > f(2)$
$\Rightarrow$ не може бити

6° $f''(x) = ?$

$x \in (1,2)\cup(2,+\infty)$ $\quad f''(x) = \left(\frac{3x-4}{3\sqrt[3]{(x-2)(x-1)^2}}\right)' =$

$$= \frac{3\cdot 3\sqrt[3]{(x-2)(x-1)^2} - (3x-4)\cdot\cancel{3}\cdot\frac{1}{\cancel{3}}\cdot\frac{1}{((x-2)(x-1)^2)^{2/3}}\cdot\left((x-1)^2+(x-2)\cdot 2(x-1)\right)}{9\cdot(x-2)^{2/3}(x-1)^{4/3}}$$

$$= \frac{1}{9(x-2)^{2/3}(x-1)^{4/3}}\cdot\frac{1}{(x-2)^{2/3}(x-1)^{4/3}}\cdot\left(9\cdot(x-2)(x-1)^2-(3x-4)(x-1)(3x-5)\right)$$

$$= \frac{1}{9(x-2)^{4/3}(x-1)^{8/3}}\cdot(x-1)\cdot\left(9(x^2-3x+2)-(9x^2-9\cdot 3x+20)\right)$$

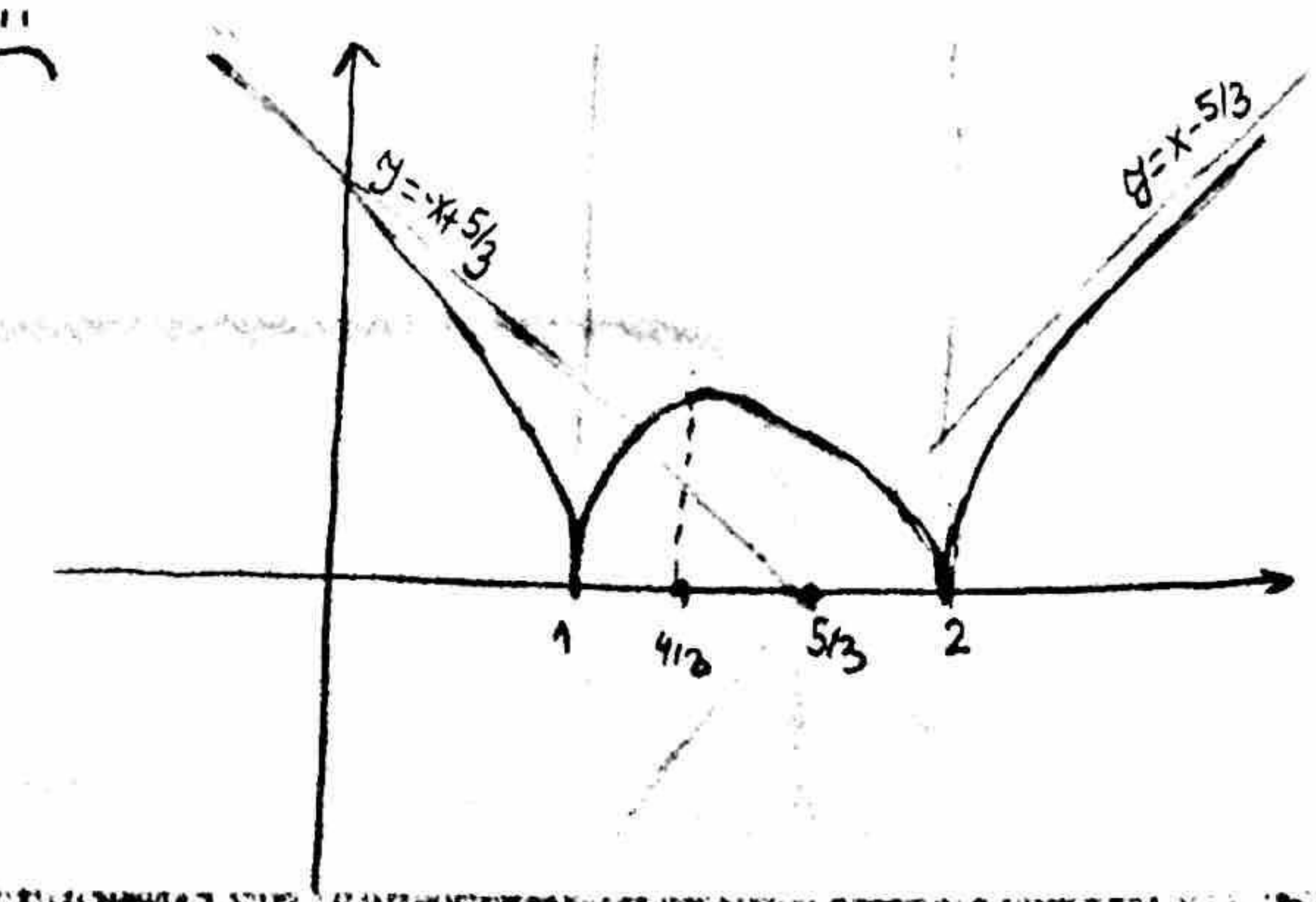
$$= \frac{1}{9(x-2)^{4/3}(x-1)^{5/3}}\cdot\left(9x^2-18x+18-9x^2+18x-20\right)$$

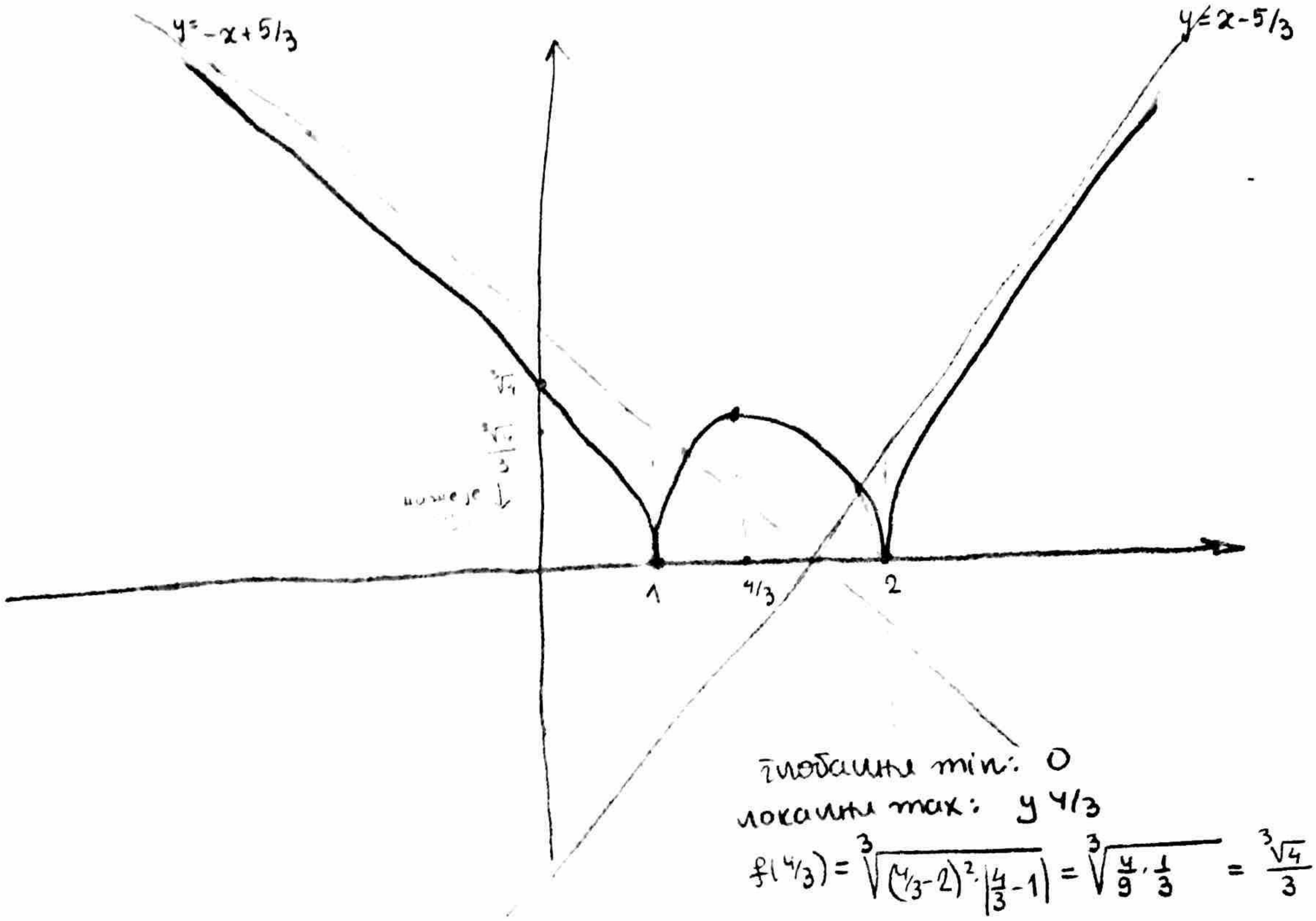
$$= \frac{-2}{9(x-2)^{4/3}(x-1)^{5/3}} \qquad <0 \iff x>1,\ x\neq 2$$

$x\in(-\infty,1)$: само $-$: $\quad f''(x) = \dfrac{2}{9(x-2)^{4/3}(x-1)^{5/3}} \quad <0$ за $x<1$

$\Rightarrow$

7° Γ_f



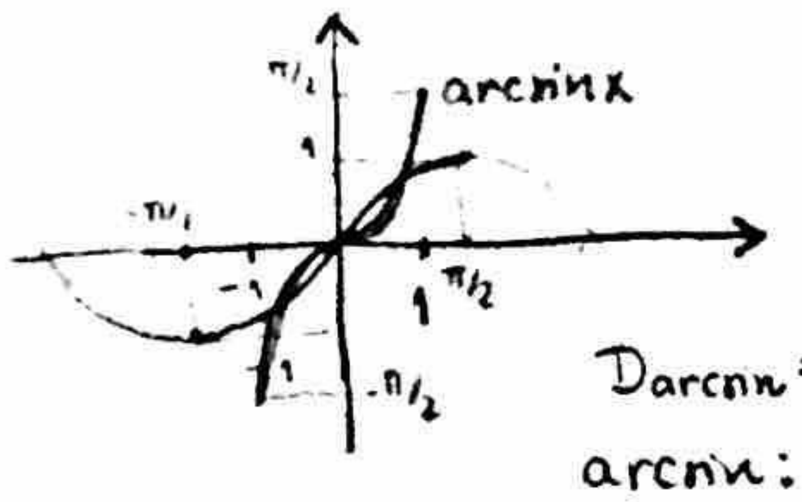


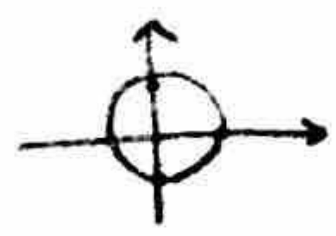
глобални min: 0

локални max: у 4/3

$f(4/3) = \sqrt[3]{(4/3-2)^2 \cdot |4/3-1|} = \sqrt[3]{\frac{4}{9}\cdot\frac{1}{3}} = \frac{\sqrt[3]{4}}{3}$ □

$\boxed{2.}$ $f(x) = \arcsin \frac{x^2}{\sqrt{2x^4-2x^2+1}}$





$D_{\arcsin} = [-1,1]$

$\arcsin: [-1,1] \to [-\pi/2, \pi/2]$

1° D_f: $x \in D_f \Leftrightarrow$

$\frac{x^2}{\sqrt{2x^4-2x^2+1}} \in [-1,1]$

$\Leftrightarrow x^2 \le \sqrt{2x^4-2x^2+1}$

$\Leftrightarrow x^4 \le 2x^4-2x^2+1 \Leftrightarrow (x^2-1)^2 \ge 0 \Leftrightarrow \top$ $\boxed{D_f = \mathbb{R}}$

2° f је парна $\Rightarrow$ довољно је посматрати на $[0,+\infty)$

3° нуле, знак: $\arcsin t = 0 \Leftrightarrow t = 0 \Leftrightarrow \boxed{x=0}$

$\frac{x^2}{\sqrt{\dots}} \ge 0 \Rightarrow \boxed{f(x) \ge 0, \forall x \in D_f}$

4° асимптоте:

$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \arcsin \underbrace{\frac{x^2}{\sqrt{2x^4-2x^2+1}}}_{\to \frac{1}{\sqrt{2}}} = \pi/4$ $\Rightarrow \boxed{y = \pi/4 \text{ Х.А. } x\to+\infty}$

због парности и кад $x \to -\infty$

За вежбу

$f(x) = \operatorname{arctg} x + \frac{2x}{1+x^2}$

непарна

5° $f'(x) = \dfrac{1}{\sqrt{1-\dfrac{x^4}{2x^4-2x^2+1}}} \cdot \dfrac{2x\cdot\sqrt{2x^4-2x^2+1} - x^2\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{\ }}\cdot(8x^3-4x)}{2x^4-2x^2+1}$

$= \dfrac{1}{\sqrt{\dfrac{x^4-2x^2+1}{\underbrace{2x^4-2x^2+1}_{>0}}}} \cdot \dfrac{1}{\sqrt{\ }} \cdot \dfrac{1}{2x^4-2x^2+1} \cdot \left(2x\cdot(2x^4-2x^2+1) - \frac{1}{2}x^2(8x^3-4x)\right)$

$= \dfrac{1}{\sqrt{(x^2-1)^2}} \cdot \dfrac{1}{2x^4-2x^2+1} \cdot (4x^5-4x^3+2x-4x^5+2x^3)$ $2x-2x^3$

! $\longrightarrow$ $x \neq 1, -1$

у њима није дб!

$= \dfrac{1}{|x^2-1|} \cdot \dfrac{1}{2x^4-2x^2+1} \cdot 2x\cdot(1-x^2) = \dfrac{2x\cdot \mathrm{sgn}(1-x^2)}{2x^4-2x^2+1}$

$= \begin{cases} \dfrac{-2x}{2x^4-2x^2+1}, & x\in(1,+\infty) \\ \dfrac{2x}{2x^4-2x^2+1}, & x\in[0,1) \end{cases}$ (парност ...)

$0 \quad + \quad 1 \quad -$; $\nearrow$ $\searrow$

max 1
(сл. -1 је max)

$\lim\limits_{x\to1+} f'(x) = -2$

$\lim\limits_{x\to1-} f'(x) = 2$

6° $f''(x) = \ldots = \begin{cases} \dfrac{2(6x^4-2x^2-1)}{(2x^4-2x^2+1)^2}, & x>1 \\ -2\,\dfrac{(6x^4-2x^2-1)}{(2x^4-2x^2+1)^2}, & 0\le x<1 \end{cases}$

$6x^4-2x^2-1=0 \quad x^2=t \quad t_{12} = \dfrac{2\pm\sqrt{28}}{12} = \dfrac{1\pm\sqrt7}{6} > 0 \Rightarrow t = \dfrac{1+\sqrt7}{6}$ t

$t=x^2$: $\frac{1-\sqrt7}{6} \quad 0 \quad - \quad \frac{1+\sqrt7}{6} \quad +$

$\Rightarrow x_{12} = \pm\sqrt{\dfrac{1+\sqrt7}{6}} \qquad x>0$

$\Rightarrow \boxed{x = \sqrt{\dfrac{1+\sqrt7}{6}}} =: \alpha$

$\alpha<1 \Rightarrow$ знамо за $x>1$:

$f''(x)>0 \Rightarrow \cup \quad x>1$

• за $x\in[0,1)$ f'' је -ово $\Rightarrow f''(x)>0 \Leftrightarrow 6x^4-2x^2-1<0 \Leftrightarrow t\in\left(\dfrac{1-\sqrt7}{6}, \dfrac{1+\sqrt7}{6}\right)$

$\Leftrightarrow x^2\in\left[0, \dfrac{1+\sqrt7}{6}\right) \Leftrightarrow x\in[0,\alpha)$

$0 \quad + \quad \alpha \quad - \quad 1$

f: $\cup$ $\cap$

$\Rightarrow \boxed{\begin{array}{l} f''(x)>0 \Leftrightarrow x\in[0,\alpha) \\ f''(x)<0 \Leftrightarrow x\in(\alpha,1) \end{array}}$

7° Γ_f: до пола па пресликамо:

$f(1) = \arcsin\dfrac{1}{\sqrt1} = \pi/2$