nastavak:

$$\frac{\sin^2\alpha}{B(s)} = \frac{\sin^2\alpha_0}{B_{min}}$$

- uslov za „zahvat" čestice $B_K \le B_{max}$

$$\frac{m v_{\parallel,0}^2}{2} + M B_{min} \le M B_{max}$$

$$v_{\parallel,0}^2 \le \sqrt{\frac{2}{m} M \left(B_{max} - B_{min}\right)}$$

$M = \text{const.}$ $\qquad M \approx \frac{W_\perp}{B(s)}$

$v_\perp = v \sin\alpha$ $\qquad v = \sqrt{\frac{2W}{m}} \Rightarrow$ $\qquad W_\perp = W \sin^2\alpha$

$v_\parallel = v \cos\alpha$ $\qquad\qquad\qquad\qquad W_\parallel = W \cos^2\alpha$

$$M = \frac{W \sin^2\alpha}{B(s)} = \frac{W \sin^2\alpha_0}{B_{min}}$$

$$\underset{=}{v_{\parallel,0}} \le \sqrt{\frac{2}{m} \frac{W \sin^2\alpha_0}{B_{min}} \left(B_{max} - B_{min}\right)}$$

$$\underset{=}{v_{\parallel,0}} = v \cos\alpha_0 = \sqrt{\frac{2W}{m}} \cos\alpha_0$$

$$\cancel{\frac{2W}{m}} \cos^2\alpha_0 \le \cancel{\frac{2W}{m}} \sin^2\alpha_0 \left(\frac{B_{max}}{B_{min}} - 1\right)$$

$$1 - \cancel{\sin^2\alpha_0} \le \sin^2\alpha_0 \frac{B_{max}}{B_{min}} - \cancel{\sin^2\alpha_0}$$

$$\boxed{\sin^2\alpha_0 \geq \frac{B_{min}}{B_{max}}}$$ uvjet da bi čestica bila zahvaćena u mag. klopku

$$\frac{B_{min}}{B_{max}} = \sin^2\alpha_{loss}$$ - konstanta mag. ogledala

- za neispunjen uvjet $\Rightarrow$ konus gubitka

(1) naći uvjet za α_0 za dip. mag. polje Zemlje.

$$B_{min} = \frac{B_\oplus}{L^3} \qquad \frac{\sin^2\alpha_0}{B_{min}} = \frac{\sin^2 90^\circ}{B_K}$$

$$\sin^2\alpha_0 = \frac{B_{min}}{B_K} = \frac{\cancel{\frac{B_\oplus}{L^3}}}{\cancel{\frac{B_\oplus}{L^3}}\,\frac{\sqrt{1+3\sin^2\lambda_m^{(K)}}}{\cos^6\lambda_m^{(K)}}}$$

$$\boxed{\sin^2\alpha_0 = \frac{\cos^6\lambda_m^{(K)}}{\sqrt{1+3\sin^2\lambda_m^{(K)}}}}$$

(2) $R_C = ?$ uz poznati intenz. mag. polja Zemlje.

$$B = \frac{\mu_0}{4\pi}\frac{M_\oplus}{r^3}\sqrt{1+3\sin^2\lambda_m}$$

$$\left\{ \frac{\nabla_\perp B}{B} \propto \frac{\vec{e}_n}{R_c} \right\}$$

$$\nabla_\perp B = (\nabla B)_\perp = (\vec{b} \times \nabla B) \times \vec{b}$$

$$\circledast \quad \nabla \times \vec{B} = \nabla \times (B\vec{b}) = \nabla B \times \vec{b} + B(\nabla \times \vec{b}) = \mu_0 \vec{j}$$

$$\vec{b} \times \nabla B = B(\nabla \times \vec{b}) - \mu_0 \vec{j}$$

$$\nabla(\vec{b} \cdot \vec{b}) = 0$$

$$\nabla(\vec{b} \cdot \vec{b}) \Rightarrow \vec{b} \times (\nabla \times \vec{b}) = -(\vec{b} \cdot \nabla)\vec{b}$$

$$\nabla_\perp B = \left(B(\nabla \times \vec{b}) - \mu_0 \vec{j} \right) \times \vec{b}$$

$$= B(\nabla \times \vec{b}) \times \vec{b} - \mu_0 \vec{j} \times \vec{b}$$

$$= B \underbrace{(\vec{b} \cdot \nabla)\vec{b}}_{\frac{\vec{e}_n}{R_c}} - \mu_0 \vec{j} \times \vec{b}$$

$$\nabla_\perp B = B \frac{\vec{e}_n}{R_c} - \mu_0 \vec{j} \times \frac{\vec{B}}{B}$$

$$\left\{ \frac{\nabla_\perp B}{B} = \frac{\vec{e}_n}{R_c} - \frac{\mu_0 \vec{j} \times \vec{B}}{B^2} \right\}$$

$*$ PPS. $\vec{j} \approx \vec{0}$

$$\frac{\nabla_\perp B}{B} \approx \frac{\vec{e}_n}{R_c} \qquad /\!/\!/$$

$$\frac{1}{R_c} = \left\| \frac{\nabla_\perp B}{B} \right\|$$

$$(\nabla B)_\perp \parallel \vec{e}_r \quad \Rightarrow \quad \frac{1}{B}\nabla_\perp B = \frac{1}{B}\frac{\partial B}{\partial r}$$

$$\frac{\partial B}{\partial r} = \frac{\mu_0 M_\oplus}{4\pi}\sqrt{1+3\sin^2\lambda_m}\,(-3)\,r^{-4}$$

$$\frac{1}{R_c} = \left\| \frac{\cancel{4\pi} r^3}{\cancel{\mu_0}\cancel{M_\oplus}} \frac{1}{\cancel{\sqrt{1+3\sin^2\lambda_m}}} \cdot \frac{\cancel{\mu_0}\cancel{M_\oplus}}{\cancel{4\pi}}\cancel{\sqrt{1+3\sin^2\lambda_m}}\frac{(-3)}{r^{\cancel{4}\,3}} \right\|$$

$$\frac{1}{R_c} = \left\| \frac{3}{r} \right\| \quad \Rightarrow \quad \boxed{R_c = \frac{r}{3}}$$

Kinetička teorija i makroskopski modeli

* čestica $\vec{r}, \vec{v}$ (u 3D) $\rightarrow$ $\left.x, y, z, v_x, v_y, v_z\right\}$ 6D

$\rightarrow$ μ-prostor : tačka $\rightarrow$ din. stanje u trenutku t

putanja $\rightarrow$ evolucija dinamičkog stanja

* el. zapr. $d^3\vec{r}\, d^3\vec{v} = dx\,dy\,dz\,dv_x\,dv_y\,dv_z$

$\Rightarrow$? usrednjena jednočestična fazna gustina: $f_\alpha(\vec{r}, \vec{v}, t)$

$d^3\vec{r}\, d^3\vec{v}$ - dovoljno mala i dovoljno velika

α - tip čestice (e^-, p^+ ....)

* $f_\alpha(\vec{r}, \vec{v}, t)$
  1) $\vec{v} \rightarrow \infty \Rightarrow f_\alpha \rightarrow 0$
  2) zavisi od $\vec{r}$ $\rightarrow$ nehomogena
  3) zavisi od $\vec{v}$ $\rightarrow$ anizotropna

VELIČINE:

1) koncentracija (α - tip):

$$n_\alpha(\vec{r}, t) = \int_{V_{\vec{v}}} f_\alpha(\vec{r}, \vec{v}, t)\, d^3\vec{v}$$

$$\int_{V_{\vec{v}}} d^3\vec{v} = \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z$$

2) gustina (α-tip):

$$\rho_\alpha = m_\alpha n_\alpha$$

⊛ srednja vrednost skalarne f-je:

$g = 1$

$$\langle g(\vec{r},\vec{v},t)\rangle_\alpha (\vec{r},t) = \frac{1}{n_\alpha(\vec{r},t)} \int_{V_{\vec{v}}} g(\vec{r},\vec{v},t) f_\alpha(\vec{r},\vec{v},t) d^3\vec{v}$$

$$1 = \frac{1}{n_\alpha} \int 1 \cdot f_\alpha d^3\vec{v} \Rightarrow \int f_\alpha d^3\vec{v} = n_\alpha$$

3) srednja brzina (makroskopska) (α-tip.):

$$\vec{u}_\alpha(\vec{r},t) = \langle \vec{v} \rangle_\alpha (\vec{r},t) = \frac{1}{n_\alpha(\vec{r},t)} \int_{V_{\vec{v}}} \vec{v} f_\alpha(\vec{r},\vec{v},t) d^3\vec{v}$$

4) neuređena (haotična) brzina (α-tip):

$$\vec{w}_\alpha = \vec{v} - \vec{u}_\alpha$$

$$\langle \vec{w}_\alpha \rangle_\alpha = \langle \vec{v} - \vec{u}_\alpha \rangle_\alpha = \langle \vec{v} \rangle_\alpha - \langle \vec{u}_\alpha \rangle_\alpha =$$

$$= \vec{u}_\alpha - \vec{u}_\alpha = 0$$

5) gustina plazme:

$$\rho = \sum_\alpha{}' \rho_\alpha$$

6) srednja brzina plazme

$$\rho \vec{u} = \sum_\alpha \rho_\alpha \vec{u}_\alpha \qquad \vec{u} = \frac{1}{\rho} \sum_\alpha \rho_\alpha \vec{u}_\alpha$$

7) Brzina difuzije (α-tip):

$$\vec{u}_{diff,\alpha} = \vec{u}_\alpha - \vec{u}$$

$$\sum_\alpha{}' \rho_\alpha \vec{u}_{diff,\alpha} = \sum_\alpha{}' \rho_\alpha \vec{u}_\alpha - \sum_\alpha{}' \rho_\alpha \vec{u} =$$

$$= \rho\vec{u} - \vec{u}\sum_\alpha{}' \rho_\alpha = \rho\vec{u} - \rho\vec{u} = 0$$

8) hidrodinamički tenzor napora: $\hat{P}_\alpha$

$$\hat{P}_\alpha = m_\alpha \int_{V_{\vec{v}}} \vec{w}_\alpha \otimes \vec{w}_\alpha \, f_\alpha(\vec{r},\vec{v},t)\, d^3\vec{v}$$

$$\hat{P}_\alpha = m_\alpha n_\alpha \langle \vec{w}_\alpha \otimes \vec{w}_\alpha \rangle_\alpha$$

$$\begin{bmatrix} w_{ii} & w_{ij} & w_{ik} \\ w_{ji} & w_{jj} & w_{jk} \\ w_{ki} & w_{kj} & w_{kk} \end{bmatrix}$$

9) skalarni (izotropni) pritisak

$$p_\alpha = \frac{1}{3} \mathrm{Tr}\, \hat{P}_\alpha = \frac{1}{3} \rho_\alpha \langle w_\alpha^2 \rangle_\alpha$$

10) ukupni tenzor napona:

$$\hat{P} = \sum_\alpha{}' \rho_\alpha \langle (\vec{v}-\vec{u}) \otimes (\vec{v}-\vec{u}) \rangle_\alpha$$

$$\hat{P} = \sum_\alpha{}' \hat{P}_\alpha + \sum_\alpha{}' \rho_\alpha \vec{u}_{diff,\alpha} \otimes \vec{u}_{diff,\alpha}$$

Kinetička jdna plazme ($N_D >> 1$)

$$\frac{d\mathcal{K}_\alpha}{dt} = 0 \qquad \mathcal{K}_\alpha \text{ - mikroskopska fazna gustoća}$$

- usrednjimo po ansamblu:

$$\boxed{\underbrace{\frac{df_\alpha(\vec{r},\vec{v},t)}{dt}}_{\text{kolektivne interakcije}} = \underbrace{I_\alpha}_{\text{kratkodometne interakcije}}}$$

osnovna jdna plazme

* jdna Vlasova ($I_\alpha = 0$)

$$\boxed{\frac{df_\alpha}{dt} = \frac{\partial f_\alpha}{\partial t} + \vec{v}\cdot\nabla f_\alpha + \frac{q_\alpha}{m_\alpha}\left(\vec{E} + \vec{v}\times\vec{B}\right)\cdot\nabla_{\vec{v}} f_\alpha = 0}$$

$$\nabla \equiv \nabla_{\vec{r}} \qquad \nabla_{\vec{v}} \equiv \nabla_{\vec{v}}$$

1. od osnovne kin. jdne u fiz. plazme izvesti JDNU KONTINUITETA. Zanemariti dodatne sile i pps. $I_\alpha = 0$

jdna kontinuiteta: $\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{u}) = 0$

$$I_\alpha = 0 \Rightarrow \frac{df_\alpha}{dt} = 0$$

$$\frac{df_\alpha}{dt} = \frac{\partial f_\alpha}{\partial t} + \vec{v}\cdot\nabla f_\alpha + \underbrace{\frac{q_\alpha}{m_\alpha}\left(\vec{E}+\vec{v}\times\vec{B}\right)}_{\vec{a}}\cdot\nabla_{\vec{v}} f_\alpha = 0$$

(*) tražimo „nulti" moment $\Rightarrow \Big/ \int_{V_{\vec{v}}} d^3\vec{v}$

$$\underbrace{\int_{V_{\vec{v}}} \frac{\partial f_\alpha}{\partial t}\, d^3\vec{v}}_{I_{11}} + \underbrace{\int_{V_{\vec{v}}} \vec{v}\cdot\nabla f_\alpha\, d^3\vec{v}}_{I_{12}} + \underbrace{\int_{V_{\vec{v}}} \vec{a}\cdot\nabla_{\vec{v}} f_\alpha\, d^3\vec{v}}_{I_{13}} = 0$$

$$I_{11} = \int_{V_{\vec{v}}} \frac{\partial f_\alpha}{\partial t}\, d^3\vec{v} = \frac{\partial}{\partial t}\int_{V_{\vec{v}}} f_\alpha\, d^3\vec{v} = \frac{\partial}{\partial t} n_\alpha = \frac{\partial n_\alpha}{\partial t}$$

$$I_{12} = \int_{V_{\vec{v}}} \vec{v}\cdot\nabla f_\alpha\, d^3\vec{v} = \int_{V_{\vec{v}}} \nabla_{\vec{r}}(\vec{v} f_\alpha)\, d^3\vec{v} = \nabla\int_{V_{\vec{v}}} \vec{v} f_\alpha\, d^3\vec{v} \;\ominus$$

$$\nabla(\vec{v} f_\alpha) = f_\alpha \underbrace{\nabla\cdot\vec{v}}_{} + \vec{v}\cdot\nabla f_\alpha = \vec{v}\cdot\nabla f_\alpha$$

$$\nabla_{\vec{r}}\cdot\vec{v} = 0$$

! $\vec{r}$ i $\vec{v}$ nezavisne

$$I_{12} \ominus \nabla\cdot(n_\alpha \vec{u}_\alpha)$$

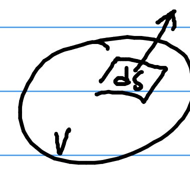
$$I_{13} = \int_{V_{\vec{v}}} \vec{a}\cdot\nabla_{\vec{v}} f_\alpha\, d^3\vec{v} \;\ominus^*$$

$$\nabla_{\vec{v}}(\vec{a} f_\alpha) = f_\alpha \nabla_{\vec{v}}\cdot\vec{a} + \vec{a}\cdot\nabla_{\vec{v}} f_\alpha$$

$$\nabla_{\vec v}(\vec a f_\alpha) - f_\alpha \nabla_{\vec v}\cdot\vec a = \vec a\cdot\nabla_{\vec v} f_\alpha$$

$$\overset{*}{=} \int_{V_{\vec v}} \nabla_{\vec v}(\vec a f_\alpha)\, d^3\vec v - \int_{V_{\vec v}} f_\alpha \nabla_{\vec v}\cdot\vec a =$$

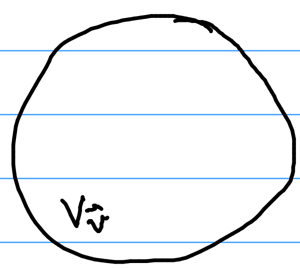
$$\int_V \nabla\cdot\vec A\, dV = \oint_{S_V=\partial V} \vec A\cdot d\vec S$$



$$= \underbrace{\oint_{S_{\vec v}} f_\alpha(\vec a\cdot d\vec S_{\vec v})}_{1} - \underbrace{\int_{V_{\vec v}} f_\alpha \nabla_{\vec v}\left(\frac{q_\alpha}{m_\alpha}(\vec E - \vec v\times\vec B)\right) d^3\vec v}_{2} =$$

$$f_\alpha = f_\alpha(\vec r,\vec v,t) \quad ; \quad \vec E = \vec E(\vec r,t) \; ; \; \vec B = \vec B(\vec r,t)$$

1



$$V_{\vec v}: \; v_x \in (-\infty, \infty)$$
$$v_y \in (-\infty, \infty)$$
$$v_z \in (-\infty, \infty)$$

$$\Rightarrow S_{\vec v} \Rightarrow \pm\infty$$

$$! \; v \to \infty \Rightarrow f_\alpha \to 0$$

$$\Rightarrow \oint_{S_{\vec v}} f_\alpha(\vec a\cdot d\vec S_v) \to 0$$

(2) $\nabla_{\vec{v}} \cdot \vec{E}(\vec{r}, t) = 0$

$$\nabla_{\vec{v}} \cdot (\vec{v} \times \vec{B}) = \nabla_{\vec{v}} \cdot \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{vmatrix} =$$

$$= \left[\frac{\partial}{\partial v_x} \quad \frac{\partial}{\partial v_y} \quad \frac{\partial}{\partial v_z} \right] \begin{bmatrix} v_y B_z - v_z B_y \\ v_z B_x - v_x B_z \\ v_x B_y - v_y B_x \end{bmatrix} =$$

$$= \frac{\partial}{\partial v_x}\left(v_y B_z - v_z B_y \right) + \frac{\partial}{\partial v_y}[\ldots] + \frac{\partial}{\partial v_z}[\ldots] = 0$$

$\Rightarrow I_{13} = 0$

- - - - - - - - - -

$$I_{11} + I_{12} + I_{13} = 0$$

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \vec{u}_\alpha) = 0 \qquad / \cdot m_\alpha$$

$$\frac{\partial \rho_\alpha}{\partial t} + \nabla \cdot (\rho_\alpha \vec{u}_\alpha) = 0 \qquad / \sum_\alpha$$

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla (\rho \vec{u}) = 0}$$

⊛ DODATAK UZ ZADATAK: $I_\alpha \neq 0$

$$\frac{\partial \rho_\alpha}{\partial t} + \nabla \cdot (\rho_\alpha \vec{u}_\alpha) = \left| m_\alpha \int_{V_{\vec{v}}} I_\alpha \, d^3\vec{v} \right. \equiv C_{1\alpha}$$

$C_{1,\alpha}$ → stvaranje i nestajanje čestica

= jonizacija i rekombinacija

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = \sum_{\alpha} C_{1,\alpha}}$$

$\sum_{\alpha} C_{1,\alpha} = 0$

zbog zakona održanja ukupne mase sistema