

$$\frac{\alpha \wedge \beta}{\alpha} (\wedge^L_E)$$

$$\frac{\alpha \wedge \beta}{\beta} (\wedge^D_E)$$

$$\frac{\alpha \quad \beta}{\alpha \wedge \beta} (\wedge^U)$$

$$\frac{\neg\neg\alpha}{\alpha} (\neg\neg^E)$$

$$\frac{\alpha}{\neg\neg\alpha} (\neg\neg^U)$$

$$\frac{\alpha \Rightarrow \beta \quad \alpha}{\beta} (\Rightarrow^E)$$

$$\frac{\alpha \Rightarrow \beta \quad \neg\beta}{\neg\alpha} (\text{MT})$$

$$\frac{\begin{array}{|l} \alpha \\ \vdots \\ \beta \end{array}}{\alpha \Rightarrow \beta} (\Rightarrow^U)$$

$$\frac{\alpha \Rightarrow \beta \quad \beta \Rightarrow \alpha}{\alpha \Leftrightarrow \beta} (\Leftrightarrow^U)$$

$$\frac{\alpha \Leftrightarrow \beta}{\alpha \Rightarrow \beta} (\Leftrightarrow^L_D)$$

$$\frac{\alpha \Leftrightarrow \beta}{\beta \Rightarrow \alpha} (\Leftrightarrow^D_L)$$

$$\frac{\alpha \quad \neg\alpha}{\perp} (\neg^E)$$

$$\frac{\begin{array}{|l} \alpha \\ \vdots \\ \perp \end{array}}{\neg\alpha} (\neg^U)$$

$$\frac{\perp}{\alpha} (\perp^E)$$

$$\frac{\alpha}{\alpha \vee \beta} (\vee^L_U)$$

$$\frac{\beta}{\alpha \vee \beta} (\vee^D_U)$$

$$\frac{\alpha \vee \beta \quad \begin{array}{|l} \alpha \\ \vdots \\ \gamma \end{array} \quad \begin{array}{|l} \beta \\ \vdots \\ \gamma \end{array}}{\gamma} (\vee^E)$$

$$\frac{\alpha \vee \beta \quad \neg\alpha}{\beta} (\text{DS})$$

$$\frac{\alpha \vee \beta \quad \neg\beta}{\alpha} (\text{DS})$$

$$\frac{\neg(\alpha \Rightarrow \beta)}{\alpha} (\text{NI})$$

$$\frac{\neg(\alpha \Rightarrow \beta)}{\neg\beta} (\text{NI})$$

$$\frac{\alpha \Rightarrow \beta \quad \beta \Rightarrow \gamma}{\alpha \Rightarrow \gamma} (\text{T})$$

$$\frac{\alpha \Rightarrow \beta}{\neg\beta \Rightarrow \neg\alpha} (\text{K})$$

$$\frac{\neg\alpha \Rightarrow \neg\beta}{\beta \Rightarrow \alpha} (\text{K})$$

$$\frac{}{\alpha \vee \neg\alpha} (\text{TND})$$

$$\frac{\neg\alpha \vee \neg\beta}{\neg(\alpha \wedge \beta)} (\text{DM})$$

$$\frac{\neg(\alpha \wedge \beta)}{\neg\alpha \vee \neg\beta} (\text{DM})$$

$$\frac{\neg\alpha \wedge \neg\beta}{\neg(\alpha \vee \beta)} (\text{DM})$$

$$\frac{\neg(\alpha \vee \beta)}{\neg\alpha \wedge \neg\beta} (\text{DM})$$

$$\frac{\forall x\alpha}{\alpha[v/x]} (\forall^E)$$

$$\frac{\begin{array}{|l} v \\ \vdots \\ \alpha[v/x] \end{array}}{\forall x\alpha} (\forall^U)$$

$$\frac{\alpha[v/x]}{\exists x\alpha} (\exists^E)$$

$$\frac{\exists x\alpha \quad \begin{array}{|l} v \quad \alpha[v/x] \\ \vdots \\ \gamma \end{array}}{\gamma} (\exists^U)$$

$$\frac{\neg\forall x\alpha}{\exists x\neg\alpha} (\text{DM})$$

$$\frac{\neg\exists x\alpha}{\forall x\neg\alpha} (\text{DM})$$