① OK metoda, letelica

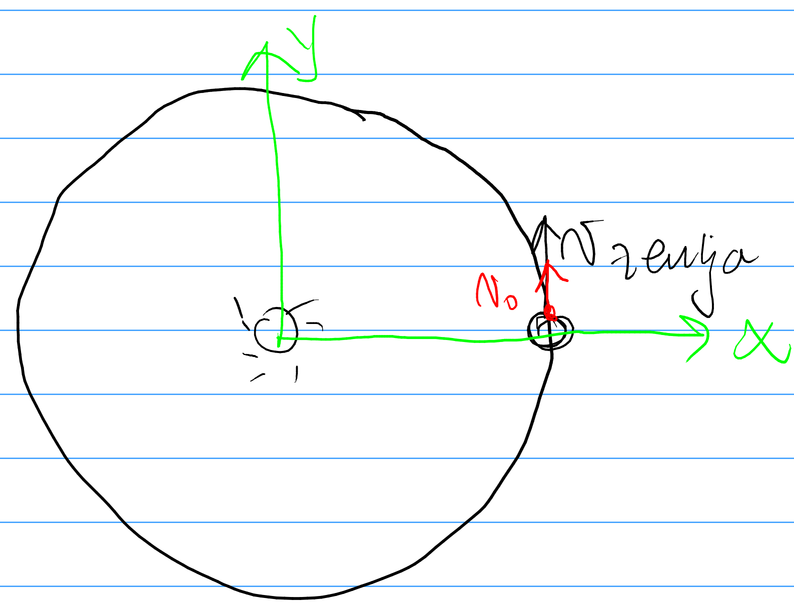
$v_0 \in \{0, 1, 2, \ldots, 10\}$ km/s

- u pravcu zemljine heliocentr. brzine
- samo grav. Sunca
- 1 julijanska godina

a) $dt_0 = const. \rightarrow$ vreme 1 (v_0)

$dt \propto \frac{1}{a} \rightarrow$ vreme 2 (v_0)

odnos = ?



$$a_x = -\frac{\gamma M_S}{\left(\sqrt{x^2+y^2}\right)^3} x \quad ; \quad a_y = -\frac{\gamma M_S}{\left(\sqrt{x^2+y^2}\right)^3} y$$

$$dt \propto \frac{1}{a} \qquad a = \sqrt{a_x^2 + a_y^2}$$

$$dt \propto \sqrt{x^2+y^2} = r \qquad a = -\frac{\gamma M_s}{\sqrt{x^2+y^2}}$$

$$\boxed{dt = dt_0 \frac{\sqrt{x^2+y^2}}{\sqrt{x_0^2+y_0^2}} = \boxed{dt_0 \frac{\sqrt{x^2+y^2}}{aj}}}$$

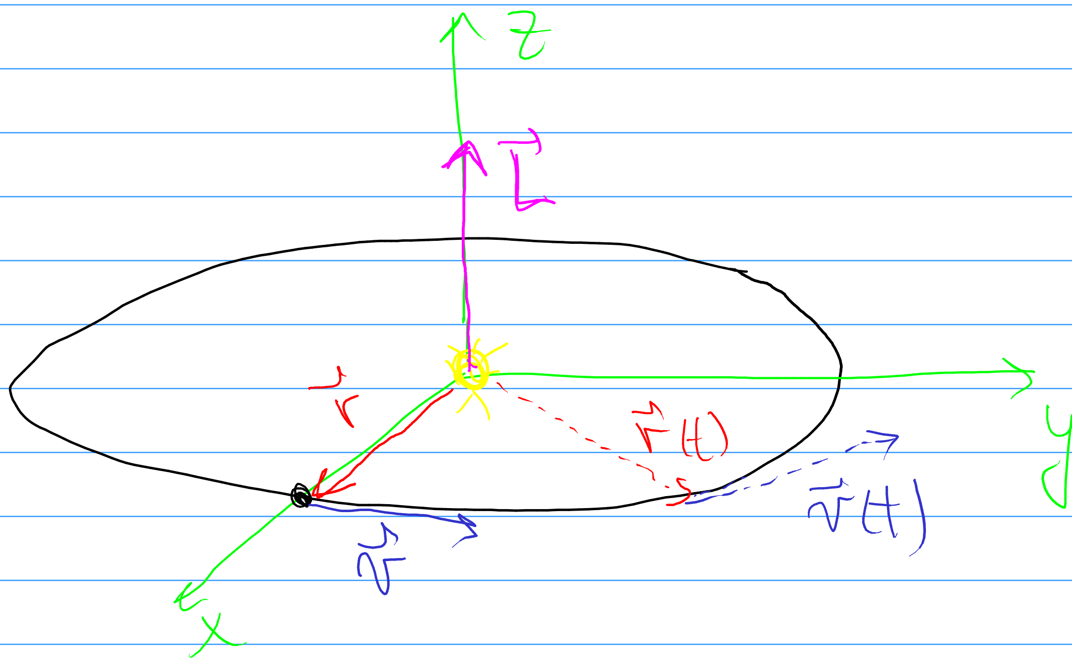
$x_0 = 1aj$, $y_0 = 0$

b) Kako se menja moment količine kretanja letelice u odnosu na heliocentar (uzeti $v_0 = 10$ km/s i $dt = const.$)

moment impulsa

def: $\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} =$

$= m\, \vec{r} \times \vec{v}$



u našem slučaju (kretanje u ravni):

$\vec{r}$ i $\vec{v}$ y xOy

$\Rightarrow \vec{L} = \text{const.}$

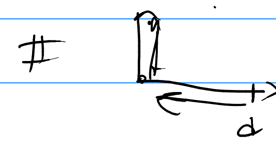
$\Rightarrow$ letelica = mat. tačka

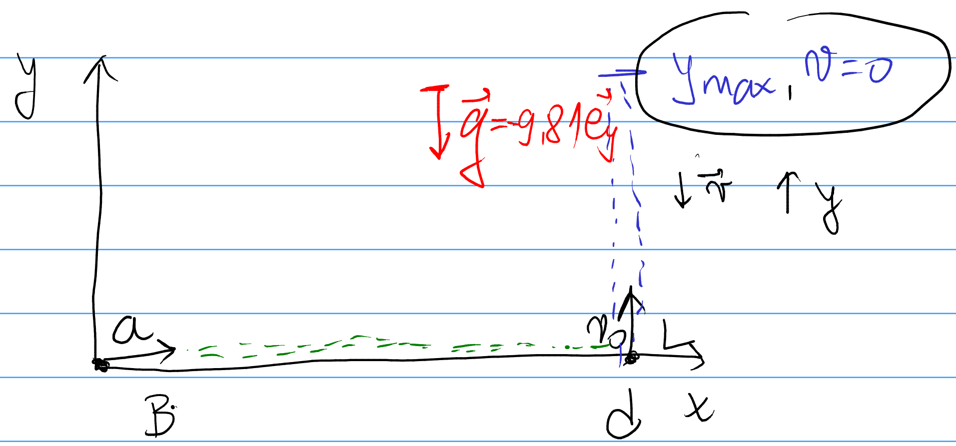
$\vec{L} = \vec{r} \times \vec{p}$

② Biciklista: $a = bt^2$, $v_0 = 0$

lopta: d, $\uparrow v_0$

$b = ?$





lopta:

1) litac naviše:

poč. uslovi

$x_0 = d = const.$

$y_0 = 0$

$v_{0y} = v_0$

$v_{0x} = 0$

$a_y = -g$

$$a_y = \frac{dv_y}{dt} = -g$$

$$dv_y = -g\,dt$$

$$\int_{v_0}^{v_y(t)} dv_y = -g \int_0^t dt$$

$$v_y(t) - v_0 = -gt$$

$$\boxed{v_y(t) = v_0 - gt}$$

$$\frac{dy}{dt} = v_y(t) = v_0 - gt$$

$$dy = (v_0 - gt)\,dt$$

$$\int_0^{y(t)} dy = \int_0^t (v_0 - gt)\,dt$$

$$\boxed{y(t) = v_0 t - \frac{1}{2} g t^2} \uparrow$$

2) slobodni pad

poč. uvjeti:

$x_0 = d$

$y_0 = y_{max}$

$v_x = 0$

$v_y = 0$

$a_y = g$

$$a_y = g = \frac{dv_y}{dt}$$

$$dv_y = g\,dt$$

$$\int_0^{v_y(t)} dv_y = g \int_0^t dt$$

$$\boxed{v_y(t) = g t} \downarrow$$

$$\boxed{\frac{dy}{dt}} = \boxed{v_y(t)} = -g t$$

$$dy = -g t\,dt$$

$$\int_{y_{max}}^{y(t)} dy = -g \int_0^t t\,dt$$

$$y(t) - y_{max} = -\frac{1}{2} g t^2$$

$$\boxed{y(t) = y_{max} - \frac{1}{2} g t^2} \downarrow$$

- za točku y_{max}: $v_y(t) = 0 \Rightarrow v_0 = g \tau_\uparrow$

$$\boxed{\tau_\uparrow = \frac{v_0}{g}}$$

$$y_{max} = y(\tau_\uparrow) = v_0 \tau_\uparrow - \frac{1}{2} g \tau_\uparrow^2$$

$$\boxed{y_{max}} = \frac{v_0^2}{g} - \frac{1}{2} \cancel{g} \frac{v_0^2}{g^{\cancel{2}}} = \boxed{\frac{v_0^2}{2g}}$$

- pad (zaustavljanje): $y(t) = 0$

$$y(t) = \frac{v_0^2}{2g} - \frac{1}{2} g t^2 = 0$$

$$\frac{v_0^2}{\cancel{2}g} = \frac{1}{\cancel{2}} g \tau_\downarrow^2$$

$$\tau_\downarrow = \sqrt{\frac{v_0^2}{g^2}} \Rightarrow \boxed{\tau_\downarrow = \frac{v_0}{g}}$$

lopta:

$$\tau_{TOT} = \tau_\uparrow + \tau_\downarrow = \frac{2v_0}{g}$$

biciklista: $a = bt^2$

poč. uslovi: $x_0 = y_0 = v_0 = 0$

$$a = \frac{dv_x}{dt} = bt^2$$

$$dv_x = bt^2 dt$$

$$\int_0^{v_x(t)} dv_x = b \int_0^t t^2 dt$$

$$\boxed{v_x(t) = b\frac{1}{3}t^3}$$

$$\frac{dx}{dt} = v_x(t) = \frac{b}{3}t^3$$

$$\int_0^{x(t)} dx = \frac{b}{3} \int_0^t t^3 dt$$

$$\boxed{x(t) = \frac{b}{12} t^4}$$

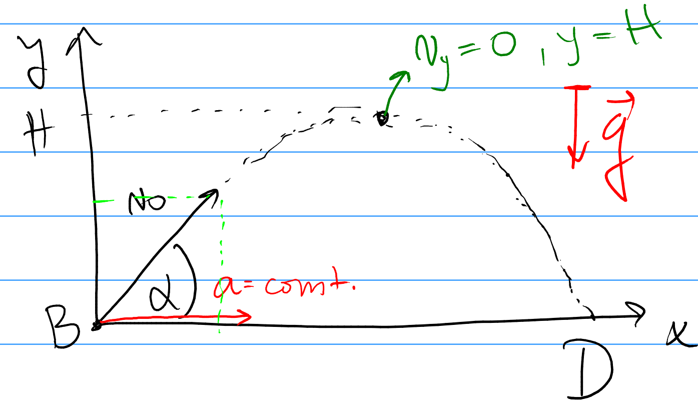
• hvatanje lopte: d, t_{TOT}

$$x(t_{TOT}) = d = \frac{b}{12} t_{TOT}^4 = \frac{b}{12}\left(\frac{2v_0}{g}\right)^4$$

$$d = \frac{b}{3}\,\frac{4v_0^4}{g^4}$$

$$\boxed{b = \frac{3}{4}\,\frac{d g^4}{v_0^4}}$$

③ Bacač v_0, α, $a = const = ?$



• kugla: kosi hitac

poč. uslovi:

$x_0 = 0$, $y_0 = 0$ $\qquad a_x = 0$

$v_{0x} = v_0 \cos\alpha \qquad v_{0y} = v_0 \sin\alpha$; $\quad a_y = -g$

x:

$$a_x = 0 = \frac{dv_x}{dt}$$

$v_x(t) = \text{const.}$ (važi i za $t=0$)

$$\boxed{v_x(t) = v_{0x} = v_0 \cos\alpha}$$

$$\frac{dx}{dt} = v_x(t) = v_0 \cos\alpha$$

$$\int_0^{x(t)} dx = v_0 \cos\alpha \int_0^t dt$$

$$\boxed{x(t) = v_0 \cos\alpha \, t}$$

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y:

$$a_y = -g$$

$$\frac{dv_y}{dt} = -g$$

$$\int_{v_{0y}}^{v_y(t)} dv_y = -g \int_0^t dt$$

$$v_y(t) - v_{0y} = -gt$$

$$\boxed{v_y(t) = v_0 \sin\alpha - gt}$$

$$\frac{dy}{dt} = v_y(t) = v_0 \sin\alpha - gt$$

$$\int_0^{y(t)} dy = \int_0^t (v_0 \sin\alpha - gt)\, dt$$

$$\boxed{y(t) = v_0 \sin\alpha \, t - \frac{1}{2} g t^2}$$

• određivanje dometa D:

$$y(t)=0$$

$$v_0 \sin\alpha\, t - \frac{1}{2} g t^2 = 0$$

$$t^2 - \frac{2 v_0 \sin\alpha}{g} t = 0$$

$$t_{1,2} = \frac{\frac{2 v_0 \sin\alpha}{g} \pm \sqrt{\left(\frac{2 v_0 \sin\alpha}{g}\right)^2 - 4\cdot 1\cdot 0}}{2}$$

$$t_1 = \frac{2 v_0 \sin\alpha}{g} \qquad t_2 = 0$$

$$\boxed{\tau = \frac{2 v_0 \sin\alpha}{g}}$$

• $$\boxed{D = x(\tau) = v_0 \cos\alpha\, \tau = \frac{2 v_0^2 \sin\alpha \cos\alpha}{g}}$$

⊗ Bacač : $a = \text{const} = ?$ poč. uslovi: $x_0 = y_0 = v_0 = 0$

$$a = \frac{dv}{dt} \Rightarrow \boxed{v(t) = at}$$

$$v(t) = \frac{ds}{dt} \Rightarrow \int_0^{s(t)} ds = v(t)\,dt = \int_0^{t} at\,dt$$

$$\boxed{s(t) = a\,\frac{1}{2}\,t^2}$$

• trenutak hvatanja kugle:

$$\underbrace{s(t) = D \quad , \quad t = \tau}$$

$$\frac{\cancel{2}\cancel{v_0^2}\cancel{\sin\alpha}\cos\alpha}{\cancel{g}} = D = s(\tau) = \frac{1}{\cancel{2}}a\left(\frac{\cancel{2}\cancel{v_0}\sin\alpha}{g}\right)^{\cancel{2}}$$

$$\cos\alpha = a\,\frac{\sin\alpha}{g}$$

$$\boxed{a = g \cdot ctg(\alpha)}$$