(1) putanja satelita $h = 200\,km$

$v_0 = \sqrt{\frac{\gamma M}{R+h}} \approx 7{,}78\,km/s$

uzimajući 1) grav. polje Zemlje

2) otpor atmosfere: F_o

$BC = 250\,kg/m^2$

a) životni vek? $\boxed{> 6 \text{ dana}}$

b) $v(t),\ h(t) = ?$

c) $\rho_0 \to 2\rho_0 \longrightarrow \boxed{> 3 \text{ dana!}}$

d) $L(t) = ?$ $\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times \vec{v}$ $E_k = \frac{1}{2} m v^2$

e) $E(t) = ?$ $E = E_k + E_p$ $T + V$ $\to BC = \frac{m}{C_o S}$ $E_p = -\frac{\gamma M m}{r}$

$F_o = -\frac{1}{2} C_o S \rho v^2, \quad a_o = -\frac{1}{2} \frac{\rho v^2}{BC}$

$\vec{F}_o = -\frac{1}{2} C_o S \rho v^2 \frac{\vec{v}}{v}$

$\rho_0 = 1{,}2255\,\frac{kg}{m^3}$

$\rho = \rho_0\, e^{-\frac{h}{H}}$ $H = 8500\,m$

$$\vec{a} = \vec{g} + \vec{a}_0 \rightarrow a = g - a_0$$

$$\vec{a} = \underbrace{-\frac{\gamma M}{r^2}\,\frac{\vec{r}}{r}}_{\text{grav. polje Zemlje}} - \underbrace{\frac{1}{2}\,\frac{\rho v^2}{BC}\,\frac{\vec{v}}{v}}_{\text{otpor atmosfere}}$$

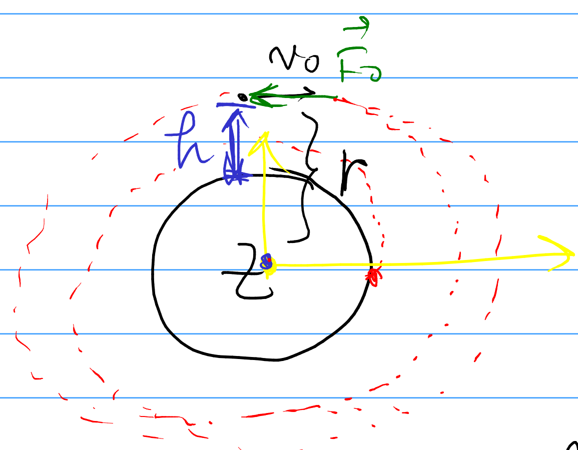
$\vec{r} = x\vec{e}_x + y\vec{e}_y$ $\qquad$ $\vec{v} = v_x\vec{e}_x + v_y\vec{e}_y$

$\vec{e}_x$:

$$a_x = -\frac{\gamma M}{\sqrt{x^2+y^2}^{\,3}}\,x - \frac{1}{2}\,\frac{\rho}{BC}\,v\,v_x$$

$$\rho = \rho_0\, e^{-\frac{h}{H}} \qquad v = \sqrt{v_x^2 + v_y^2}$$

2D



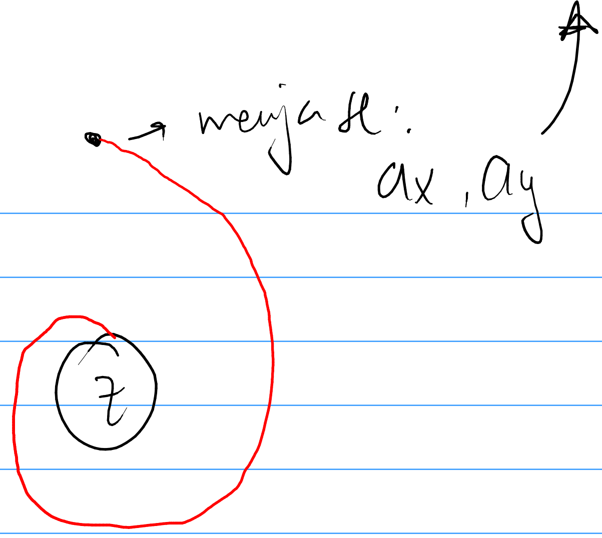
g $\quad R_h = R + h$

$g \propto \frac{1}{r}$

$$r = h + R = \sqrt{x^2 + y^2}$$

- menja nam se:

ρ, v, h, t, a_x, a_y, v_x, v_y, x, y



$x[-1]$ = poslednji član u listi

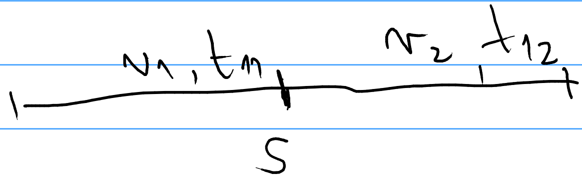
ZADACI KOJI SE REŠAVAJU ANALITIČKI

(1) 1. : 1/2 puta v_1 ; 1/2 puta v_2

2. : 1/2 vremena v_1 ; 1/2 vremena v_2

koji putnik prvi stiže?

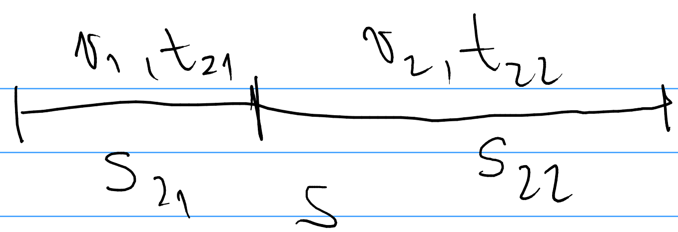
1.:



$$v_1 = \frac{s/2}{t_{11}} \qquad v_2 = \frac{s/2}{t_{12}}$$

(1) $$t_1 = t_{11} + t_{12} = \frac{s}{2}\left(\frac{1}{v_1} + \frac{1}{v_2}\right) = \frac{s}{2}\,\frac{v_1+v_2}{v_1 v_2}$$

(2)



$t_2 = 2t_{21} = 2t_{22}$

$\downarrow$

$$v_1 = \frac{S_{21}}{t_2/2} \qquad v_2 = \frac{S_{22}}{t_2/2}$$

$$S = S_{21} + S_{22} = \frac{t_2}{2}(v_1 + v_2) \qquad (2)$$

$$\frac{2t_1}{v_1 + v_2} v_1 v_2 = \frac{t_2}{2}(v_1 + v_2)$$

$$\Rightarrow \frac{t_1}{t_2} = \frac{1}{4} \frac{(v_1 + v_2)^2}{v_1 v_2}$$

pps. $t_1 < t_2$

$$\frac{t_1}{t_2} < 1$$

$$\frac{1}{4} \frac{(v_1 + v_2)^2}{v_1 v_2} < 1$$

$$v_1^2 + 2v_1v_2 + v_2^2 < 4v_1v_2$$

$$(v_1 - v_2)^2 < 0 \quad ↯$$

$$\Rightarrow t_1 \not< t_2$$

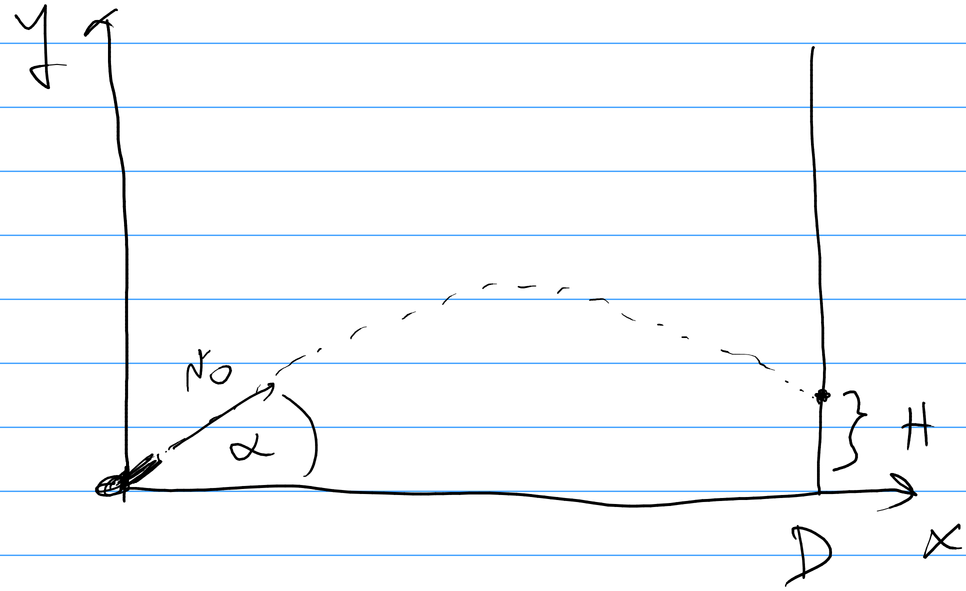
$$\Rightarrow t_2 < t_1$$

drugi putnik stiže pre prvog
$(v_1 \neq v_2)$

$\Rightarrow v_1 = v_2$

$\Rightarrow$ putnici stižu istovremeno.

(2) Na kojoj visini će granata da udari u vert. stenu na udalj. 4km od topa? Granata: $v_0 = 400\frac{m}{s}$
$\alpha = 14°$



jdne za kosi hitac:

$$x(t) = v_0 \cos\alpha \, t$$

$$y(t) = v_0 \sin\alpha \, t - \frac{1}{2} g t^2$$

jdne stene:

$$x = D \qquad y > 0$$

granata:

$$x(t) = D \Rightarrow \tau$$

$$x(\tau) = v_0 \cos\alpha \, \tau = D \Rightarrow \boxed{\tau = \frac{D}{v_0 \cos\alpha}}$$

$$y(\tau) = v_0 \sin\alpha\, \tau - \frac{1}{2} g \tau^2 = H$$

$$H = v_0 \sin\alpha \frac{D}{v_0 \cos\alpha} - \frac{1}{2} g \frac{D^2}{v_0^2 \cos^2\alpha}$$

$$\boxed{H = D \operatorname{tg}\alpha - \frac{g D^2}{2 v_0^2 \cos^2}}$$

$$H = 476\,m$$

3 $\alpha = ?$

a) $H = D$

b) $D = H$ (hitac naviše, v_0 ista)

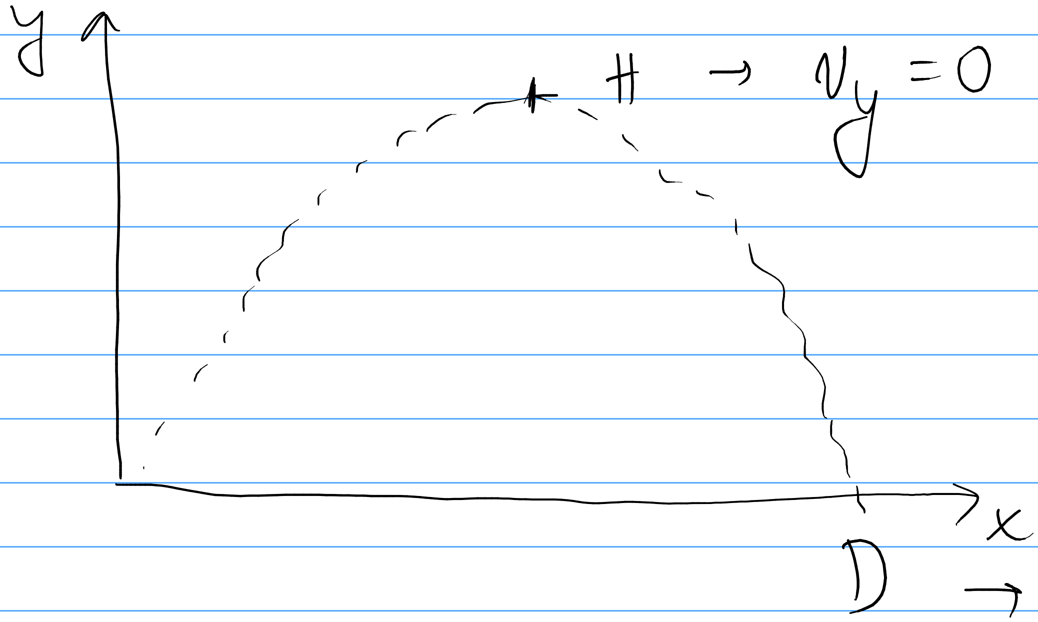
<u>kosi hitac:</u>

$$x(t) = v_0 \cos\alpha\, t$$

$$y(t) = v_0 \sin\alpha\, t - \frac{1}{2} g t^2$$

$$v_x(t) = v_0 \cos\alpha$$

$$v_y(t) = v_0 \sin\alpha - gt$$



$H \rightarrow v_y = 0 \Rightarrow T_1$

$D \rightarrow y = 0$

$$v_y(T_1) = 0 = v_0 \sin\alpha - gT_1 \Rightarrow \boxed{T_1 = \frac{v_0 \sin\alpha}{g}}$$

$$y_{max} = y(T_1) = v_0 \sin\alpha \, \frac{v_0 \sin\alpha}{g} - \frac{1}{2} g \frac{v_0^2 \sin^2\alpha}{g^2}$$

$$\boxed{y_{max} = \frac{v_0^2 \sin^2\alpha}{2g} = H}$$

$$t_{\uparrow} = t_{\downarrow} \Rightarrow t_{TOT} = t_{\uparrow} + t_{\downarrow} = \frac{2 v_0 \sin\alpha}{g}$$

$$x_{max} = D = v_0 \cos\alpha \, t_{TOT}$$

$$D = v_0 \cos\alpha \cdot \frac{2 v_0 \sin\alpha}{g}$$

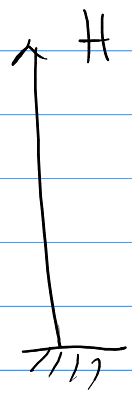
$$\boxed{D = \frac{2 v_0^2 \sin\alpha \cos\alpha}{g}}$$

$$\Rightarrow \frac{\cancel{v_0^2} \sin^{\cancel{2}}\alpha}{2\cancel{g}} = \frac{2 \cancel{v_0^2} \cancel{\sin\alpha} \cos\alpha}{\cancel{g}}$$

$$tg\alpha = 4$$

$$\boxed{\alpha = arctg(4)}$$

$$\alpha = 75^{\circ},96$$

b) 

Jdne za hitac naviše:

$$x(t) = \text{const.}$$

$$v_x(t) = 0$$

$$y(t) = v_0 t - \frac{1}{2} g t^2$$

$$v_y(t) = v_0 - g t$$

$$y_{max}(\alpha = 90^\circ) \doteq \frac{v_0^2}{2g}$$

$$x_{max} = \frac{\cancel{v_0^2}}{2\cancel{g}} = \frac{2\cancel{v_0^2} \sin\alpha \cos\alpha}{\cancel{g}}$$

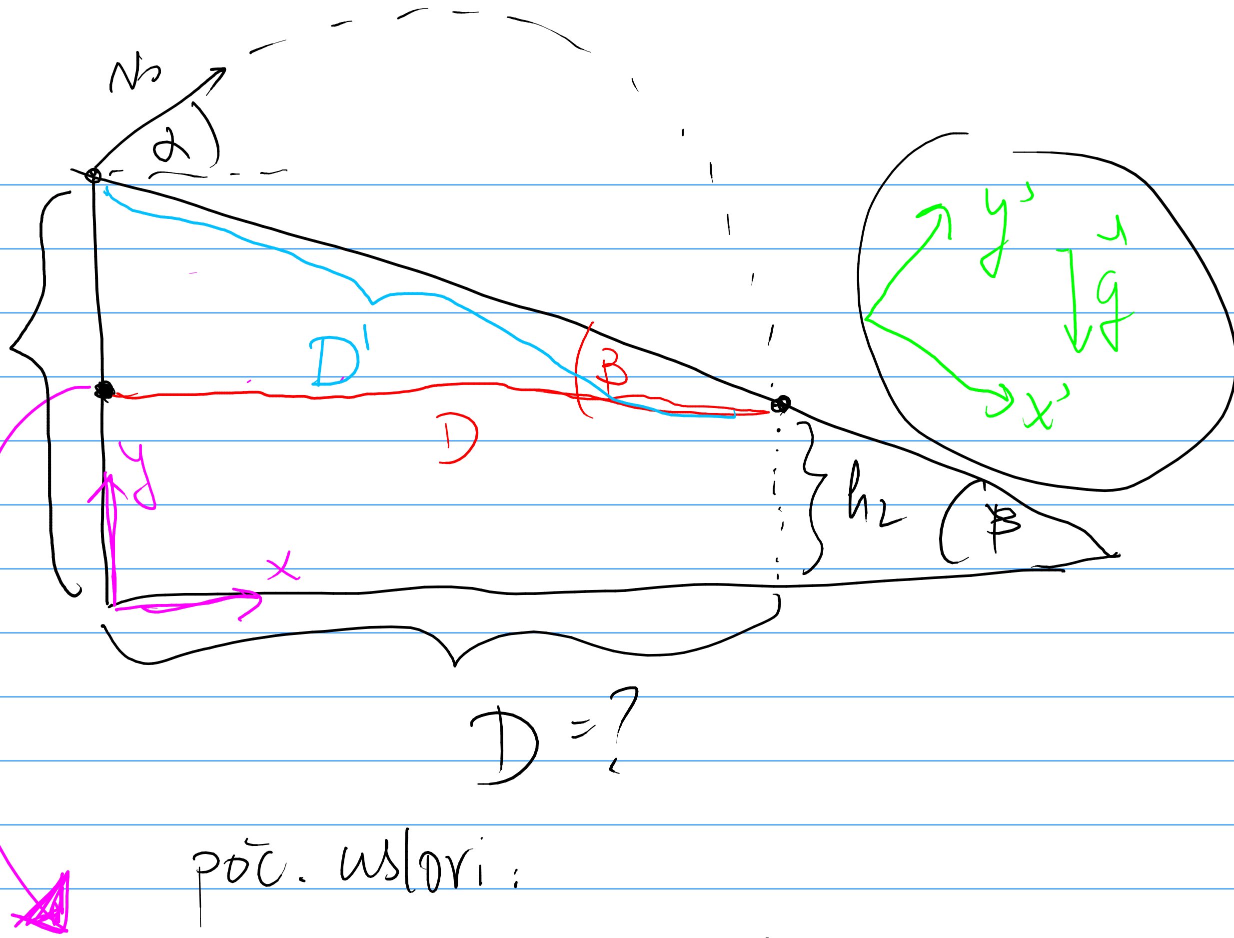
$$2\sin\alpha\cos\alpha = \sin(2\alpha) = \frac{1}{2}$$

$$\boxed{\alpha = \frac{1}{2}\arcsin\left(\frac{1}{2}\right)}$$

$$2\alpha = 30^\circ$$

$$\boxed{\alpha = 15^\circ}$$

4



poč. uslovi:

$x_0 = 0$ $\qquad y_0 = h_1$

krajnji uslovi:

$x = D$ $\qquad y = h_2$

$v_x = v_0 \cos\alpha$ $\qquad v_y = v_0 \sin\alpha - gt$

$x = v_0 \cos\alpha \, t$ $\qquad y = v_0 \sin\alpha \, t - \frac{1}{2} g t^2 + h_1$

$$\frac{h_1 - h_2}{D} = \operatorname{tg}\beta$$

τ - vreme kretanja

$$x(\tau) = D = v_0 \cos\alpha \, \tau \Rightarrow \tau = \frac{D}{v_0 \cos\alpha}$$

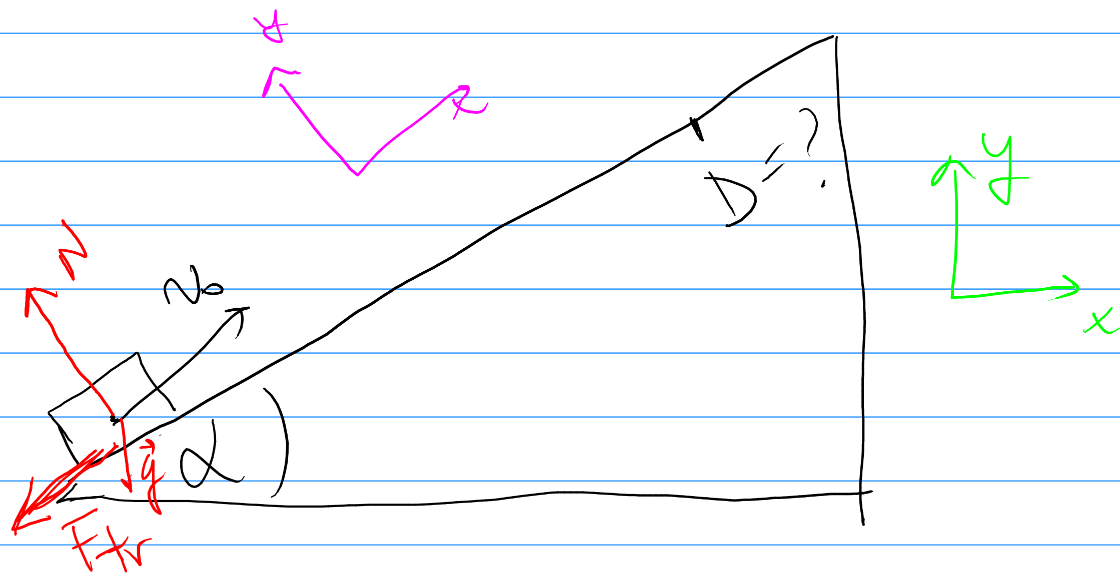
$$y(\tau) = h_2 = v_0 \sin\alpha \frac{D}{v_0 \cos\alpha} - \frac{1}{2} g \frac{D^2}{v_0^2 \cos^2\alpha} + h_1$$

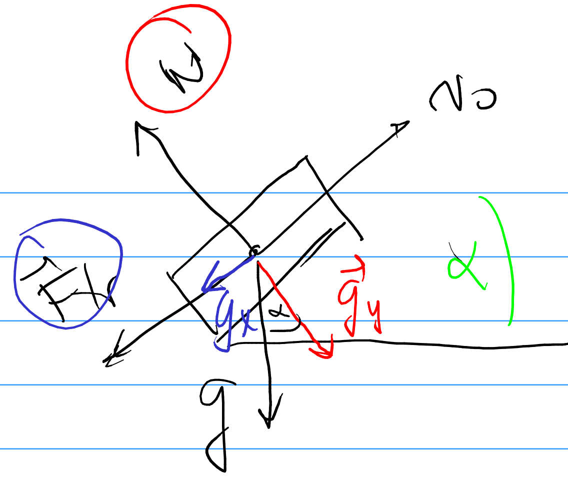
$$D \, tg\alpha - \frac{1}{2} g \frac{D^2}{v_0^2 \cos^2\alpha} = h_2 - h_1$$

$$tg\alpha - \frac{1}{2} \frac{gD}{v_0^2 \cos^2\alpha} = -tg\beta$$

$$\boxed{D = (tg\alpha + tg\beta) \frac{2 v_0^2 \cos^2\alpha}{g}}$$

4





$g_x = mg\sin\alpha$

$g_y = mg\cos\alpha$

$F_{tr} = \mu N$

$\vec{e}_x$:

$$m\vec{a}_x = \vec{F}_{tr} + \vec{g}_x$$

$$ma_x = -F_{tr} - g_x \quad (1)$$

$\vec{e}_y$:

$$m\vec{a}_y = \vec{N} + \vec{g}_y$$

$$ma_y = N - g_y \quad (2)$$

iz (2) : telo se ne kreće u pravcu y ose

$\Rightarrow$ sile su u ravnoteži

$\Rightarrow a_y = 0$

$$N - mg\cos\alpha = 0 \Rightarrow \boxed{N = mg\cos\alpha}$$

iz (1):

$$ma_x = -\mu N - mg\sin\alpha$$

$$ma_x = -\mu mg\cos\alpha - mg\sin\alpha$$

$$a_x = -g(\mu\cos\alpha + \sin\alpha) = \frac{d^2x}{dt^2} = \frac{dv_x}{dt}$$

• poč. uvjeti : $x(0) = 0$, $v_x(0) = v_0$

$$a_x = \frac{dv_x}{dt}$$

$$dv_x = a_x\, dt$$

$$\int_{v_0}^{v_x(t)} dv_x = \int_0^t a_x dt = -g(\mu\cos\alpha + \sin\alpha)t$$

$$\boxed{v_x(t) = v_0 - g(\mu\cos\alpha + \sin\alpha)t}$$

$x(t)$

$$\int_0^{x(t)} dx = \int_0^t v_x(t)\,dt$$

$$\boxed{x(t) = v_0 t - g(\mu\cos\alpha + \sin\alpha)\frac{t^2}{2}}$$

$$v_x(\tau) = 0 = v_0 - g(\mu\cos\alpha + \sin\alpha)\tau$$

$$\tau = \frac{v_0}{g(\mu\cos\alpha + \sin\alpha)}$$

$$D = x(\tau) = \frac{v_0^2}{g(\mu\cos\alpha + \sin\alpha)} -$$

$$- \frac{\cancel{g}\,\cancel{(\mu\cos\alpha + \sin\alpha)}}{2} \; \frac{v_0^2}{g^{\cancel{2}}(\mu\cos\alpha + \sin\alpha)^{\cancel{2}}}$$

$$\boxed{D = \frac{v_0^2}{2g(\mu\cos\alpha + \sin\alpha)}}$$