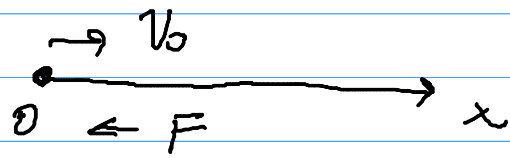
telo se kreće duž x-ose

(1) pređeni put = ? $\quad v_0 \quad F_0 = -mkv\,, \; k>0$



$$ma = F_0$$

$$\cancel{m}a = -\cancel{m}kv$$

$$\frac{dv}{dt} = -kv$$

$$\int_{v_0}^{v(t)} \frac{dv}{v} = \int_0^t -k\,dt$$

$$\ln \frac{v(t)}{v_0} = -kt$$

$$\boxed{v(t) = v_0\, e^{-kt}}$$

zaustavljanje:

$v(t) = 0$

$\Rightarrow t \to \infty$

$$\frac{dx}{dt} = v(t) = v_0\, e^{-kt}$$

$$\int_0^{x(t)} dx = \int_0^t v_0\, e^{-kt}\, dt$$

$$\xi = -kt \; ; \; d\xi = -k\,dt$$

$$x(t) = \int_0^{-kt} v_0 e^{\xi} \left(-\frac{1}{k}\right) d\xi = -\frac{v_0}{k} e^{\xi} \Big|_0^{-kt}$$

$$\boxed{x(t) = \frac{v_0}{k}\left(1 - e^{-kt}\right)}$$

- pređeni put do zaustavljanja:

$t \to \infty$

$$\boxed{x(t \to \infty) = \frac{v_0}{k}}$$

2 —||— $F_0 = -mkv^2$

$$ma = F_0 = -mkv^2$$

$$\frac{dv}{dt} = -kv^2$$

$$\int -\frac{dx}{x^2} = \frac{1}{x}$$

$$\int_{v_0}^{v(t)} -\frac{dv}{v^2} = \int_0^t k\,dt$$

$$\left(\frac{1}{x}\right)' = -1\, x^{-2}$$

$$\frac{1}{v(t)} - \frac{1}{v_0} = kt$$

$$\frac{1}{v(t)} = kt + \frac{1}{v_0}$$

$$v(t) = \frac{1}{\frac{v_0 kt + 1}{v_0}}$$

$$\boxed{v(t) = \frac{v_0}{v_0 kt + 1}}$$

zaustavljanje:

$v(t) = 0$

$t \to \infty$

$$\frac{dx}{dt} = v(t) = \frac{v_0}{v_0 kt + 1}$$

$$\int_0^{x(t)} dx = \int_0^t \frac{v_0}{v_0 kt + 1}\, dt$$

$$\xi = v_0 kt + 1$$

$$d\xi = v_0 k\, dt$$

$$x(t) = \int_1^{v_0 k t+1} \cancel{v_0} \frac{1}{\xi} \frac{d\xi}{\cancel{v_0} k} = \frac{1}{k} \ln \xi \Big|_1^{v_0 k t + 1}$$

$$x(t) = \frac{1}{k} \left[\ln(v_0 k t + 1) - \ln 1 \right]$$

$$\boxed{x(t) = \frac{1}{k} \ln(v_0 k t + 1)}$$

- pređeni put: $t \to \infty$

$$x(t \to \infty) \to \infty$$

(3) Naći pređeni put ako se telo kreće duž x-ose sa $v_0 = 0$ iz $x_0 = 0$, deluje sila $F = m(\alpha - \beta t)$

$\alpha, \beta > 0$

DOMAĆI

ROK ZA DOMAĆE: 26. maj (zaključno sa 26.)

④ grav. polje zemlje ($g = const$) i sila otpora $F_o = -mkv$, poč. brzina v_0

$v(t) = ?$; prodiskutovati slučajeve za parametar v_0

$$ma = mg - mkv$$

$$a = g - kv$$

$$\frac{dv}{dt} = g - kv$$

$$\int_{v_0}^{v(t)} \frac{dv}{g-kv} = \int_0^t dt = t$$

$$\xi = g - kv$$

$$d\xi = -k\,dv$$

$$\int_{g-kv_0}^{g-kv(t)} -\frac{1}{k}\frac{d\xi}{\xi} = t$$

$$-\frac{1}{k}\ln\left(\frac{g-kv(t)}{g-kv_0}\right) = t$$

$$\frac{g-kv(t)}{g-kv_0} = e^{-kt}$$

$$v(t) = \frac{g}{k} - \frac{1}{k}(g-kv_0)\,e^{-kt}$$

$$\boxed{v(t) = \frac{g}{k} - \left(\frac{g}{k} - v_0\right)e^{-kt}}$$

- analiza:

$$t \to \infty \quad \Rightarrow \quad v(t\to\infty) = \frac{g}{k}$$

„terminalna brzina"

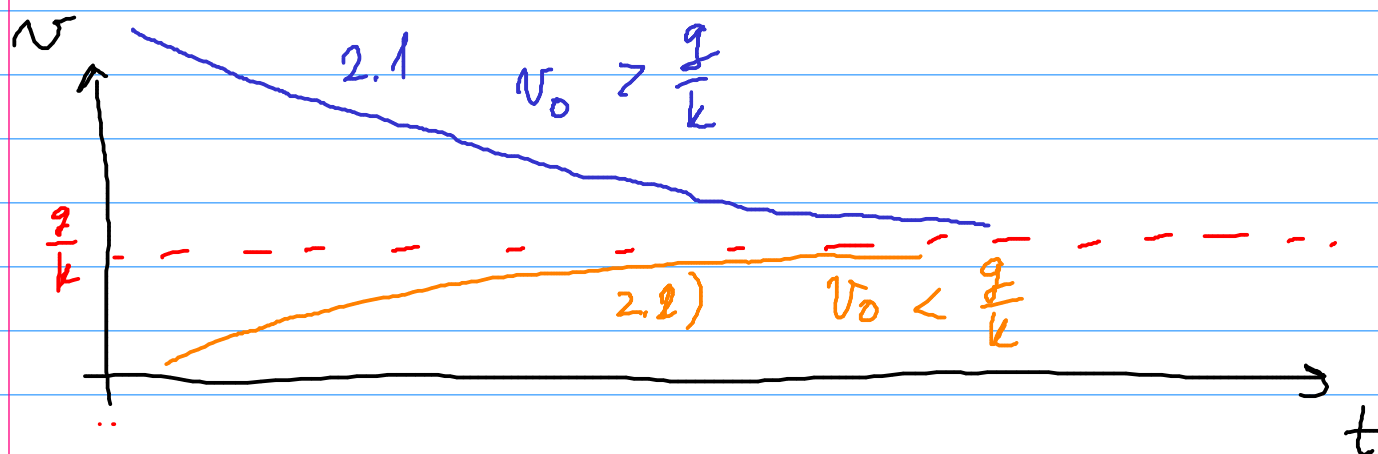
• v_0 - parametar

1) $v_0 = \frac{g}{k}$ $\Rightarrow$ $v(t) = const = \frac{g}{k}$

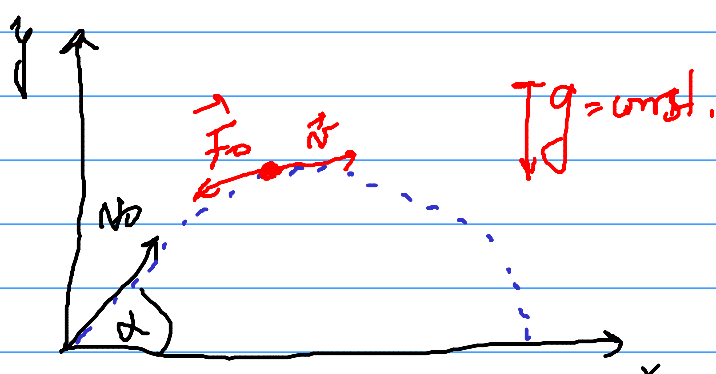
2) $v_0 \neq \frac{g}{k}$

2.1) $v_0 > \frac{g}{k}$ $\quad v(t) \downarrow$ ka $\frac{g}{k}$

2.2) $v_0 < \frac{g}{k}$ $\quad v(t) \uparrow$ ka $\frac{g}{k}$



(5) $x(t), y(t) = ?$ $\quad \alpha, v_0$ i $\vec{F}_o = -mk\vec{v}$



$\vec{v}_0$:

$v_{0x} = v_0 \cos\alpha$

$v_{0y} = v_0 \sin\alpha$

$$\vec{F}_0 = -mk(v_x\vec{e}_x + v_y\vec{e}_y)$$

$$\vec{g} = -g\vec{e}_y$$

$$m\vec{a} = m\vec{g} + \vec{F}_0$$

x: $ma_x = -mkv_x$

y: $ma_y = -mkv_y - mg$

$$a_x = -kv_x$$

[...]

$$v_x(t) = v_{0x}\, e^{-kt}$$

$$v_x(t) = v_0 \cos\alpha\, e^{-kt}$$

$$\boxed{x(t) = \frac{v_0\cos\alpha}{k}\left(1 - e^{-kt}\right)}$$

y: $a_y = -g - kv_y = -(g + kv_y)$

$$\frac{dv_y}{dt} = -(g + kv_y)$$

$$\int_{v_{0y}}^{v_y(t)} \frac{dv_y}{g + kv_y} = \int_0^t - dt$$

$$\xi = g + kv_y$$

$$d\xi = k\,dv_y$$

$$\int_{g+kv_0\sin\alpha}^{g+kv_y(t)} \frac{1}{k} \frac{d\xi}{\xi} = -t$$

$$\ln\left(\frac{g + kv_y(t)}{g + kv_0\sin\alpha}\right) = -kt$$

$$g + kv_y(t) = (g + kv_0\sin\alpha)\, e^{-kt}$$

$$\boxed{v_y(t) = -\frac{g}{k} + \left(\frac{g}{k} + v_0\sin\alpha\right) e^{-kt}}$$

$$\frac{dy}{dt} = -\frac{g}{k} + \left(\frac{g}{k} + v_0 \sin\alpha\right) e^{-kt}$$

$$\boxed{y(t) = -\frac{g}{k} t + \frac{1}{k}\left(\frac{g}{k} + v_0 \sin\alpha\right)\left(1 - e^{-kt}\right)}$$