• $f(x) = 2\ln|e^x-1| - \ln|e^x-2|$

1° D_f : $x\neq 0,\ x\neq \ln 2$; $D_f = (-\infty,0)\cup(0,\ln 2)\cup(\ln 2,+\infty)$

2° // (није домен симетричан)

3° Нуле и знак: $f(x)>0 \iff \ln\frac{(e^x-1)^2}{|e^x-2|}>0 \iff \frac{(e^x-1)^2}{|e^x-2|}>1$

1) $e^x>2$: $\iff (e^x-1)^2>e^x-2 \iff \underbrace{(e^x)^2-3e^x+3>0}_{\text{увек тачно}}$ $\quad D=9-12<0$

$\Rightarrow \boxed{x\in(\ln 2,+\infty)}$

2) $e^x<2$: $\iff (e^x-1)^2>2-e^x \iff (e^x)^2-e^x-1>0$ $\quad t_{12}=\frac{1\pm\sqrt5}{2}$

$\iff e^x\in(-\infty,\frac{1-\sqrt5}{2})\cup(\frac{1+\sqrt5}{2},+\infty)$

$e^x\in(\frac{1+\sqrt5}{2},2)$ $\quad \boxed{x\in(\underbrace{\ln\frac{1+\sqrt5}{2}}_{\alpha},\ln 2)}$

слично, овде ставимо „=" да бисмо добили нуле, и $f<0$ ван ...

НУЛА: $\boxed{x=\ln\frac{1+\sqrt5}{2}}=\alpha$

дакле: f:

	0		α (НУЛА)		$\ln 2$	
$-$	$\times$	$-$	0	$+$	$\times$	$+$

4° АСИМПТОТИКА:

$-\infty$: $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} 2\ln|\underbrace{e^x}_{\to 0}-1| - \ln|\underbrace{e^x}_{\to 0}-2| = 2\ln 1-\ln 2=-\ln 2$

$\boxed{y=-\ln 2 \text{ х.а. } x\to-\infty}$

$+\infty$: $\lim\limits_{x\to+\infty} f(x)=\lim\limits_{x\to+\infty}\ln\frac{(e^x-1)^2}{|e^x-2|}=+\infty \to$ нема х.а. ☺

Снађемо се: $x\to+\infty$:

$f(x)=2\ln e^x+2\ln(1-\frac{1}{e^x})-\ln e^x-\ln(1-\frac{2}{e^x})=$

$=\ln e^x+\underbrace{2\ln(\underbrace{1-\frac{1}{e^x}}_{\to 1,\ x\to+\infty})-\ln(\underbrace{1-\frac{2}{e^x}}_{\to 1,\ x\to+\infty})}_{\to 0,\ x\to+\infty}$ $\quad$ + може ту ?

$=x+o(1),\ x\to+\infty$

$\Rightarrow$ по деф. $\boxed{y=x \text{ је к.а. за } x\to+\infty}$

0: $\lim\limits_{x\to 0} f(x)=\lim\limits_{x\to 0} 2\ln|\underbrace{e^x-1}_{\to 0+}|-\underbrace{\ln|\underbrace{e^x-2}_{\to 1}|}_{\to 0}=-\infty$ $\quad\boxed{\text{В.А. } x=0}$

$\ln 2$: $\lim\limits_{x\to\ln 2} f(x)=\lim\limits_{x\to\ln 2} \underbrace{2\ln|\underbrace{e^x-1}_{\to 1}|}_{\to 0}-\underbrace{\ln|\underbrace{e^x-2}_{\to 0+}|}_{\to-\infty}=+\infty$ $\quad\boxed{\text{В.А. } x=\ln 2}$

!

5° $f'(x) = 2\cdot\frac{1}{|e^x-1|}\cdot sgn(e^x-1)\cdot e^x - \frac{1}{|e^x-2|}\cdot sgn(e^x-2)\cdot e^x$ $\quad f(x) = 2\ln|e^x-1| - \ln|e^x-2|$

$= \frac{2e^x}{e^x-1} - \frac{e^x}{e^x-2} = \frac{e^x(e^x-3)}{(e^x-1)(e^x-2)}, \quad x \neq 0, \ln 2$ $\quad \ln|f(x)|' = \frac{f'(x)}{f(x)}$

јер: $= \frac{1}{|f(x)|}\cdot sgn(f(x))\cdot f'(x)$

за $f(x) \neq 0$

		0	ln2	ln3	
e^x-3	−	−	−	+	
e^x-1	−	+	+	+	
e^x-2	−	−	+	+	
f':	−	+	−	+	
f:	↘	↗	↘	↗	

$x = \ln 3$ локални минимум

[illegible]

6° $f''(x) = \left(\frac{2e^x}{e^x-1} - \frac{e^x}{e^x-2}\right)' = \frac{2e^x(e^x-1) - 2e^x\cdot e^x}{(e^x-1)^2} - \frac{e^x(e^x-2) - e^x\cdot e^x}{(e^x-2)^2}$

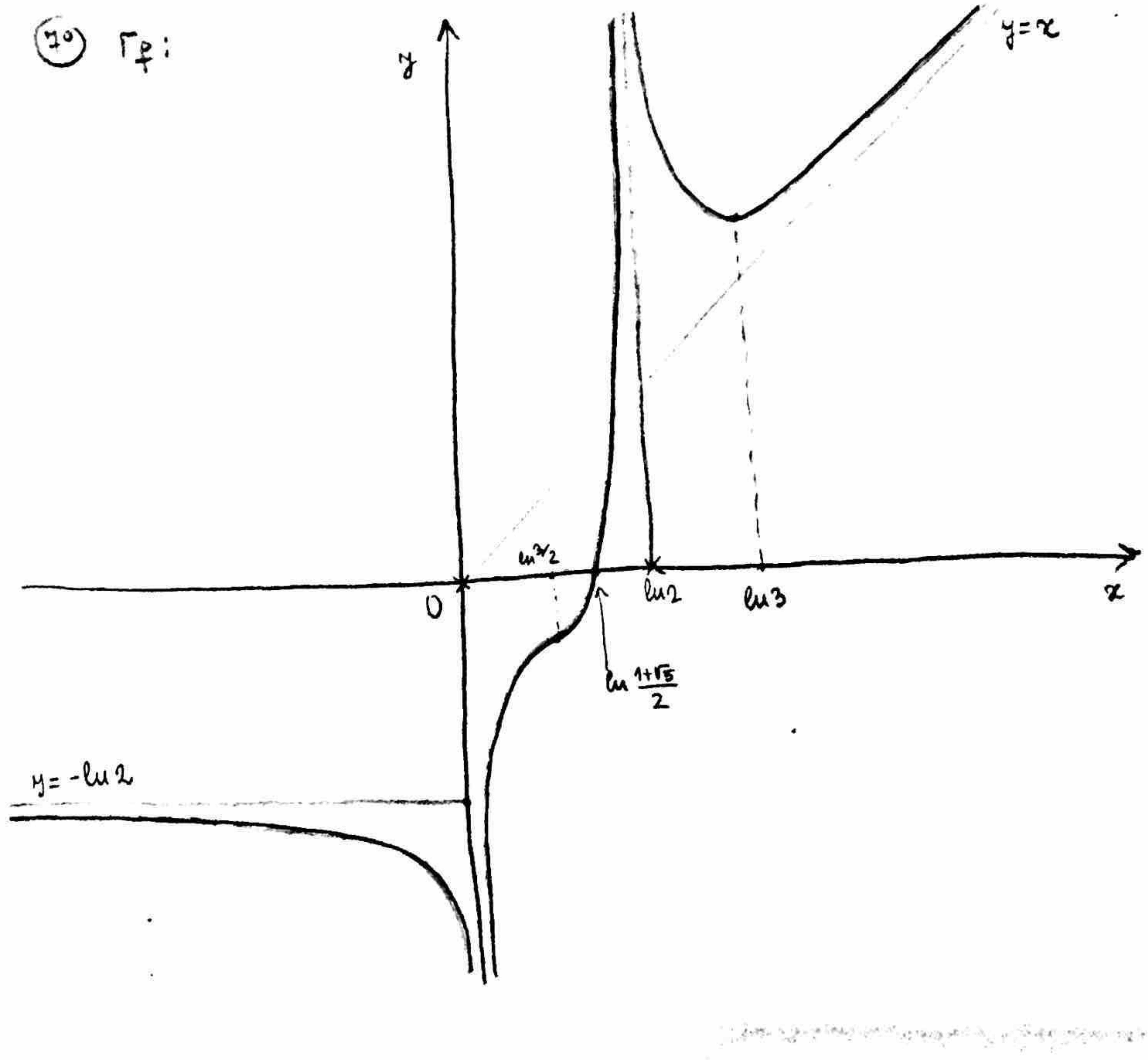
$= \frac{-2e^x}{(e^x-1)^2} + \frac{2e^x}{(e^x-2)^2} = \frac{2e^x}{(e^x-1)^2(e^x-2)^2}\cdot\left((e^x-1)^2-(e^x-2)^2\right) = \frac{2e^x\cdot(2e^x-3)}{(e^x-1)^2(e^x-2)^2},$

$x \neq 0, \ln 2$

		0		$\ln 3/2$		$\ln 2$	
f'':	−	×	−		+	×	+
f:	∩		∩		∪		∪

$\ln 3/2$ превојна тачка

7° Γ_f:



• $f(x) = \operatorname{arctg} x + \dfrac{2x}{1+x^2}$

(1°) $D_f = \mathbb{R}$.

(2°) НЕПАРНА ЈЕ - па довољно испитивати на $[0,+\infty)$ и онда само пресликати центр. сим. у одн. на 0

(3°) нуле и знак:

$x>0 \Rightarrow \operatorname{arctg} x > 0,\ 2x>0 \Rightarrow f(x)=0$

$\Rightarrow$ на $[0,+\infty)$ једина нула је $\underline{x=0}$

$\boxed{f(x)>0,\ \forall x\in(0,+\infty)}$

f: $-$ $+$ (0)

због непарности: $\boxed{f(x)<0,\ \forall x\in(0,+\infty)}$

(4°) АСИМПТОТИКА:

$\lim\limits_{x\to+\infty} f(x) = \pi/2$ $\left(\overset{\text{нец.}}{\Rightarrow} \lim\limits_{x\to-\infty} f(x) = -\pi/2\right)$

Х.А. $y=\pi/2$ Х.А. $y=-\pi/2$

(5°) $f'(x) = \dfrac{1}{1+x^2} + \dfrac{2(1+x^2)-2x\cdot 2x}{(1+x^2)^2} = \dfrac{1+x^2+2-2x^2}{(1+x^2)^2} = \dfrac{3-x^2}{(1+x^2)^2}$

f': $[0$, $+$, $\sqrt3$, $-$

f: $\nearrow$ $\searrow$

$\sqrt3$ лок. max. $\Rightarrow$ $\searrow$ $-\sqrt3$ $\nearrow$ $0]$ f — лок. min

(6°) $f''(x) = \dfrac{1}{(1+x^2)^4}\cdot\left(-2x\cdot(1+x^2)^2 - (3-x^2)\,2\cdot(1+x^2)\cdot 2x\right) = \dfrac{2x}{(1+x^2)^3}\cdot\left(-1-x^2-2(3-x^2)\right)$

$f''(x) = \dfrac{2x\cdot(x^2-7)}{(1+x^2)^3}$

f'': $[0$, $-$, $\sqrt7$, $+$

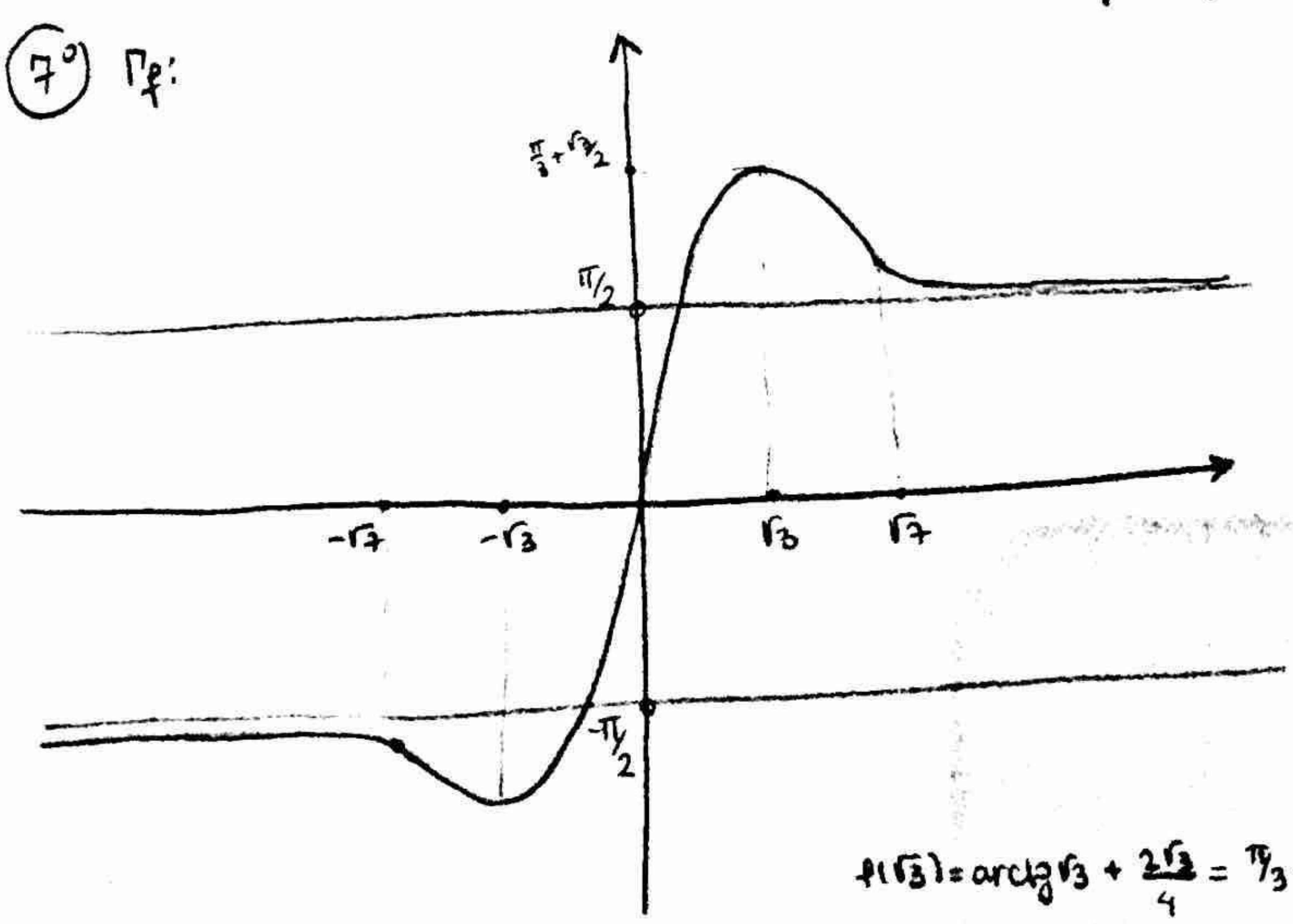
f: ⌒ ⌣

$\sqrt7$ - преводјна т. (и $-\sqrt7$)

центр. симетр. $\Rightarrow$ ⌒ $-\sqrt7$ ⌣ $0]$

или директно проверимо лако f'': $-$ $-\sqrt7$ $+$ $0]$

(7°) Γ_f:



$f(\sqrt3) = \operatorname{arctg}\sqrt3 + \dfrac{2\sqrt3}{4} = \pi/3 + \sqrt3/2$

☺ конвексно се лепи уз асимптоту хориз. $\Rightarrow$ одозго !

објасни ове случајеве...

$$\cancel{f'(x) = \frac{2}{1+x^2}\left(\operatorname{sgn}(1-x^2) - \operatorname{sgn} x\right)}$$

	-1		0		1	
$\operatorname{sgn}(1-x^2)$	$-$		$+$	$+$		$-$
$\operatorname{sgn} x$	$-$		$-$	$+$		$+$
разл.	0		2	0		-2

7

$$= \begin{cases} \frac{4}{1+x^2}, & x \in (-1,0) \\ 0, & x \in (-\infty,-1) \cup (0,1) \\ -\frac{4}{1+x^2}, & x \in (1,+\infty) \end{cases}$$

f': 0 on $(-\infty,-1)$, $+$ on $(-1,0)$, 0 on $(0,1)$, $-$ on $(1,+\infty)$

f: const на $(-\infty,-1)$ ↗ const на $(0,1)$ ↘

збот непр. у -1 const на $(-\infty,-1]$

$= f(-1) = 2\underbrace{\operatorname{arctg} 1}_{\pi/4} + \underbrace{\arcsin -1}_{-\pi/2} = 0$

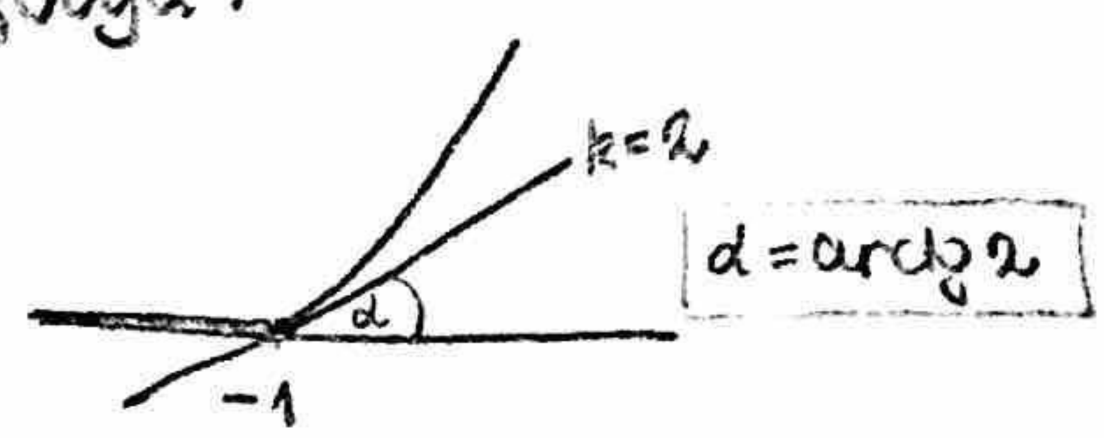
збот непр. у 1 const на $[0,1]$

$= f(1) = 2\underbrace{\operatorname{arctg} 1}_{\pi/4} + \underbrace{\arcsin 1}_{\pi/2} = \pi$

Граничне вредности извода:

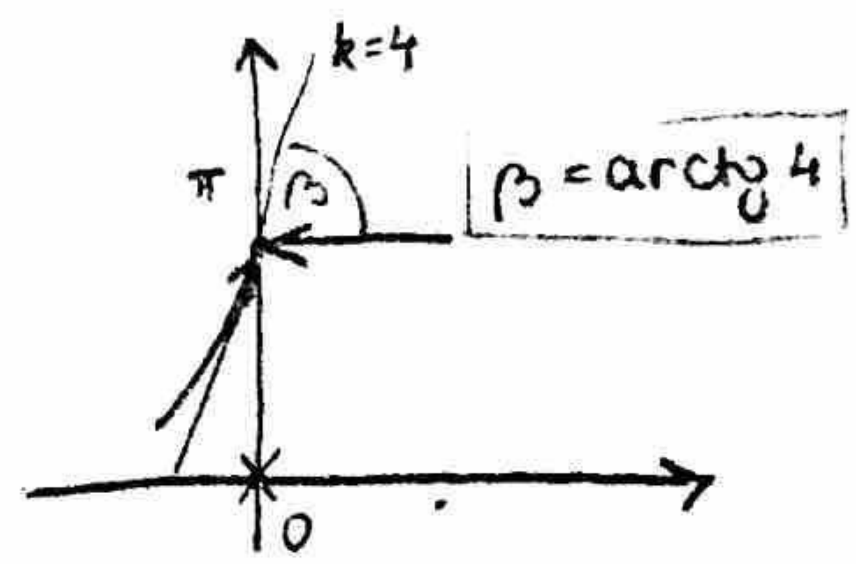
-1: $\lim\limits_{x\to -1_-} f'(x) = 0$

$\lim\limits_{x\to -1_+} \underbrace{f'(x)}_{\text{јер } \frac{4}{1+x^2}} = 2$



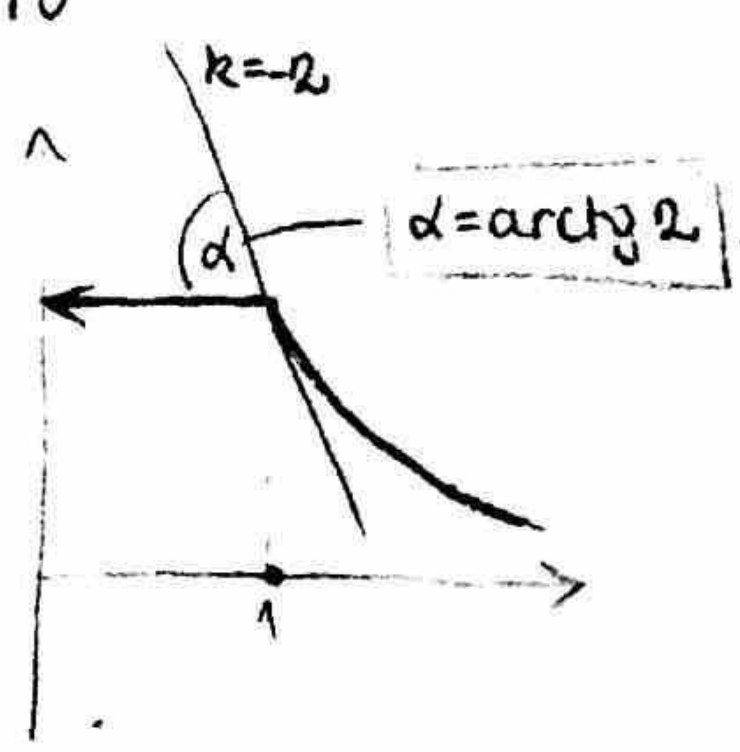
0: $\lim\limits_{x\to 0_-} \underbrace{f'(x)}_{\text{јер } \frac{4}{1+x^2}} = 4$

$\lim\limits_{x\to 0_+} f'(x) = 0$



1: $\lim\limits_{x\to 1_+} \underbrace{f'(x)}_{-\frac{4}{1+x^2}} = -2$

$\lim\limits_{x\to 1_-} \underbrace{f'(x)}_{0} = 0$



6° $f''(x)$: $\left(\frac{4}{1+x^2}\right)' = \frac{-4}{(1+x^2)^2}\cdot 2x$

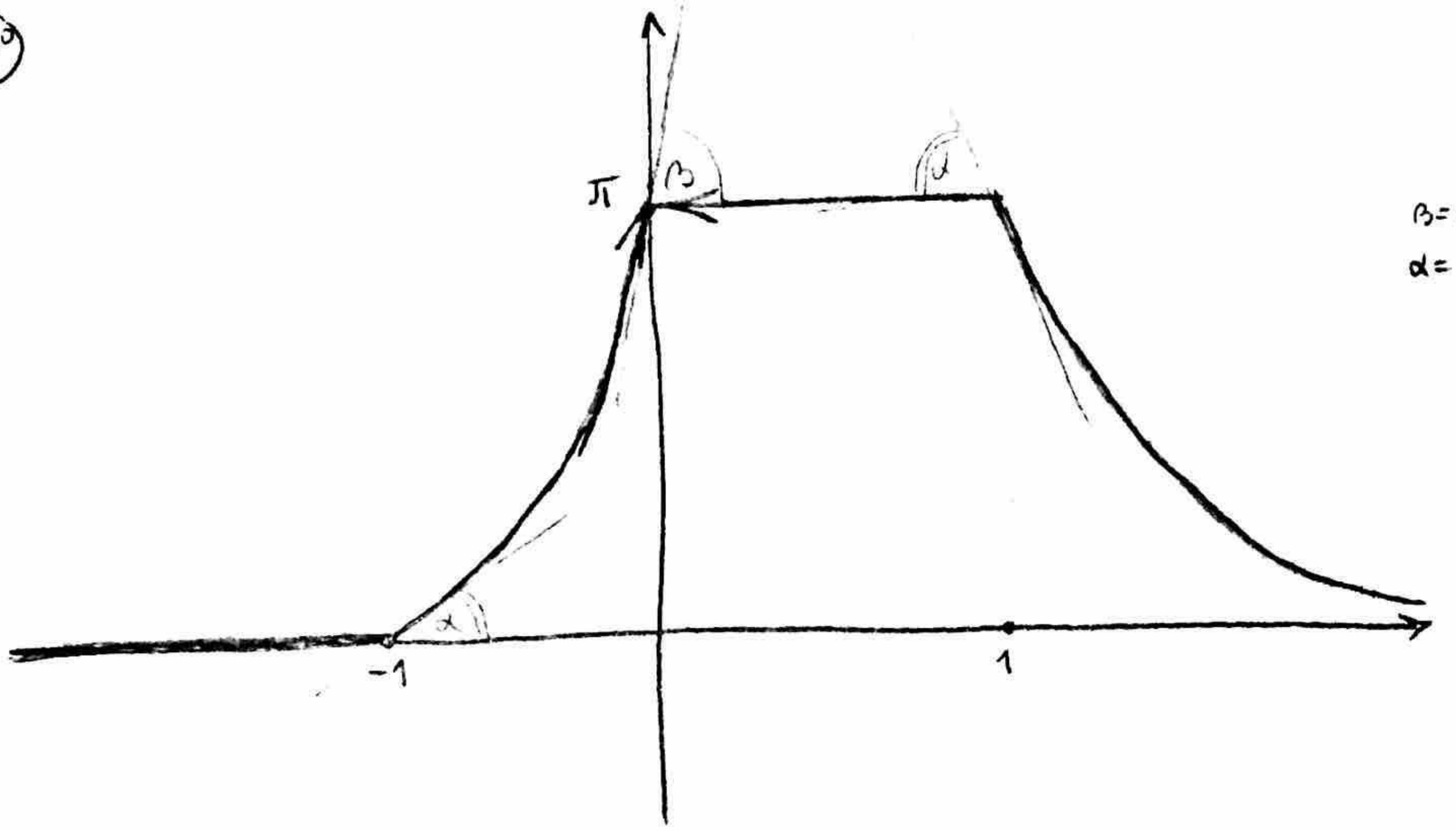
$$f''(x) = \begin{cases} \frac{-8x}{(1+x^2)^2}, & x \in (-1,0) \\ 0, & x \in (-\infty,-1) \cup (0,1) \\ \frac{8x}{(1+x^2)^2}, & x \in (1,+\infty) \end{cases}$$

$x \neq 1, -1, 0$

f'': 0 on $(-\infty,-1)$, $+$ on $(-1,0)$, 0 on $(0,1)$, $+$ on $(1,+\infty)$

f: већ знамо из const; ⌣; const; ⌣

7°



$\beta = \mathrm{arctg}\, 4$
$\alpha = \mathrm{arctg}\, 2$

3° НУЛЕ И ЗНАК

$\forall x \in (-\infty, -1]$

$f(x) > 0,\ x > -1$
$f(x) = 0,\ x \leq -1$

• $f(x) = \sqrt{1+x^2} + \ln \dfrac{1-\sqrt{x^2+1}}{x}$

1° D_f: $x \neq 0$ и $\underbrace{\dfrac{1-\sqrt{x^2+1}}{x}}_{<0 \text{ увек за } x\neq 0} > 0 \Rightarrow x < 0$

$\boxed{D_f = (-\infty, 0)}$

2° //

3° знак, нуле ? оставимо за касније :)

4° АСИМПТОТИКА:

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}\Big(\underbrace{\sqrt{1+x^2}}_{\to 1} + \overbrace{\ln \underbrace{\frac{1-\sqrt{x^2+1}}{x}}_{\to 0}}^{\to -\infty}\Big) = -\infty$$

јер: $$\frac{1-\sqrt{x^2+1}}{x} = -\underbrace{\frac{(1+x^2)^{1/2}-1}{x^2}}_{\to 1/2,\ x\to 0}\cdot\underbrace{(-x)}_{\to 0} \to 0$$

$\Rightarrow$ $\boxed{x=0 \text{ В.А. са леве стране}}$

$-\infty$: $$f(x) = \sqrt{1+x^2} + \ln\Big(\frac{1}{x} - \frac{\sqrt{x^2+1}}{x}\Big) = \sqrt{x^2\big(1+\tfrac{1}{x^2}\big)} + \ln\Big(\frac{1}{x} + \sqrt{1+\frac{1}{x^2}}\Big) =$$

унутра $x<0$! $|x| = -x$, $x<0$ $\big(1+\frac{1}{x^2}\big)^{1/2}$

$$= -x\cdot\Big(1+\frac{1}{x^2}\Big)^{1/2} + \ln\Big(\frac{1}{x} + 1 + \frac{1}{2}\cdot\frac{1}{x^2} + o\Big(\frac{1}{x^2}\Big)\Big) = -x\cdot\Big(1+\frac{1}{2}\cdot\frac{1}{x^2} + o\Big(\frac{1}{x^2}\Big)\Big) + \ln\Big(1+\underbrace{\frac{1}{x}+\frac{1}{2x^2}+o\Big(\frac{1}{x^2}\Big)}\Big)$$

$$= -x - \frac{1}{2x} + o\Big(\frac{1}{x}\Big) + \underbrace{\frac{1}{x} + \frac{1}{2x^2} + o\Big(\frac{1}{x^2}\Big)}_{t} + \underbrace{o\Big(\frac{1}{x}\Big)}_{o(t)} = -x + \frac{1}{2}\cdot\underbrace{\frac{1}{x}}_{<0} + o\Big(\frac{1}{x}\Big)$$

$\Rightarrow$ $\boxed{y=-x \text{ је К.А., } x\to-\infty}$, Γ_f испод ње

5° $f'(x) = \frac{1}{2} \cdot \frac{2x}{\sqrt{1+x^2}} + \frac{1}{\frac{1-\sqrt{x^2+1}}{x}} \cdot \frac{-\frac{1}{2}\frac{2x}{\sqrt{1+x^2}} \cdot x - (1-\sqrt{1+x^2}) \cdot 1}{x^2}$, $x \in (-\infty, 0)$

$$= \frac{x}{\sqrt{1+x^2}} + \frac{\frac{-x^2}{\sqrt{1+x^2}} - 1 + \sqrt{1+x^2}}{x \cdot (1-\sqrt{1+x^2})} = \frac{x}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+x^2}} \cdot \frac{\cancel{-x^2} - \sqrt{1+x^2} + 1 \cancel{+ x^2}}{x(1-\sqrt{1+x^2})}$$

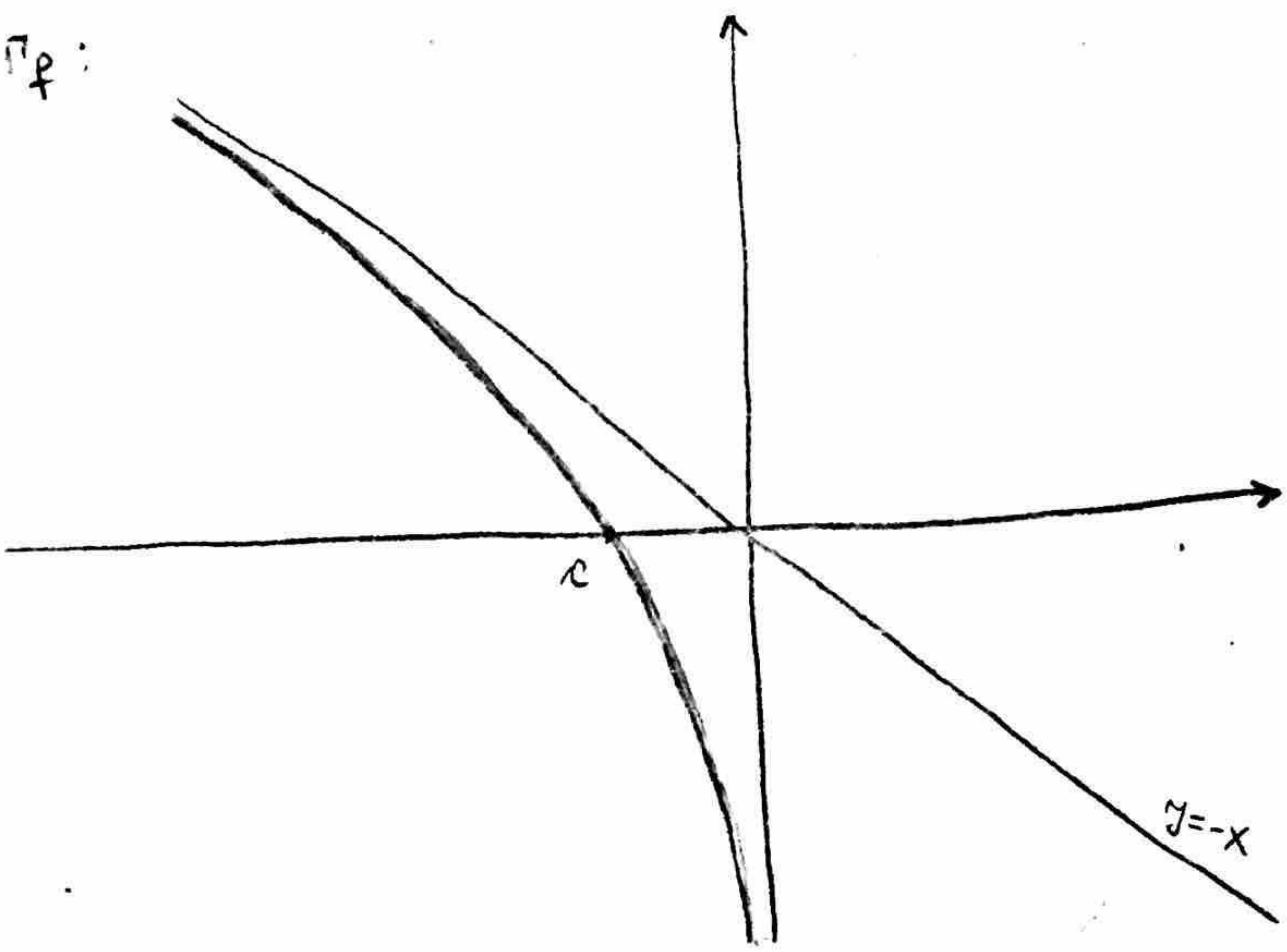
$$= \frac{x}{\sqrt{1+x^2}} + \frac{1}{x\sqrt{1+x^2}} = \frac{x^2+1}{x\sqrt{1+x^2}} = \frac{\sqrt{x^2+1}}{x}$$

$f'(x) = \frac{\sqrt{x^2+1}}{x} \underset{\text{на } D_f}{<} 0 \quad \Rightarrow \quad f \searrow$ на $(-\infty, 0)$

6° $f''(x) = \frac{\frac{1}{2} \cdot \frac{2x}{\sqrt{x^2+1}} \cdot x - \sqrt{x^2+1}}{x^2} = \frac{x^2 - (x^2+1)}{x^2\sqrt{x^2+1}} = \frac{-1}{x^2\sqrt{x^2+1}} < 0$ на $(-\infty, 0)$

$\Rightarrow f: \cap$

7° Γ_f :



3° нуле, знак:

$f \searrow$; $f \xrightarrow[x \to 0]{} -\infty$; $f \sim -x, \; x \to -\infty$

$\Rightarrow \exists 1\, c, \; f(c) = 0$

$f(x) < 0, \; x \in (c, 0)$

$f(x) > 0, \; x \in (-\infty, 0)$