→ • orbitalni metod

• kinetičkom teorijom

• hidrodinamičkim metodom

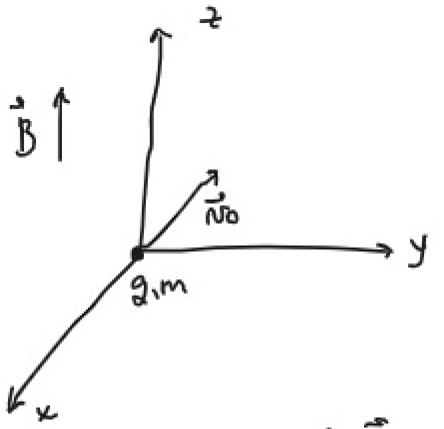
* ORBITALNI METOD

* $\vec{B}, \vec{E}$ – spolj. polje i posmatramo kretanje izolovane čestice

* razređene plazme

$\underset{q}{\circ} \xrightarrow{\vec{a}}$? $\vec{r}$

① Nerel. nael. čestica (m i q), homogeno i stacionarno mag. polje $\vec{B} = B\vec{e}_z$. U početnom trenutku $\vec{r}(t=0) = \vec{0}$, a brzina $\vec{v}_0(t=0) = v_\perp^0 \vec{e}_y + v_\parallel^0 \vec{e}_z$. Odrediti jdne kretanja + diskusija. Pokazati W = const. (kin. energija) u stac. $\vec{B}$.



$$m\dot{\vec{v}} = q(\vec{v} \times \vec{B})$$

$$\vec{v} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}, \quad \dot{\vec{v}} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix}, \quad \vec{B} = \begin{bmatrix} 0 \\ 0 \\ B \end{bmatrix}$$

$$\vec{v} \times \vec{B} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \dot{x} & \dot{y} & \dot{z} \\ 0 & 0 & B \end{vmatrix} = \dot{y}B\vec{e}_x - \dot{x}B\vec{e}_y = \begin{bmatrix} \dot{y}B \\ -\dot{x}B \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} m\ddot{x} &= q\dot{y}B \\ m\ddot{y} &= -q\dot{x}B \\ m\ddot{z} &= 0 \end{aligned}\right\} \quad \boxed{\begin{aligned} \ddot{x} &= \frac{qB}{m}\dot{y} \\ \ddot{y} &= -\frac{qB}{m}\dot{x} \\ \ddot{z} &= 0 \end{aligned}} \qquad \boxed{\omega_c = \frac{qB}{m}}$$

(I) $\ddot{z} = 0 \qquad \dfrac{dv_z}{dt} = 0 \qquad v_z = \text{const.} \qquad \boxed{v_z(t) = v_\parallel^0}$

$$z(t) = \int_0^t v_z(t)\,dt = v_\parallel^0 t \qquad \boxed{z(t) = v_\parallel^0 t}$$

(II) $\dot{x} = \omega_c \dot{y}$ $\xrightarrow{d/dt}$ $\dddot{x} = \omega_c \ddot{y} = -\omega_c^2 \dot{x}$

$\ddot{y} = -\omega_c \dot{x}$

$$\boxed{\dddot{x} + \omega_c^2 \dot{x} = 0}$$

$$\lambda^3 + \omega_c^2 \lambda = 0$$

$a \pm ib$

$$\lambda_1 = 0 \qquad \lambda_{2,3} = 0 \pm i\omega_c$$

$$x(t) = C_1 e^{\lambda_1 t} + C_2 e^{at}\cos(bt) + C_3 e^{at}\sin(bt)$$

$$x(t) = C_1 + C_2\cos(\omega_c t) + C_3\sin(\omega_c t)$$

$\vec{r}(t=0) = \vec{0}$

$x(t=0) = 0$ $\qquad x(t=0) = \underline{C_1 + C_2 = 0}$ (1)

$v_x = \dot{x}(t=0) = 0$

$$\boxed{C_1 = \frac{v_\perp^0}{\omega_c}}$$

$$\dot{x}(t) = -C_2\omega_c\sin(\omega_c t) + C_3\omega_c\cos(\omega_c t)$$

$\dot{x}(t=0) = C_3\omega_c = 0$ $\qquad \boxed{C_3 = 0}$ (2)

$$\ddot{x}(t) = -C_2\omega_c^2\cos(\omega_c t) - \underbrace{C_3\omega_c^2\sin(\omega_c t)}_{0}$$

$\left.\begin{array}{l}\ddot{x}(t=0) = -C_2\omega_c^2 \\ \ddot{x} = \omega_c \dot{y} \underset{t=0}{=} \omega_c v_\perp^0\end{array}\right\}$ $\qquad -C_2\omega_c^{\cancel{2}} = \cancel{\omega_c} v_\perp^0$

$$\boxed{C_2 = -\frac{v_\perp^0}{\omega_c}}$$

$$x(t) = \frac{v_\perp^0}{\omega_c} + \left(-\frac{v_\perp^0}{\omega_c}\right)\cos(\omega_c t) \qquad \frac{v_\perp^0}{\omega_c} = r_c$$

$$\boxed{x(t) = r_c\left(1 - \cos(\omega_c t)\right)} \qquad \boxed{\dot{x}(t) = r_c\omega_c\sin(\omega_c t)}$$

(III) $\dot{y} = -\omega_c \dot{x}$ $\xrightarrow{d/dt}$ $\ddot{y} = -\omega_c \ddot{x} = -\omega_c^2 \dot{y}$

$\ddot{x} = \omega_c \dot{y}$

$$\boxed{\dddot{y} + \omega_c^2 \dot{y} = 0} \qquad \lambda^3 + \omega_c \lambda = 0$$

$\lambda_1 = 0$, $\lambda_{2,3} = 0 \pm i\omega$

$$y(t) = C_4 + C_5\cos(\omega_c t) + C_6\sin(\omega_c t)$$

poč.usl. $y(t=0)=0$

$v_y=\dot{y}(t=0)=v_\perp^0$

$$y(t)=C_4+C_5=0 \qquad \boxed{C_4=0}$$

$$\dot{y}(t)=-C_5\omega_c\sin(\omega_c t)+C_6\omega_c\cos(\omega_c t)$$

$$\dot{y}(t=0)=C_6\omega_c=v_\perp^0 \rightarrow \boxed{C_6=\frac{v_\perp^0}{\omega_c}=r_c}$$

$$\ddot{y}(t)=-C_5\omega_c^2\cos(\omega_c t)-C_6\omega_c^2\sin(\omega_c t)$$

$$\ddot{y}=-\omega_c\dot{x}=0$$

$t=0$

$$\ddot{y}(t=0)=-C_5\omega_c^2=0 \qquad \boxed{C_5=0}$$

$$\boxed{y(t)=r_c\sin(\omega_c t)}$$

$$\boxed{\dot{y}(t)=r_c\omega_c\cos(\omega_c t)}$$

$$\boxed{x(t)=r_c\left(1-\cos(\omega_c t)\right)} \qquad \boxed{y(t)=r_c\sin(\omega_c t)}$$

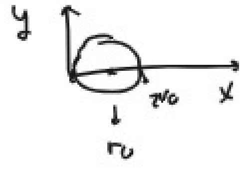
* kretanje u xOy ravni:

$$x(t)-r_c=-r_c\cos(\omega_c t)$$
$$y(t)=r_c\sin(\omega_c t)$$

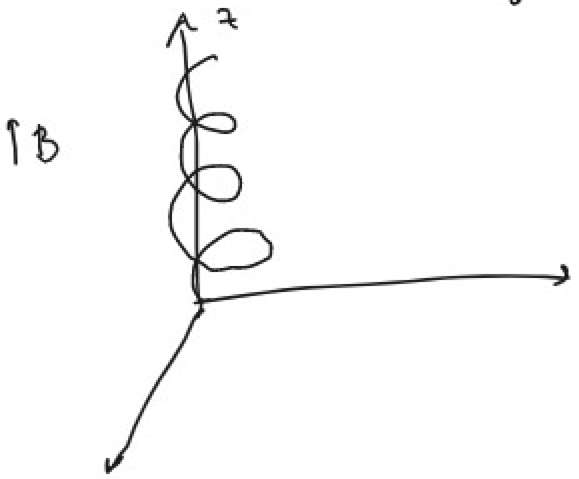
$/^2$ $\oplus$

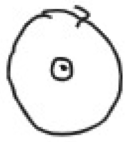
$$(x(t)-r_c)^2+(y(t))^2=r_c^2\cos^2(\omega_c t)+r_c^2\sin^2(\omega_c t)$$

$$\boxed{(x(t)-r_c)^2+(y(t))^2=r_c^2}$$ jdna kružnice



$$\boxed{z(t)=v_\parallel^0 t}$$



* 1) $q>0$ (npr. p^+) 

2) $q<0$ (e^-) 

(*)

$$m\ddot{\vec r} = q(\dot{\vec r} \times \vec B(\vec r)) \quad / \cdot \dot{\vec r}$$

$$\underbrace{m\ddot{\vec r}\cdot\dot{\vec r}} = q\underbrace{(\dot{\vec r}\times\vec B)\cdot\dot{\vec r}} = \vec 0$$

$$\frac{d}{dt}(\underbrace{\dot{\vec r}\cdot\dot{\vec r}}) = 2\dot{\vec r}\cdot\ddot{\vec r} = \frac{d}{dt}(\dot r^2)$$

$$m\,\frac{1}{2}\frac{d}{dt}(\dot r^2) = \frac{d}{dt}\left(\frac{1}{2}m\dot r^2\right) = \frac{d}{dt}(W) = 0$$

$W = \text{const.}$

(2) m, q, $\vec B = B\vec e_z$ $\quad \vec F = \vec F_\parallel + \vec F_\perp = F_\perp \vec e_x + F_\parallel \vec e_z$

$\vec r(t=0) = \vec 0$ $\quad \vec v(t=0) = v_\perp^0 \vec e_y + v_\parallel^0 \vec e_z$

$$m\dot{\vec v} = q(\vec v \times \vec B) + \vec F$$

$$m\begin{bmatrix}\ddot x\\ \ddot y\\ \ddot z\end{bmatrix} = q\begin{bmatrix}\dot y B\\ -\dot x B\\ 0\end{bmatrix} + \begin{bmatrix}F_\perp\\ 0\\ F_\parallel\end{bmatrix} = \begin{bmatrix}q\dot y B + F_\perp\\ -q\dot x B\\ F_\parallel\end{bmatrix}$$

$$\ddot{x} = \frac{qB}{m}\dot{y} + \frac{F_\perp}{m}$$

$$\ddot{y} = -\frac{qB}{m}\dot{x}$$

$$\ddot{z} = \frac{F_\parallel}{m}$$

(I) $\ddot{z} = \frac{F_\parallel}{m}$ $\quad \frac{d\dot{z}}{dt} = \frac{F_\parallel}{m} \quad \longrightarrow \quad \dot{z}(t) = \int_0^t \frac{F_\parallel}{m} dt = \frac{F_\parallel}{m} t + \dot{z}(0)$

$$\boxed{\dot{z}(t) = \frac{F_\parallel}{m} t + v_\parallel^0}$$

$$z(t) = \int_0^t \dot{z}(t)\, dt + z(0)$$

$$\boxed{z(t) = \frac{1}{2}\frac{F_\parallel}{m} t^2 + v_\parallel^0 t}$$

(II) $\ddot{x} = \omega_c \dot{y} + \underline{\underline{\frac{F_\perp}{m}}} \xrightarrow{d/dt} \dddot{x} + \omega_c^2 \dot{x} = 0$

$\ddot{y} = -\omega_c \dot{x}$

$$x(t) = C_1 \left(1 - \cos(\omega_c t)\right) \qquad C_1 = -C_2$$

$$\dot{x}(t) = C_1 \omega_c \sin(\omega_c t)$$

$$\ddot{x}(t) = C_1 \omega_c^2 \cos(\omega_c t)$$

$$\left.\begin{aligned} \ddot{x}(t=0) &= C_1 \omega_c^2 \\ \ddot{x}(t) &= \omega_c \dot{y} + \frac{F_\perp}{m} \\ \ddot{x}(t=0) &= \omega_c v_\perp^0 + \frac{F_\perp}{m} \end{aligned}\right\} \quad C_1 = \frac{\omega_c v_\perp^0 + \frac{F_\perp}{m}}{\omega_c^2}$$

$$C_1 = \frac{v_\perp^0}{\omega_c} + \frac{F_\perp}{m\omega_c^2}$$

$$\boxed{x(t) = \left(\frac{v_\perp^0}{\omega_c} + \frac{F_\perp}{m\omega_c^2}\right)\left(1 - \cos(\omega_c t)\right)}$$

III

$$\ddot{y} = -\omega_c \dot{x} \xrightarrow{d/dt} \dddot{y} = -\omega_c \ddot{x}$$

$$\ddot{x} = \omega_c \dot{y} + \frac{F_\perp}{m} \qquad \boxed{\dddot{y} + \omega_c^2 \dot{y} - \omega_c \frac{F_\perp}{m} = 0}$$

$$y(t) = C_4 + C_5 \cos(\omega_c t) + C_6 \sin(\omega_c t) - \frac{F_\perp}{m\omega_c} t = 0$$

$$y(t=0) = C_4 + C_5 = 0$$

$$\dot{y}(t) = -C_5 \omega_c \sin(\omega_c t) + C_6 \omega_c \cos(\omega_c t) - \frac{F_\perp}{m\omega_c}$$

p.u.
$$\left.\begin{aligned} \dot{y}(t=0) &= v_\perp^0 \\ \dot{y}(t=0) &= C_6 \omega_c - \frac{F_\perp}{m\omega_c} \end{aligned}\right\} \quad \boxed{C_6 = \frac{v_\perp^0}{\omega_c} + \frac{F_\perp}{m\omega_c^2}}$$

$$\ddot{y}(t) = -C_5 \omega_c^2 \cos(\omega_c t) - C_6 \omega_c^2 \sin(\omega_c t)$$

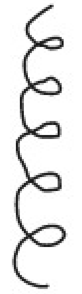
$$\ddot{y}(t=0) = -C_5 \omega_c^2$$

$$\ddot{y} = -\omega_c \dot{x} \underset{t=0}{=} 0 \qquad C_5 = C_4 = 0$$

$$\boxed{y(t) = \left(\frac{v_\perp^0}{\omega_c} + \frac{F_\perp}{m\omega_c^2}\right) \sin(\omega_c t) - \frac{F_\perp}{m\omega_c} t} \qquad \dot{y}(t) = \frac{dy}{dt}$$

$$\begin{cases} x(t) = \left(\dfrac{v_\perp^0}{\omega_c} + \dfrac{F_\perp}{m\omega_c^2}\right)(1 - \cos(\omega_c t)) \\ y(t) = \left(\dfrac{v_\perp^0}{\omega_c} + \dfrac{F_\perp}{m\omega_c^2}\right)\sin(\omega_c t) - \dfrac{F_\perp}{m\omega_c} t \end{cases}$$

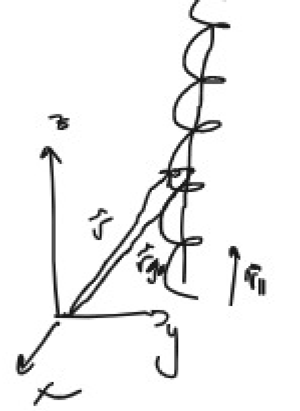
$$z(t) = \frac{1}{2}\frac{F_\parallel}{m} t^2 + v_\parallel^0 t \qquad (\text{prošli tad } z = v_\parallel^0 t)$$



↳ korak zavojnice nije konstantan

* $x(t)$ i $y(t)$ ⇒ nema više kružnog kretanja u xOy ravni

⊛ kretanje predstavimo preko: $\vec{r} = \vec{r}_{gc} + \vec{r}_c$



$$\vec{v} = \vec{v}_{gc} + \vec{v}_c = \underbrace{\vec{v}_{\parallel} + \vec{V}_D}_{\vec{v}_{gc}} + \vec{v}_c$$

$$\vec{V}_D = -\frac{F_\perp}{m\omega_c}\vec{e}_y$$

uopšteno: $$\vec{V}_D = \frac{1}{q}\frac{\vec{F}_\perp \times \vec{B}}{B^2} \quad \text{(nerel. slučaj)}$$

$\vec{F}_\perp \parallel \vec{e}_x$, $\vec{B} \parallel \vec{e}_z$ $\rightarrow$ $\vec{e}_x \times \vec{e}_z = -\vec{e}_y$

⊛ pređemo u sistem reference vezanom za vodeći centar

GALILEJEVE TRANSFORMACIJA

$$\tilde{\vec{r}} = \vec{r} - \vec{v}_{gc} t$$

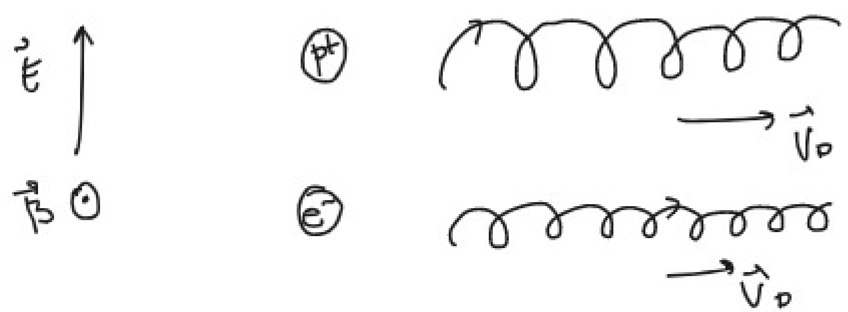
⇒ kretanje u xOy ravni opet kao ciklotronska rotacija

$$r_c = \frac{v_\perp^0}{\omega_c} + \frac{F_\perp}{m\omega_c^2} \qquad (r_c, 0, 0)$$

- $\tilde{v}_\perp = v_c = v_\perp^0 + \frac{F_\perp}{m\omega_c}$
- $\vec{v}_{\parallel} = 0$

<u>primer</u>: $\vec{F}$: 1) $\vec{E}$

$$\vec{V}_D = \frac{\vec{E}_\perp \times \vec{B}}{B^2}$$



2) $\vec{g}$ - $\vec{v}_D = \frac{m}{q}\frac{\vec{g}\times\vec{B}}{B^2}$

(3) isto ko i (2), (+) $v_\perp^0 = -\frac{F_\perp}{m\omega_c}$

$$x(t) = \left(\frac{v_\perp^0}{\omega_c} + \frac{F_\perp}{m\omega_c^2}\right)(1-\cos(\omega_c t))$$

$$y(t) = \left(\frac{v_\perp^0}{\omega_c} + \frac{F_\perp}{m\omega_c^2}\right)\sin(\omega_c t) - \frac{F_\perp}{m\omega_c}t$$

$$z(t) = \frac{1}{2}\frac{F_\parallel}{m}t^2 + v_\parallel^0 t$$

$$\begin{cases} x(t) = 0 \\ y(t) = -\frac{F_\perp}{m\omega_c}t \end{cases}$$

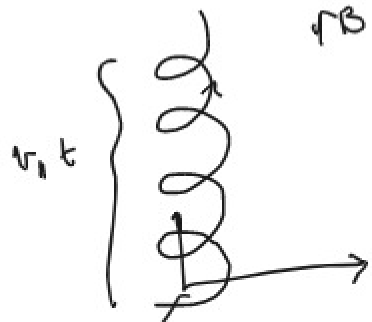
kretanje postaje ravnomjerno pravolinijsko $\Rightarrow$ gubi se ciklotronska rotacija

(4) isti uslovi ko u (1): $tg\,\alpha = \frac{v_\perp}{v_\parallel}$, $\alpha = \angle(\vec{v},\vec{B})$
odrediti R_c

$$x(t) = r_c(1-\cos(\omega_c t))$$

$$y(t) = r_c\sin(\omega_c t)$$

$$z(t) = v_\parallel^0 t$$



$$R_c = \frac{1}{k} \quad , k = \frac{\|\dot{\vec r}(t) \times \ddot{\vec r}(t)\|}{\|\dot{\vec r}(t)\|^3}$$

$$\dot{\vec r} \times \ddot{\vec r} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \vec{e}_z \\ r_c w_c \sin(w_c t) & r_c w_c \cos(w_c t) & v_\parallel^0 \\ r_c w_c^2 \cos(w_c t) & -r_c w_c^2 \sin(w_c t) & 0 \end{vmatrix}$$

$$= \begin{bmatrix} +v_\parallel^0 r_c w_c^2 \sin(w_c t) \\ v_\parallel^0 r_c w_c^2 \cos(w_c t) \\ -r_c^2 w_c^3 \sin^2(w_c t) - r_c^2 w_c^3 \cos^2(w_c t) \end{bmatrix}$$

$$\|\dot{\vec r} \times \ddot{\vec r}\| = \sqrt{v_\parallel^{0\,2} r_c^2 w_c^4 + r^4 w_c^6}$$

$$= r_c w_c^2 \sqrt{v_\parallel^{0\,2} + r_c^2 w_c^2}$$

$$\|\dot{\vec r}\| = \sqrt{r_c^2 w_c^2 + v_\parallel^{0\,2}}$$

$$k = \frac{r_c w_c^2 \sqrt{v_\parallel^{0\,2} + r_c^2 w_c^2}}{(r_c^2 w_c^2 + v_\parallel^{0\,2}) \sqrt{v_\parallel^{0\,2} + r_c^2 w_c^2}} = \frac{r_c w_c^2}{r_c^2 w_c^2 + v_\parallel^{0\,2}}$$

$$R_c = \frac{r_c^2 w_c^2 + v_\parallel^{0\,2}}{r_c w^2} \qquad r_c w_c = v_\perp^0$$

$$w_c = \frac{v_\perp^0}{r_c}$$

$$R_c = r_c \frac{v_\perp^{0\,2} + v_\parallel^{0\,2}}{v_\perp^{0\,2}} = r_c \left[1 + \left(\frac{v_\parallel^0}{v_\perp^0} \right)^2 \right]$$

⊛ $\alpha = \measuredangle(\hat{v}, \vec{B}) \qquad \operatorname{tg} \alpha = \frac{v_\perp}{v_\parallel} = \frac{v_\perp^0}{v_\parallel^0}$

$$R_c = r_c\left(1 + \frac{1}{tg^2\alpha}\right) = r_c\left(\frac{\sin^2\alpha + \cos^2\alpha}{\sin^2\alpha}\right)$$

$$\boxed{R_c = \frac{r_c}{\sin^2\alpha}}$$

$\sin\alpha \in [0, 1]$ $\quad R_c > r_c$

- spec. sluč.

1) $\sin^2\alpha = 0 \rightarrow \alpha = 0 \rightarrow \boxed{R_c \rightarrow \infty}$

$tg\alpha = \frac{v_\perp}{v_\parallel} \rightarrow v_\perp = 0, \; v_\parallel \neq 0$

- $\measuredangle(\vec{v}, \vec{B}) = 0 \rightarrow$ pravolinijsko kretanje

2) $\sin^2\alpha = 1 \rightarrow \alpha = \frac{\pi}{2} \rightarrow \boxed{R_c = r_c}$

$v_\perp \neq 0 \;\land\; \underline{v_\parallel = 0}$

- $\measuredangle(\vec{v}, \vec{B}) = \frac{\pi}{2}$ $\quad \boxed{v_\parallel = 0}$

$\Rightarrow$ kružno kretanje