(1) Polazeći od osn. kin. jdne u fizici plazme → ZO naelektrisanja

ZON: $\frac{\partial \rho^{el}_\alpha}{\partial t} + \nabla \cdot \vec{j}_\alpha = 0$

• postupak kao u prethodnom zadatku (za jdnu kontinuiteta)

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \vec{u}_\alpha) = 0 \quad / \cdot q_\alpha \qquad \rho^{el}_\alpha = q_\alpha \cdot n_\alpha$$

$$\frac{\partial \rho^{el}_\alpha}{\partial t} + \nabla \cdot (\rho^{el} \vec{u}_\alpha) = 0 \qquad \vec{j}_\alpha = \rho^{el}_\alpha \cdot \vec{u}_\alpha$$

$$\frac{\partial \rho^{el}_\alpha}{\partial t} + \nabla \cdot \vec{j}_\alpha = 0$$

• Napomena: imamo makroskopsku neutralnost:

$$n_e \approx n_p \rightarrow \rho^{el} \approx 0 \qquad \nabla \cdot \vec{j} \approx 0$$

$$\rightarrow \nabla \times \vec{B} = \mu_0 \vec{j} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} \qquad / \nabla \cdot$$

$$\underbrace{\nabla \cdot (\nabla \times \vec{B})}_{div(rot \vec{B}) = 0} = \mu_0 \nabla \cdot \vec{j} + \frac{1}{c^2} \frac{\partial}{\partial t} (\nabla \cdot \vec{E}) \qquad \frac{\rho^{el}}{\varepsilon_0}$$

$$0 = \mu_0 \nabla \cdot \vec{j} + (\approx 0) \Rightarrow \nabla \cdot \vec{j} \approx 0$$

(2) Osnovna kin. jdna u fiz. plazme → jdna kretanja (ZOI)

(ZOI) jdna kretanja: $\rho \frac{d\vec{u}}{dt} = - \nabla \cdot \hat{P} + \vec{j} \times \vec{B}$

$$\frac{\partial f_\alpha}{\partial t} + (\vec{v} \cdot \nabla f_\alpha) + \underbrace{\frac{q_\alpha}{m_\alpha} (\vec{E} + \vec{v} \times \vec{B})}_{\vec{a}} \cdot \nabla_{\vec{v}} f_\alpha = I_\alpha \qquad / \cdot \int_{V_{\vec{v}}} m_\alpha \vec{v} \, d^3\vec{v}$$

$$\underbrace{m_\alpha \int_{V_{\vec{v}}} \frac{\partial f_\alpha}{\partial t} \vec{v}\, d^3\vec{v}}_{I_{21}} + \underbrace{m_\alpha \int_{V_{\vec{v}}} (\vec{v}\cdot\nabla f_\alpha)\vec{v}\, d^3\vec{v}}_{I_{22}} + \underbrace{\int_{V_{\vec{v}}} m_\alpha(\vec{a}\cdot\nabla_{\vec{v}} f_\alpha)\vec{v}\, d^3\vec{v}}_{I_{23}} = \underbrace{m_\alpha \int_{V_{\vec{v}}} I_\alpha \vec{v}\, d^3\vec{v}}_{\vec{C}_{2,\alpha}}$$

$$I_{21} = m_\alpha \int_{V_{\vec{v}}} \frac{\partial f_\alpha}{\partial t} \vec{v}\, d^3\vec{v} = \frac{\partial}{\partial t}\left(\int_{V_{\vec{v}}} m_\alpha f_\alpha \vec{v}\, d^3\vec{v}\right) = \frac{\partial}{\partial t}\left(m_\alpha \langle \vec{v}\rangle_\alpha n_\alpha\right) =$$

$$= \frac{\partial}{\partial t}\left(m_\alpha n_\alpha \vec{u}_\alpha\right) = \boxed{\frac{\partial}{\partial t}(\rho_\alpha \vec{u}_\alpha)}$$

$$I_{22} = m_\alpha \int_{V_{\vec{v}}} (\vec{v}\cdot\nabla f_\alpha)\vec{v}\, d^3\vec{v} \; \ominus$$

$$\otimes\; \vec{A}(\vec{B}\cdot\vec{C}) = (\vec{A}\otimes\vec{B})\cdot\vec{C}$$

$$\nabla\cdot(\psi\, \vec{e}\otimes\vec{c}) = \nabla\psi\cdot(\vec{e}\otimes\vec{c}) + \psi\nabla\cdot(\vec{e}\otimes\vec{c})$$

$$\vec{e}\cdot(\nabla\psi\otimes\vec{c}) = (\vec{e}\cdot\nabla\psi)\vec{c} = (\nabla\psi\cdot\vec{e})\vec{c} = \nabla\psi\cdot(\vec{e}\otimes\vec{c})$$

$$\nabla\cdot(\psi\, \vec{e}\otimes\vec{c}) = \vec{e}\cdot(\nabla\psi\otimes\vec{c}) + \psi\nabla\cdot(\vec{e}\otimes\vec{c})$$

$$\boxed{\vec{e}\cdot(\nabla\psi\otimes\vec{c}) = \nabla\cdot(\psi\, \vec{e}\otimes\vec{c}) - \psi\nabla\cdot(\vec{e}\otimes\vec{c})}$$

$$\ominus\; m_\alpha \int_{V_{\vec{v}}} \left[\nabla\cdot(f_\alpha \vec{v}\otimes\vec{v}) - f_\alpha\nabla\cdot(\vec{v}\otimes\vec{v})\right] d^3\vec{v} =$$

$$= \int_{V_{\vec{v}}} \nabla\cdot(m_\alpha f_\alpha \vec{v}\otimes\vec{v})\, d^3\vec{v} - \int_{V_{\vec{v}}} m_\alpha f_\alpha \nabla\cdot(\vec{v}\otimes\vec{v})\, d^3\vec{v} =$$

$\nabla \equiv \nabla_{\vec{r}}$ $\qquad$ $\nabla_{\vec{r}}\cdot(\vec{v}\otimes\vec{v}) = \vec{0}$

$$= \nabla \cdot \int_{V_{\vec{v}}} m_\alpha f_\alpha \, \vec{v} \otimes \vec{v} \, d^3\vec{v} = \boxed{\nabla \cdot \left(m_\alpha n_\alpha \langle \vec{v} \otimes \vec{v} \rangle_\alpha \right)}$$

$$I_{23} = m_\alpha \int_{V_{\vec{v}}} (\vec{a} \cdot \nabla_{\vec{v}} f_\alpha) \vec{v} \, d^3\vec{v} =$$

$$m_\alpha \vec{a} \equiv \vec{F}$$

$$(\vec{F} \cdot \nabla_{\vec{v}} f_\alpha) \vec{v} = \nabla_{\vec{v}} f_\alpha (\vec{F} \otimes \vec{v})$$

$$\nabla_{\vec{v}} \left(f_\alpha \vec{F} \otimes \vec{v} \right) = \nabla_{\vec{v}} f_\alpha \cdot (\vec{F} \otimes \vec{v}) + f_\alpha \nabla \cdot (\vec{F} \otimes \vec{v})$$

$$\Rightarrow (\vec{F} \cdot \nabla_{\vec{v}} f_\alpha) \vec{v} = \nabla_{\vec{v}} (f_\alpha \vec{F} \otimes \vec{v}) - f_\alpha \nabla \cdot (\vec{F} \otimes \vec{v})$$

$$= \underbrace{\int_{V_{\vec{v}}} \nabla_{\vec{v}} (f_\alpha \vec{F} \otimes \vec{v}) d^3\vec{v}} - \int_{V_{\vec{v}}} f_\alpha \nabla_{\vec{v}} \cdot (\vec{F} \otimes \vec{v}) d^3\vec{v} =$$

$$\int_V \nabla \cdot \vec{A} \, dV = \oint_{\partial V = S_V} \vec{A} \cdot d\vec{S} \qquad + \quad |\vec{v}| \to \infty \Rightarrow f_\alpha \to 0 \Rightarrow 0$$

$$= -\int_{V_{\vec{v}}} f_\alpha \underbrace{\nabla_{\vec{v}} \cdot (\vec{F} \otimes \vec{v})} d^3\vec{v} =$$

$$\nabla \cdot (\vec{A} \otimes \vec{B}) = (\nabla \cdot \vec{A}) \vec{B} + (\vec{A} \cdot \nabla) \vec{B} = \vec{B} (\nabla \cdot \vec{A}) + \vec{A} \cdot (\nabla \otimes \vec{B})$$

$$= -\int_{V_{\vec{v}}} f_\alpha \left[\underbrace{\vec{v} (\nabla_{\vec{v}} \cdot \vec{F})}_{1} + \underbrace{\vec{F} \cdot (\nabla_{\vec{v}} \otimes \vec{v})}_{2} \right] d^3\vec{v} =$$

① $\vec{v}\,(\nabla_{\vec{v}} \cdot \vec{F})$

$$\nabla_{\vec{v}} \cdot \vec{F} = m_\alpha \nabla_{\vec{v}} \cdot \vec{a} = q_\alpha \nabla_{\vec{v}} \cdot (\vec{E} + \vec{v} \times \vec{B})$$

$$\nabla_{\vec{v}} \cdot \vec{E}(\vec{r},t) = 0$$

$$\nabla_{\vec{v}} \cdot (\vec{v} \times \vec{B}) = \dots = \vec{0}$$

② $\nabla_{\vec{v}} \otimes \vec{v} =$

$$= \begin{bmatrix} \frac{\partial}{\partial v_x} \\ \frac{\partial}{\partial v_y} \\ \frac{\partial}{\partial v_z} \end{bmatrix} \begin{bmatrix} v_x & v_y & v_z \end{bmatrix} = \begin{bmatrix} \partial_{v_x} v_x & \partial_{v_x} v_y & \partial_{v_x} v_z \\ \partial_{v_y} v_x & \partial_{v_y} v_y & \partial_{v_y} v_z \\ \partial_{v_z} v_x & \partial_{v_z} v_y & \partial_{v_z} v_z \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \hat{I}$$

$$\vec{F} \cdot (\nabla_{\vec{v}} \otimes \vec{v}) = \underbrace{\begin{bmatrix} F_x & F_y & F_z \end{bmatrix}}_{1 \times 3} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{3 \times 3} = \begin{bmatrix} F_x & F_y & F_z \end{bmatrix} = \vec{F}$$

$$= -\int_{V_{\vec{v}}} f_\alpha \vec{F}\, d^3\vec{v} = -\int_{V_{\vec{v}}} f_\alpha q_\alpha (\vec{E} + \vec{v} \times \vec{B})\, d^3\vec{v} =$$

$$= -q_\alpha \vec{E} \underbrace{\int_{V_{\vec{v}}} f_\alpha\, d^3\vec{v}}_{n_\alpha} - q_\alpha \Big(\underbrace{\int_{V_{\vec{v}}} f_\alpha \vec{v}\, d^3\vec{v}}_{n_\alpha \vec{u}_\alpha} \Big) \times \vec{B} =$$

$$= -q_\alpha \vec{E} n_\alpha - q_\alpha n_\alpha \vec{u}_\alpha \times \vec{B} = -q_\alpha n_\alpha \left(\vec{E} + \vec{u}_\alpha \times \vec{B} \right)$$

$\Rightarrow$ Konačno: $I_{21} + I_{22} + I_{23} = \vec{C}_{2,\alpha}$

$$\frac{\partial}{\partial t}(\rho_\alpha \vec{u}_\alpha) + \nabla \cdot \left(\rho_\alpha \langle \vec{v} \otimes \vec{v} \rangle_\alpha \right) - q_\alpha n_\alpha \left(\vec{E} + \vec{u}_\alpha \times \vec{B} \right) = \vec{C}_{2\alpha}$$

$\rho_\alpha^{el} = q_\alpha n_\alpha \qquad \rho_\alpha^{el} \vec{u}_\alpha = \vec{j}_\alpha$

$$\underbrace{\frac{\partial}{\partial t}(\rho_\alpha \vec{u}_\alpha)}_{I_{31}} + \underbrace{\nabla \cdot \left(\rho_\alpha \langle \vec{v} \otimes \vec{v} \rangle_\alpha \right)}_{I_{32}} - \rho_\alpha^{el} \vec{E} - \vec{j}_\alpha \times \vec{B} = \vec{C}_{2,\alpha}$$

$$I_{31} = \frac{\partial}{\partial t}\left(\rho_\alpha \vec{u}_\alpha \right) = \frac{\partial \rho_\alpha}{\partial t} \vec{u}_\alpha + \rho_\alpha \frac{\partial \vec{u}_\alpha}{\partial t} =$$

jdne kontinuiteta: $\dfrac{\partial \rho_\alpha}{\partial t} + \nabla \cdot \left(\rho_\alpha \vec{u}_\alpha \right) = C_{1,\alpha} \quad / \cdot \vec{u}_\alpha$

$$\vec{u}_\alpha \frac{\partial \rho_\alpha}{\partial t} = \vec{u}_\alpha C_{1\alpha} - \vec{u}_\alpha \left(\nabla \cdot (\rho_\alpha \vec{u}_\alpha) \right)$$

$$= \vec{u}_\alpha C_{1\alpha} - \vec{u}_\alpha \left(\nabla \cdot (\rho_\alpha \vec{u}_\alpha) \right) + \rho_\alpha \frac{\partial \vec{u}_\alpha}{\partial t}$$

$$I_{32} = \nabla \cdot \left(\rho_\alpha \langle \vec{v} \otimes \vec{v} \rangle_\alpha \right) \ominus$$

$$\vec{v} = \vec{u}_\alpha + \vec{w}_\alpha \qquad \langle \vec{w}_\alpha \rangle_\alpha = 0$$

$$\langle \vec{v} \rangle_\alpha = \vec{u}_\alpha \qquad \langle \vec{u}_\alpha \rangle_\alpha = \vec{u}_\alpha$$

$$\langle \vec{v} \otimes \vec{v} \rangle_\alpha = \langle (\vec{u}_\alpha + \vec{w}_\alpha) \otimes (\vec{u}_\alpha + \vec{w}_\alpha) \rangle_\alpha =$$

$$= \langle \vec{u}_\alpha \otimes \vec{u}_\alpha \rangle_\alpha + \langle \vec{u}_\alpha \otimes \vec{w}_\alpha \rangle_\alpha + \langle \vec{w}_\alpha \otimes \vec{u}_\alpha \rangle_\alpha + \langle \vec{w}_\alpha \otimes \vec{w}_\alpha \rangle_\alpha$$

$$= \vec{u}_\alpha \otimes \vec{u}_\alpha + 0 + 0 + \langle \vec{w}_\alpha \otimes \vec{w}_\alpha \rangle_\alpha$$

$$= \nabla \cdot \left(\rho_\alpha \vec{u}_\alpha \otimes \vec{u}_\alpha + \rho_\alpha \langle \vec{w}_\alpha \otimes \vec{w}_\alpha \rangle_\alpha \right) =$$

$$= \nabla \cdot \left(\underbrace{(\rho_\alpha \vec{u}_\alpha) \otimes \vec{u}_\alpha}_{(*)} \right) + \nabla \cdot \underbrace{\left(\rho_\alpha \langle \vec{w}_\alpha \otimes \vec{w}_\alpha \rangle_\alpha \right)}_{\hat{P}_\alpha} \ominus$$

$(*)$ $$\nabla \cdot \left((\rho_\alpha \vec{u}_\alpha) \otimes \vec{u}_\alpha \right) = \vec{u}_\alpha \left(\nabla \cdot (\rho_\alpha \vec{u}_\alpha) \right) + \rho_\alpha \vec{u}_\alpha \cdot \left(\nabla \otimes \vec{u}_\alpha \right)$$

$$\nabla \cdot (\vec{A} \otimes \vec{B}) = \vec{B} (\nabla \cdot \vec{A}) + \vec{A} \cdot (\nabla \otimes \vec{B})$$

$$\ominus\, \vec{u}_\alpha \left(\nabla \cdot (\rho_\alpha \vec{u}_\alpha) \right) + \rho_\alpha \vec{u}_\alpha \cdot (\nabla \otimes \vec{u}_\alpha) + \nabla \cdot \hat{P}_\alpha$$

iz $(*)$ $\Rightarrow$ $$I_{31} + I_{32} = \vec{C}_{2\alpha} + \rho_\alpha^{el} \vec{E} + \vec{j}_\alpha \times \vec{B}$$

$$\vec{u}_\alpha C_{1,\alpha} - \cancel{\vec{u}_\alpha \left(\nabla \cdot (\rho_\alpha \vec{u}_\alpha) \right)} + \rho_\alpha \frac{\partial \vec{u}_\alpha}{\partial t} + \cancel{\vec{u}_\alpha \left(\nabla \cdot (\rho_\alpha \vec{u}_\alpha) \right)} + \rho_\alpha \vec{u}_\alpha \cdot$$

$$\cdot (\nabla \otimes \vec{u}_\alpha) + \nabla \cdot \hat{P}_\alpha = \vec{C}_{2,\alpha} + \rho_\alpha^{el} \vec{E} + \vec{j}_\alpha \times \vec{B}$$

$$\rho_\alpha \frac{\partial \vec{u}_\alpha}{\partial t} + \rho_\alpha \vec{u}_\alpha \cdot (\nabla \otimes \vec{u}_\alpha) = -\nabla \cdot \hat{P}_\alpha - \vec{u}_\alpha C_{1\alpha} + \vec{C}_{2\alpha} + \rho_\alpha^{el} \vec{E} + \vec{j}_\alpha \times \vec{B}$$

$$\rho_\alpha \underbrace{\left(\frac{\partial \vec{u}_\alpha}{\partial t} + \vec{u}_\alpha \cdot (\nabla \otimes \vec{u}_\alpha) \right)} = -\nabla \cdot \hat{P}_\alpha - \vec{u}_\alpha C_{1,\alpha} + \vec{C}_{2\alpha} + \rho_\alpha^{el} \vec{E} + \vec{j}_\alpha \times \vec{B}$$

$$\frac{d\vec{A}}{dt} = \frac{\partial \vec{A}}{\partial t} + \vec{v} \cdot (\nabla \otimes \vec{A})$$

$$\rho_\alpha \frac{d\vec{u}_\alpha}{dt} = -\nabla \cdot \hat{P}_\alpha - \vec{u}_\alpha C_{1\alpha} + \vec{C}_{2\alpha} + \rho_\alpha^{el} \vec{E} + \vec{j}_\alpha \times \vec{B}$$

$$\frac{\partial}{\partial t}(\rho_\alpha \vec{u}_\alpha) + \nabla \cdot (\rho_\alpha \vec{u}_\alpha \otimes \vec{u}_\alpha) =$$

$$= \rho_\alpha \frac{\partial \vec{u}_\alpha}{\partial t} + \vec{u}_\alpha \frac{\partial \rho_\alpha}{\partial t} + \vec{u}_\alpha (\nabla \cdot (\rho_\alpha \vec{u}_\alpha)) + \rho_\alpha \vec{u}_\alpha \cdot (\nabla \otimes \vec{u}_\alpha)$$

$$= \rho_\alpha \underbrace{\left(\frac{\partial \vec{u}_\alpha}{\partial t} + \vec{u}_\alpha \cdot (\nabla \otimes \vec{u}_\alpha) \right)}_{\frac{d\vec{u}_\alpha}{dt}} + \vec{u}_\alpha \underbrace{\left(\frac{\partial \rho_\alpha}{\partial t} + \nabla \cdot (\rho_\alpha \vec{u}_\alpha) \right)}_{C_{1,\alpha}} =$$

jdna. kont.

$$= \rho_\alpha \frac{d\vec{u}_\alpha}{dt} + \vec{u}_\alpha C_{1,\alpha}$$

$$\frac{\partial}{\partial t}(\rho_\alpha \vec{u}_\alpha) + \nabla \cdot (\rho_\alpha \vec{u}_\alpha \otimes \vec{u}_\alpha) = -\nabla \cdot \hat{P}_\alpha + \vec{C}_{2\alpha} + \rho_\alpha^{el} \vec{E} + \vec{j}_\alpha \times \vec{B}$$

$\sum_\alpha{}'$

$\rho = \sum_\alpha{}' \rho_\alpha \qquad \vec{j} = \sum_\alpha{}' \vec{j}_\alpha$, $\vec{u}$ - srednja makr. brzina

$$\vec{u}_{diff,\alpha} = \vec{u}_\alpha - \vec{u} \qquad \rho \vec{u} = \sum_\alpha{}' \rho_\alpha \vec{u}_\alpha$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \rightarrow \text{jdna kont.}$$

- ukupni tenzor napona: $\hat{P}$

$$\hat{P} = \sum_\alpha{}' \hat{P}_\alpha + \sum_\alpha{}' \rho_\alpha \vec{u}_{diff,\alpha} \otimes \vec{u}_{diff,\alpha}$$

$$\sum_\alpha{}' \frac{\partial}{\partial t}(\rho_\alpha u_\alpha) + \sum_\alpha{}' \nabla\cdot(\rho_\alpha \vec{u}_\alpha \otimes \vec{u}_\alpha) = -\nabla\cdot\sum_\alpha{}' \hat{P}_\alpha + \sum_\alpha{}' \vec{C}_{2\alpha} + \sum_\alpha{}' \rho_\alpha^{el}\vec{E} + \sum_\alpha{}' \vec{j}_\alpha \times \vec{B}$$

! $\rho^{el} \approx 0 \Rightarrow \rho^{el}\vec{E} \approx 0$

$$\frac{\partial}{\partial t}(\rho\vec{u}) + \sum_\alpha{}' \nabla\cdot\left(\rho_\alpha(\vec{u}_{diff,\alpha} + \vec{u})\otimes(\vec{u}_{diff,\alpha} + \vec{u})\right) = -\nabla\cdot\sum_\alpha{}' \hat{P}_\alpha + \sum_\alpha{}' \vec{C}_{2\alpha} + \vec{j}\times\vec{B}$$

$$\frac{\partial}{\partial t}(\rho\vec{u}) + \nabla\cdot\left[\sum_\alpha{}' \rho_\alpha \vec{u}_{diff,\alpha}\otimes\vec{u}_{diff,\alpha} + \underbrace{\sum_\alpha{}' \rho_\alpha \vec{u}_{diff,\alpha}}_{=0}\otimes\vec{u} + \sum_\alpha{}' \rho_\alpha \vec{u}\otimes\underbrace{\vec{u}_{diff,\alpha}}_{=0} + \sum_\alpha{}' \rho_\alpha \vec{u}\otimes\vec{u}\right] = -\nabla\cdot\sum_\alpha{}' \hat{P}_\alpha + \vec{j}\times\vec{B} + \sum_\alpha{}' \vec{C}_{2,\alpha}$$

$$\sum_\alpha{}' \rho_\alpha \vec{u}_{diff,\alpha} = 0$$

$$\frac{\partial}{\partial t}(\rho\vec{u}) + \nabla\cdot(\rho\vec{u}\otimes\vec{u}) = -\nabla\cdot\Big(\underbrace{\sum_\alpha{}' \hat{P}_\alpha + \sum_\alpha{}' \rho_\alpha \vec{u}_{diff,\alpha}\otimes\vec{u}_{diff,\alpha}}_{\hat{P}}\Big) + \vec{j}\times\vec{B} + \sum_\alpha{}' \vec{C}_{2,\alpha}$$

$$\frac{\partial}{\partial t}(\rho\vec{u}) + \nabla\cdot(\rho\vec{u}\otimes\vec{u}) = -\nabla\cdot\hat{P} + \vec{j}\times\vec{B} + \sum_\alpha{}' \vec{C}_{2,\alpha}$$

$\Rightarrow$ važi to na globalnom nivou: $\sum_\alpha{}' \vec{C}_{2,\alpha} = \vec{0}$

$$\boxed{\frac{\partial}{\partial t}(\rho\vec{u}) + \nabla\cdot(\rho\vec{u}\otimes\vec{u}) = -\nabla\cdot\hat{P} + \vec{j}\times\vec{B}}$$

$$\frac{\partial}{\partial t}(\rho\vec{u}) + \nabla\cdot(\rho\vec{u}\otimes\vec{u}) =$$

$$= \frac{\partial\rho}{\partial t}\vec{u} + \rho\frac{\partial\vec{u}}{\partial t} + \vec{u}\left(\nabla\cdot(\rho\vec{u})\right) + \rho\vec{u}\cdot(\nabla\otimes\vec{u}) =$$

$$= \rho\left(\frac{\partial\vec{u}}{\partial t} + \vec{u}\cdot(\nabla\otimes\vec{u})\right) + \vec{u}\underbrace{\left(-\nabla\cdot(\rho\vec{u}) + \nabla\cdot(\rho\vec{u})\right)}_{0} =$$

$$= \rho\frac{d\vec{u}}{dt}$$

$$\boxed{\rho\frac{d\vec{u}}{dt} = -\nabla\cdot\hat{P} + \vec{j}\times\vec{B}}$$

$\blacksquare$