



ZERO KNOWLEDGE PROOFS

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ZKP Course

Class 3: ZK SNARKs & ZK STARKs



What is a SNARK?

SNARK - **Succinct** Non-Interactive Argument of Knowledge;

SNARK: the proof is “short” and fast to “verify”;



You can find more information on these links:

[Link 1 >](#)

[Link 2 >](#)

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SNARK: the proof is “short” and fast to “verify”;

Example: I know an w such that $\text{SHA256}(w)=y$, where w is 1 GB.

Proof: few kB; Time to verify: few ms;

Proof size: $O(\log|C|, \lambda)$ Time to verify: $O(|y|, \log|C|, \lambda)$

There's More 

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ZK SNARK = Zero Knowledge + SNARK;



There's More 

What is a STARK?

STARK - Scalable **Transparent** Arguments of Knowledge.

STARK: no trusted setup;

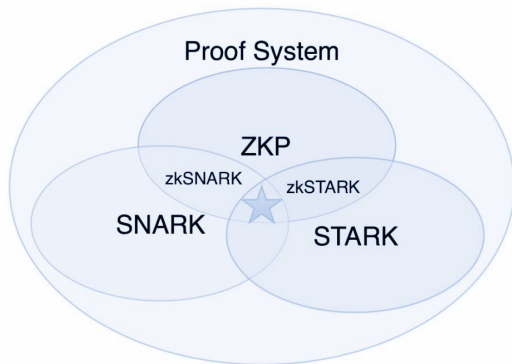
There's More 

What is a STARK?

STARK - Scalable **Transparent** Arguments of Knowledge.

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ZK SNARK = Zero Knowledge + SNARK;



	Proof size (bytes)	Verification time (ms)
Groth 16	200	3
PLONK	400	6
STARK	80*1024	10

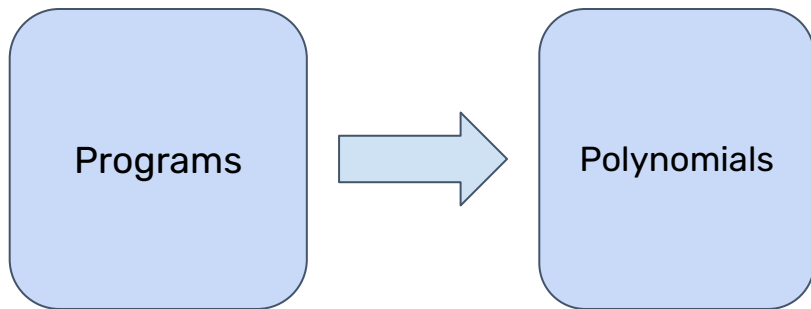


You can find more information on this link:

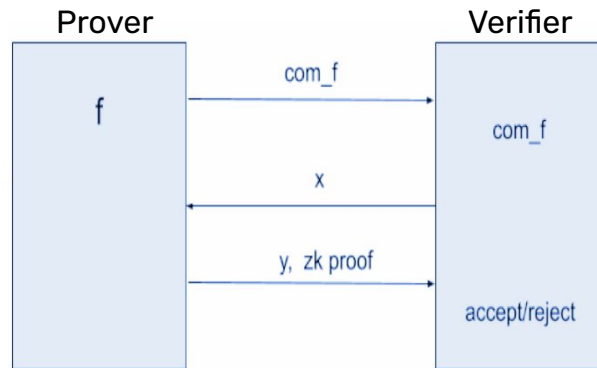
[Link 1 >](#)

Building zk SNARKs & zk STARKs

Arithmetization



Polynomial
Commitments



There's More 

Arithmetization

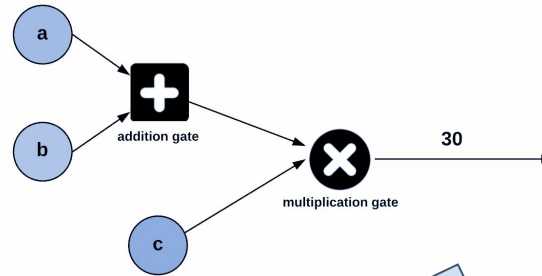
R1CS - Rank-1 Constraint Systems

AIR - Algebraic Intermediate Representation

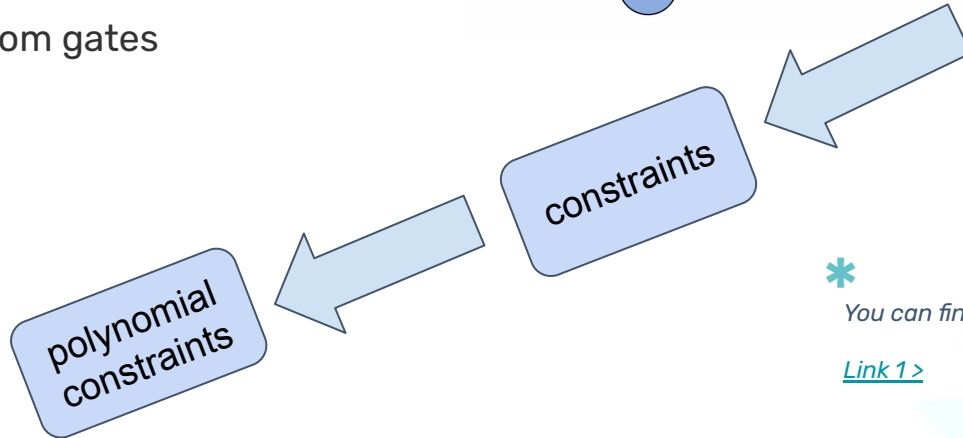
PLONK CG - PLONK custom gates

Arithmetic circuits

Execution trace



0	0	1	0	1	1
0	1	0	1	1	0
1	0	1	1	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	1	0	1	0	0
1	0	1	0	0	0
0	0	0	0	1	0
0	0	0	1	0	0
0	0	1	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0

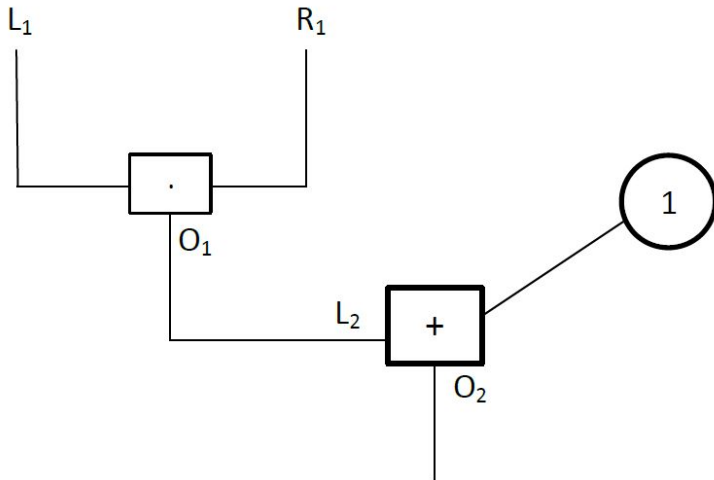


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[Link 1 >](#)

Arithmetic circuits

Example 1: I know an a such that $a \cdot a + 1 = b$, for given b .



Arithmetic circuit $C: F^n \rightarrow F$.

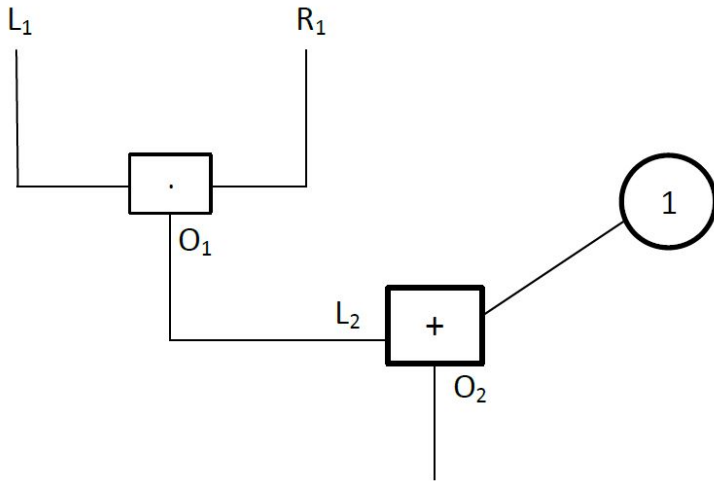
$|C|$ - number of gates in C

Example 2: $C_{\text{SHA256}}(w, y) = y - \text{SHA256}(w)$, $|C| \approx 20\,000$ gates

There's More 

Arithmetic circuits + system constraints

Example: I know an a such that $a \cdot a + 1 = b$, for given b .



System constraints:

Gate constraints:

- (1) $L_1 \cdot R_1 - O_1 = 0$
- (2) $L_2 + 1 - O_2 = 0$

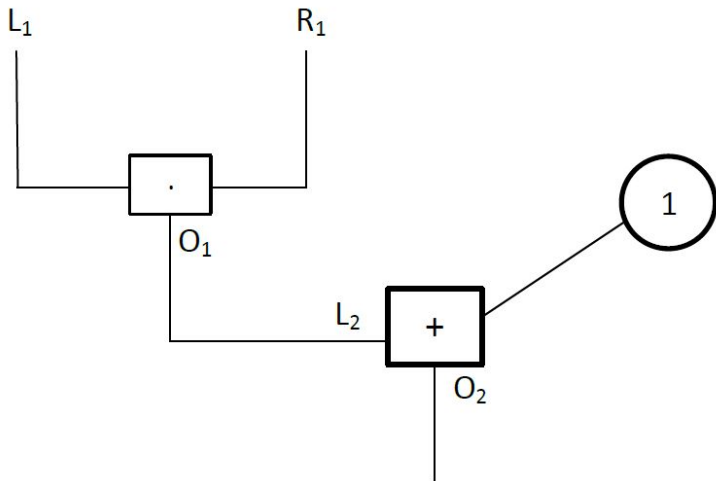
Copy constraints:

$$L_1 = R_1$$
$$O_1 = L_2$$

There's More 

Arithmetic circuits + system constraints

Example: I know an a such that $a \cdot a + 1 = b$, for given b .



System constraints:

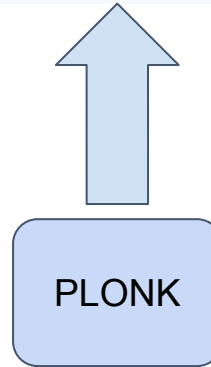
Gate constraints:

- (1) $L_1 \cdot R_1 - O_1 = 0$
- (2) $L_2 + 1 - O_2 = 0$

Copy constraints:

$$\begin{aligned} L_1 &= R_1 \\ O_1 &= L_2 \end{aligned}$$

$$L_i \cdot q_{L_i} + R_i \cdot q_{R_i} + O_i \cdot q_{O_i} + q_{C_i} + L_i \cdot R_i \cdot q_{M_i} = 0.$$

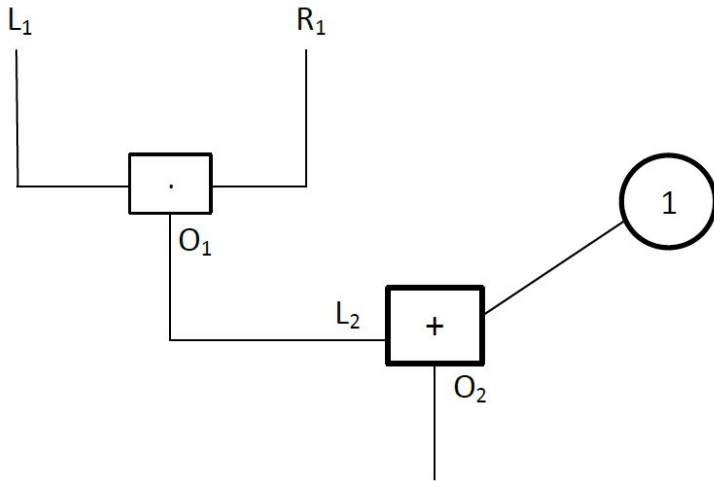


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Arithmetic circuits + polynomial constraints

Example: I know an a such that $a \cdot a + 1 = b$, for given b .



System constraints:

Gate constraints:

- (1) $L_1 \cdot R_1 - O_1 = 0$
- (2) $L_2 + 1 - O_2 = 0$

Copy constraints:

$$\begin{aligned} L_1 &= R_1 \\ O_1 &= L_2 \end{aligned}$$

$$L_i \cdot q_{L_i} + R_i \cdot q_{R_i} + O_i \cdot q_{O_i} + q_{C_i} + L_i \cdot R_i \cdot q_{M_i} = 0.$$

Define $L(x)$, $R(x)$, $O(x)$:

$$L(1) = L_1, L(2) = L_2, R(1) = R_1, R(2) = R_2, O(1) = O_1, O(2) = O_2.$$

$$f(x) = L(x) \cdot q_L(x) + R(x) \cdot q_R(x) + O(x) \cdot q_O(x) + q_C(x) + L(x) \cdot R(x) \cdot q_M(x).$$

Execution trace + constraints

Example: States for bill.

item	price	running total
Avocado	\$4.98	\$0.00
Apple	\$7.98	\$4.98
Milk	\$3.45	\$12.96
Bread	\$2.65	\$16.41
Brown Sugar	\$1.40	\$19.06
<hr/>		
total	\$20.46	\$20.46

Zoom in:

Milk	\$3.45	\$12.96
Bread	\$2.65	\$16.41

There's More 

Execution trace + constraints

Example: States for bill.

Avocado	4.98	0
Apple	7.98	4.98
Milk	3.45	12.96
Bread	2.65	16.41
Brown Sugar	1.40	19.06
Total	20.46	20.46

Constraints:

- 1) $A_{0,2} = 0$ // We start the running total from 0.
- 2) $\forall 1 \leq i \leq 5 : A_{i,2} - A_{i-1,2} - A_{i-1,1} = 0$ // Each row's running total is correct.
- 3) $A_{5,1} - A_{5,2} = 0$ // The last running total is the total sum.



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Thank you!