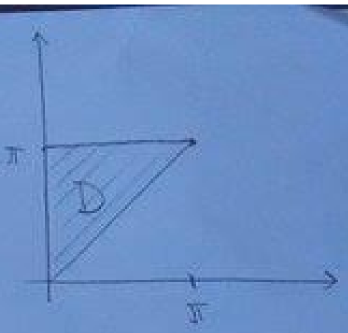
$\boxed{1}$ a) $\int_0^{\pi}\int_x^{\pi} \frac{\sin y}{y}\, dy\, dx = \int_0^{\pi}\left(\int_0^{y} \frac{\sin y}{y}\, dx \right) dy$

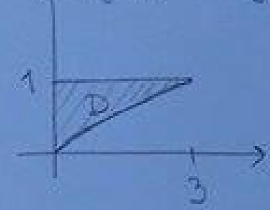


$D = \{(x,y) \in \mathbb{R}^2 \mid 0 \le x \le \pi,\ x \le y \le \pi\}$

$$= \int_0^{\pi} \frac{\sin y}{y} \cdot (y-0)\, dy$$

$$= \int_0^{\pi} \sin y\, dy = -\cos y \Big|_0^{\pi} = 2$$

b) $\int_0^{3}\int_{\sqrt{\frac{x}{3}}}^{1} e^{y^3}\, dy\, dx = \int_0^{1}\left(\int_0^{3y^2} e^{y^3}\, dx \right) dy$



$D = \{(x,y) \in \mathbb{R}^2 \mid 0 \le x \le 3,\ \sqrt{\frac{x}{3}} \le y \le 1\}$

$$= \int_0^{1} e^{y^3} \cdot (3y^2 - 0)\, dy = 3 \cdot \int_0^{1} y^2 e^{y^3}\, dy$$

$t = y^3 \to dt = 3y^2 dy$

$$= \int_0^{1} e^t\, dt = e^t \Big|_0^1 = e - 1$$

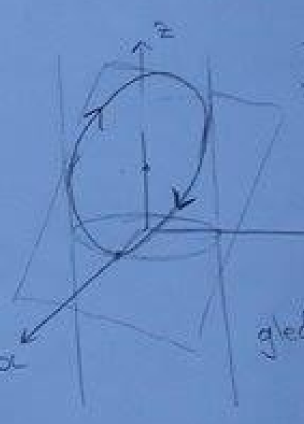
$\boxed{2}$ $\int_{\gamma} F \cdot dr$, $F(x,y,z) = (x,y,y)$

γ: presek $x^2+y^2=4$ i $x+z=2$

⟲ iz $(0,0,0)$

$x^2+y^2=4$ cilindar

$x+z=2$ ravan



Parametrizacija krive γ:

$r(t) = (2\cos t, 2\sin t, 2-2\cos t)$, $-\pi \le t \le \pi$

jer je $x^2+y^2=4$ $\quad$ $z = 2-x$

gledamo krivu odozdo

$r'(t) = (-2\sin t, 2\cos t, 2\sin t)$

$$\int_{\gamma} F \cdot dr = (-)\int_{-\pi}^{\pi} F(r(t)) \cdot r'(t)\, dt = -\int_{-\pi}^{\pi} (2\cos t, 2\sin t, 2\sin t) \cdot (-2\sin t, 2\cos t, 2\sin t)\, dt$$

$$= -\int_{-\pi}^{\pi} (\cancel{-4\sin t\cos t} + \cancel{4\sin t\cos t} + 4\sin^2 t)\, dt = -4 \cdot \int_{-\pi}^{\pi} \frac{1-\cos 2t}{2}\, dt =$$

$$= -2\left[2\pi - \frac{\sin 2t}{2}\Big|_{-\pi}^{\pi}\right] = -4\pi$$

$\boxed{3}$ $\iint_S x^2\sqrt{5-4z}\, dS$, $S: z = 1-x^2-y^2$ iznad $z=0$ ravni

Parametrizacija površi S: $r(x,y) = (x, y, 1-x^2-y^2)$, $(x,y) \in D$

$D: x^2+y^2 \le 1$

$$\|r'_x \times r'_y\| = \sqrt{1 + (z'_x)^2 + (z'_y)^2}$$

$$= \sqrt{1 + 4x^2 + 4y^2}$$

$$\iint_S x^2\sqrt{5-4z}\,dS = \iint_D x^2\sqrt{5-4(1-x^2-y^2)}\cdot\sqrt{1+4x^2+4y^2}\,dx\,dy$$

$$= \iint_D x^2(1+4x^2+4y^2)\,dx\,dy$$

$$\begin{bmatrix} x=\rho\cos\theta & 0\le\rho\le 1 \\ y=\rho\sin\theta & -\pi\le\theta\le\pi \\ J=\rho & \end{bmatrix} = \int_{-\pi}^{\pi}\left(\int_0^1 \rho^2\cos^2\theta\,(1+4\rho^2)\,\rho\,d\rho\right)d\theta$$

$$= \int_{-\pi}^{\pi}\overset{\frac{1+\cos 2\theta}{2}}{\overset{\parallel}{\cos^2\theta}}\,d\theta\cdot\int_0^1(\rho^3+4\rho^5)\,d\rho \qquad \frac{1}{4}+\frac{2}{3}=\frac{3+8}{12}$$

$$= \frac{1}{2}\left[2\pi+\underbrace{\frac{\sin 2\theta}{2}\Big|_{-\pi}^{\pi}}_{0}\right]\cdot\left[\frac{1}{4}+4\cdot\frac{1}{6}\right]$$

$$= \pi\cdot\frac{11}{12}$$

$\boxed{4.}$ $(2xy+4x^3)\,dy+(y^2+12x^2y)\,dx=0$ $\leftarrow$ prevedemo na ovaj oblik

$M(x,y)=y^2+12x^2y \Rightarrow M'_y=2y+12x^2$

$N(x,y)=2xy+4x^3 \Rightarrow N'_x=2y+12x^2$

Dakle, radi se o dif. jednačini sa totalnim diferencijalom.

$(\exists F(x,y))\quad F'_x=M(x,y)\ \wedge\ F'_y=N(x,y)$

- $F'_x=y^2+12x^2y \ /\int dx$

  $F(x,y)+\varphi(y)=y^2\cdot x+12y\cdot\frac{x^3}{3}=y^2x+4yx^3 \Rightarrow F'_y=2yx+4x^3-\varphi'(y)$

- $F'_y=2xy+4x^3$

  $2yx+4x^3-\varphi'(y)=2xy+4x^3$

  $\varphi'(y)=0$

  $\varphi(y)=\text{const.}$

Opšte rešenje jednačine je:

$F(x,y)=C$, tj.

$\underline{y^2x+4yx^3=C}$