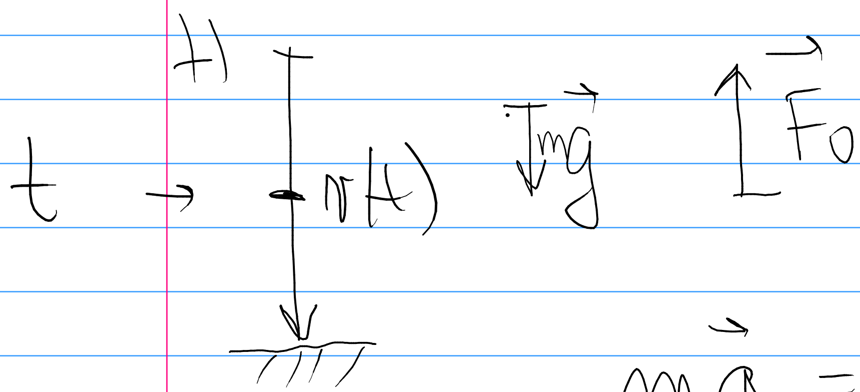
①



$$m\vec{a} = m\vec{g} + \vec{F}_o$$

$$ma = mg - mkv \quad /:m$$

$$\boxed{a = g - kv} \qquad a(v(t)) = g - kv(t)$$

$$\frac{dv}{dt} = g - kv$$

$$\int_{v_0=0}^{v(t)} \frac{dv}{g-kv} = \int_0^t dt$$

poč. uslovi:

$v_0 = 0 \quad y_0 = H$

$t_0 = 0$

$$x = g - kv \;;\; dx = -k\,dv$$

$$v_0 \to g - k\cdot 0 = g$$

$$v(t) \to g - kv(t)$$

$$\int_{g}^{g-kv(t)} -\frac{1}{k}\frac{dx}{x} = t$$

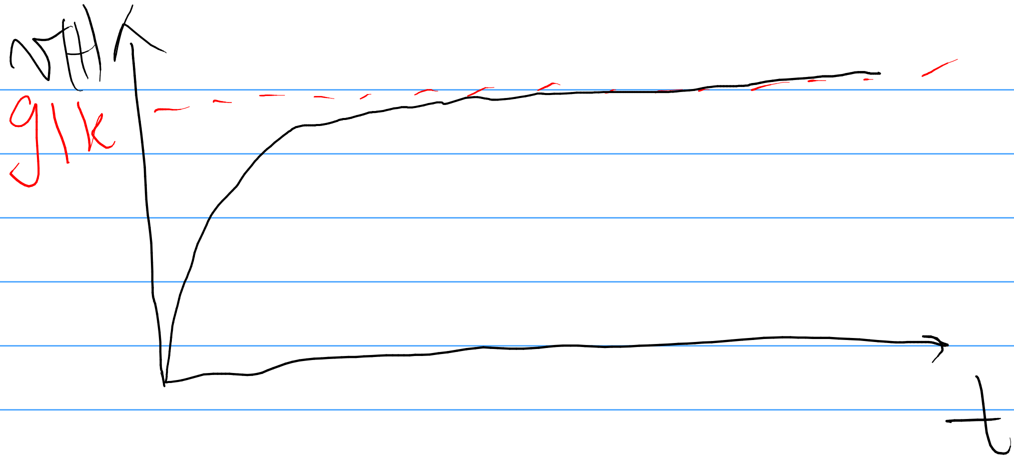
$$\ln x \Big|_{g}^{g-kv(t)} = -kt$$

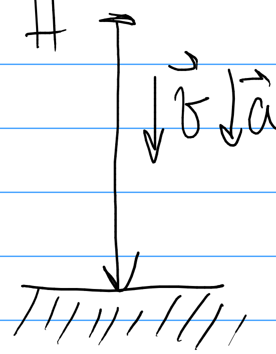
$$\ln(g-kv(t)) - \ln(g) = -kt$$

$$\ln\left(\frac{g-kv(t)}{g}\right) = -kt \qquad / e$$

$$g-kv(t) = g e^{-kt}$$

$$\boxed{v(t) = \frac{g}{k}\left(1 - e^{-kt}\right)}$$





$v(t=0) = 0$ w

$v(t \to \infty) = \frac{g}{k} = v_{as}$

OK $\begin{cases} v(t+\Delta t) = v(t) + a\Delta t \\ r(t+\Delta t) = r(t) + v(t+\Delta t)\Delta t \end{cases}$

$a = g - kv$

---

(2) $m = 250\,kg$ slobodan pad

$h[m] \in (100, \ldots, 10000)$, korak 10

$v_{final} = ?$

a) $g = 9{,}81 = \text{const.}$

b) $g = 9{,}81$ (+) $F_o = -\frac{1}{2} C_0 S \rho v^2 = m a_o$

$C_0 S = 0{,}5$

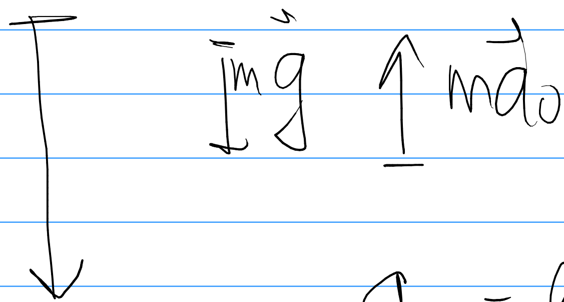
balistični koef.

$$a_o = -\frac{1}{2}\rho v^2 \cdot \frac{1}{BC}, \quad BC = \frac{m}{C_0 S}$$

$$\rho(h) = \rho_0 e^{-h/H}, \quad H = 10\,\text{km}$$

$$\rho_0 = 1{,}23\ \frac{\text{kg}}{\text{m}^3}$$

$$a_o = -\frac{1}{2}\rho_0 e^{-\frac{h}{H}} v^2 \frac{1}{BC} = a_o(h, v)$$



$$a = g - a_o$$

c) $W_o = ?$

$$W = \int_{s_1}^{s_2} \vec{F} \cdot d\vec{s} = \int_{s_1}^{s_1} F\, ds$$

$\vec{F} \parallel d\vec{s}$

(*) ZOE (zakon održanja energije)

$$E_k + E_p = const.$$

$$K + U = const$$

- konzervativne (npr. slob. pad u $\vec{g}$)

$$E_p \rightarrow E_k$$

- nekonzervativna sila $\Delta E \rightarrow W$

(−) „protivi se" kretanju

(+) „potiče"

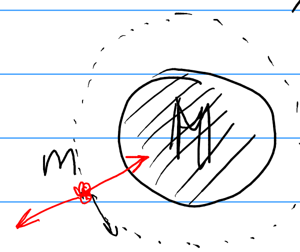
$$W = \sum_i F \cdot ds_i$$

$$ds_i = h_2 - h_1$$

(3)

• Kosmičke brzine (za $g = \gamma \frac{M}{r^2}$)

1) 1. k.b. → da orbitira



$$F_c = F_g$$

$$\frac{m v^2}{r} = \gamma \frac{mM}{r^2}$$

$$\boxed{v_1 = \sqrt{\frac{\gamma M}{r}}}$$

• za Zemlju: $M = M_Z$, $r = R_Z$

2) 2. k.b. - napusti grav. polje

(*) ZOE: $E_k = E_p$

$$\frac{1}{2} m v_2^2 = \gamma \frac{mM}{r}$$

$$v_2 = \sqrt{\frac{2\gamma M}{r}}$$

$M = M_Z$

$r = R_Z$

3) 3. kosm. brz. - da napusti Sunčev sistem

- grav. polje Sunca:

$$v_s = \sqrt{\frac{2\gamma M_s}{d}} \qquad d = 1AU$$

- v orb Zemlje

$$v_{orb} = \sqrt{\frac{\gamma M_s}{d}}$$

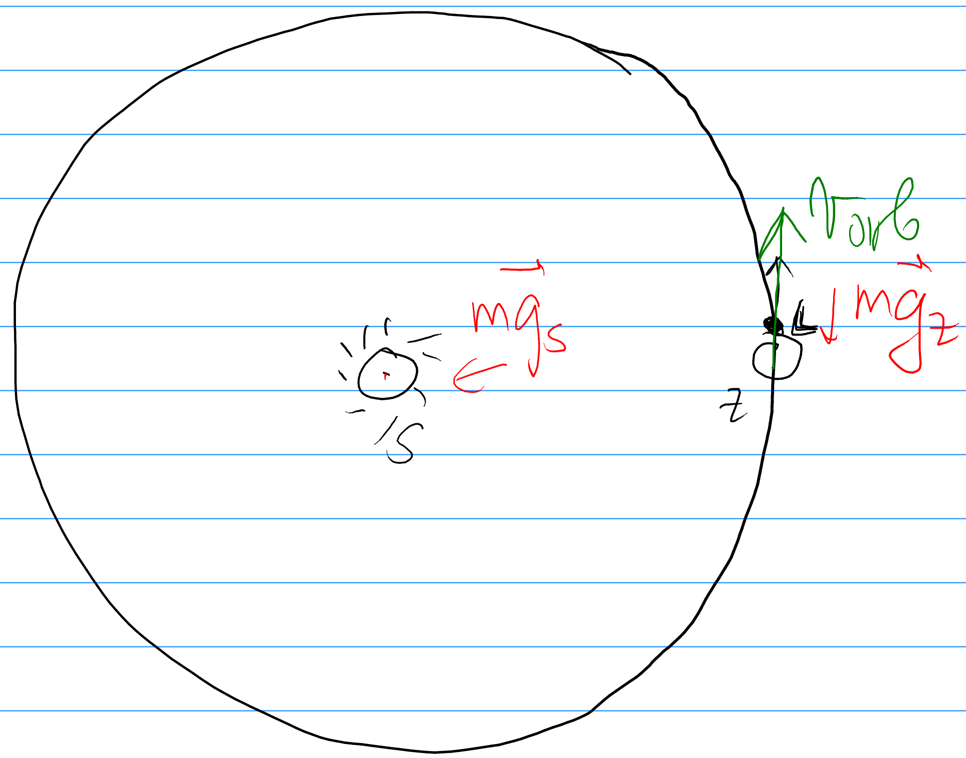
- $v_z = v_s - v_{orb}$

- da napusti i Zemlju

ZOE:

$$\frac{1}{2} m v_3^2 - \gamma \frac{m M_z}{R_z} = \frac{1}{2} m v_z^2$$

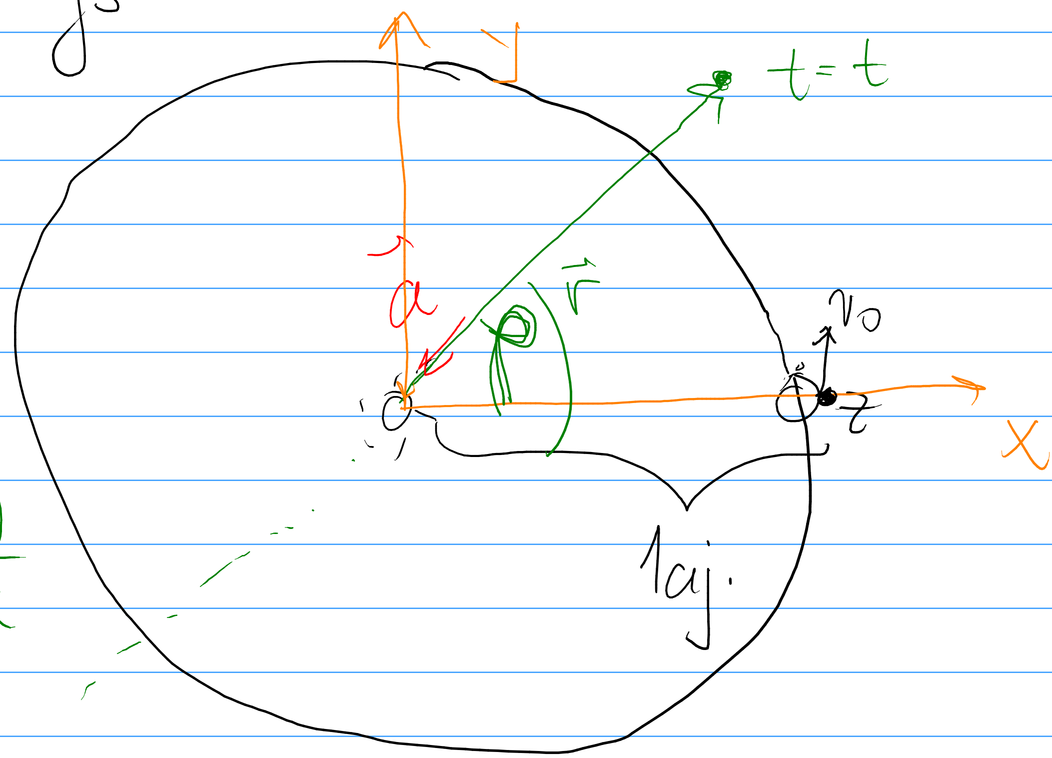
$$\Rightarrow \boxed{v_3 = \sqrt{\frac{2\gamma M_Z}{R_Z} + v_Z^2}} \quad 16{,}7\,km/s$$



3. O-K metoda → putanja letelice sa Zemlje $v_0 = 10\,km/s$ u smeru heliocentrične brzine Zemlje.

pps. kružnu putanju Z oko S ⊕ ⊙

pps. $\vec{g}_S$



$\varphi = \operatorname{arctg} \frac{y}{x}$

$-\frac{\pi}{2}, \frac{\pi}{2}$

$$\vec{g}_S = \vec{a} = -\frac{\gamma M_S}{r^2}\frac{\vec{r}}{r} = -\frac{\gamma M_S}{r^3}\vec{r}$$

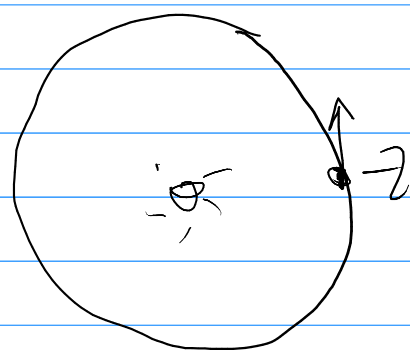
$$\vec{a} = -\frac{\gamma M_S}{r^3}(x\vec{e}_x + y\vec{e}_y)$$

$$r = \|\vec{r}\| = \sqrt{x^2+y^2}$$

$$a_x = -\frac{\gamma M_S}{\left(\sqrt{x^2+y^2}\right)^3}\cdot x$$

$$a_y = -\frac{\gamma M_s}{(x^2+y^2)^{3/2}} \cdot y$$

$$a_t = 0$$



$R_s = 1$ aj.

$$v = \omega \cdot r = \frac{s}{t} = \frac{2\pi R_s}{1 \text{god}}$$

$$1 \text{ god} = 365{,}25 \cdot 24 \cdot 60 \cdot 60$$

• numpy.arctan2(y,x) → $[-\pi, \pi)$

• numpy.arctan(y) → $[-\frac{\pi}{2}, \frac{\pi}{2}$

• $x \in (-5, 5)$ $y \in (-10, 10)$