Nastavak sa prethodnog časa ...

$$\frac{1}{2}\underbrace{\frac{\partial}{\partial t}\left(\rho_\alpha\langle v^2\rangle_\alpha\right)}_{I_{41}} + \underbrace{\frac{1}{2}\nabla\cdot\left(\rho_\alpha\langle v^2\vec{v}\rangle_\alpha\right)}_{I_{42}} - \vec{j}_\alpha\cdot\vec{E} = C_{3,\alpha}$$

$$I_{41} = \frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha\langle v^2\rangle_\alpha\right) = \frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha\langle(\vec{u}_\alpha+\vec{w}_\alpha)\cdot(\vec{u}_\alpha+\vec{w}_\alpha)\rangle_\alpha\right)$$

$$\vec{v} = \vec{u}_\alpha + \vec{w}_\alpha, \quad p_\alpha = \frac{1}{3}\rho_\alpha\langle w_\alpha^2\rangle_\alpha, \quad \langle\vec{w}_\alpha\rangle_\alpha = 0$$

$$= \frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha\langle\vec{u}_\alpha\cdot\vec{u}_\alpha + \underbrace{\vec{u}_\alpha\cdot\vec{w}_\alpha}_{0} + \underbrace{\vec{w}_\alpha\cdot\vec{u}_\alpha}_{0} + \vec{w}_\alpha\cdot\vec{w}_\alpha\rangle_\alpha\right)$$

$$= \frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha\langle u_\alpha^2 + w_\alpha^2\rangle_\alpha\right) =$$

$$= \frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha u_\alpha^2\right) + \frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha\langle w_\alpha^2\rangle_\alpha\right) =$$

$$= \frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha u_\alpha^2\right) + \frac{\partial}{\partial t}\left(\frac{3}{2}p_\alpha\right) \qquad I_{41}$$

$$I_{42} = \nabla\cdot\left(\frac{1}{2}\rho_\alpha\langle v^2\vec{v}\rangle_\alpha\right) =$$

$$\langle v^2\vec{v}\rangle_\alpha = \langle(\vec{v}\cdot\vec{v})\vec{v}\rangle_\alpha =$$

$$\vec{v} = \vec{u}_\alpha + \vec{w}_\alpha$$

$$= \langle [(\vec u_\alpha + \vec w_\alpha)\cdot(\vec u_\alpha+\vec w_\alpha)](\vec u_\alpha+\vec w_\alpha)\rangle_\alpha =$$

$$= \langle u_\alpha^2 (\vec u_\alpha + \vec w_\alpha) + 2\vec u_\alpha \cdot \vec w_\alpha(\vec u_\alpha+\vec w_\alpha) +$$
$$+ w_\alpha^2(\vec u_\alpha+\vec w_\alpha)\rangle_\alpha =$$

$\langle \vec u_\alpha\rangle_\alpha = \vec u_\alpha$ $\quad$ $\langle \vec w_\alpha \rangle_\alpha = 0$

$$= u_\alpha^2 \vec u_\alpha + \cancel{u_\alpha^2 \langle \vec w_\alpha\rangle_\alpha}^{0} + \cancel{2\langle(\vec w_\alpha\cdot\vec u_\alpha)\vec u_\alpha\rangle_\alpha}^{0} +$$
$$+ 2\langle(\vec u_\alpha\cdot\vec w_\alpha)\vec w_\alpha\rangle_\alpha + \langle w_\alpha^2\rangle_\alpha \vec u_\alpha + \langle w_\alpha^2 \vec w_\alpha\rangle_\alpha =$$

$$= u_\alpha^2\vec u_\alpha + 2\vec u_\alpha\cdot\langle \vec w_\alpha \otimes \vec w_\alpha\rangle_\alpha + \langle w_\alpha^2\rangle_\alpha \vec u_\alpha$$
$$+ \langle w_\alpha^2 \vec w_\alpha\rangle_\alpha$$

$$= \nabla\cdot\left(\frac{1}{2}\rho_\alpha\left(u_\alpha^2\vec u_\alpha + 2\vec u_\alpha\cdot\langle\vec w_\alpha\otimes\vec w_\alpha\rangle_\alpha +\right.\right.$$
$$\left.\left.+ \langle w_\alpha^2\rangle_\alpha\vec u_\alpha + \langle w_\alpha^2\vec w_\alpha\rangle_\alpha\right)\right) =^{*}$$

1) $\vec q_\alpha = \frac{1}{2}\rho_\alpha\langle w_\alpha^2\vec w_\alpha\rangle_\alpha$

2) $\hat P_\alpha = \rho_\alpha\langle \vec w_\alpha\otimes\vec w_\alpha\rangle_\alpha$

3) $3p_\alpha = \rho_\alpha \langle w_\alpha^2 \rangle_\alpha$

$$\overset{*}{=} \nabla \cdot \left(\frac{1}{2}\rho_\alpha u_\alpha^2 \vec{u}_\alpha + \hat{p}_\alpha \cdot \vec{u}_\alpha + \frac{3}{2} p_\alpha \vec{u}_\alpha + \vec{q}_\alpha \right)$$

$$= \nabla \cdot \left(\frac{1}{2}\rho_\alpha u_\alpha^2 \vec{u}_\alpha \right) + \nabla \cdot \vec{q}_\alpha + \underbrace{\frac{3}{2} p_\alpha \nabla \cdot \vec{u}_\alpha + \frac{3}{2} \nabla p_\alpha \cdot \vec{u}_\alpha}_{\nabla \cdot (T\vec{A})} +$$

$$+ \nabla \cdot \left(\hat{p}_\alpha \cdot \vec{u}_\alpha \right) \qquad I_{42}$$

$\Rightarrow I_{41} + I_{42}$

$$\boxed{\frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha u_\alpha^2\right)} + \frac{\partial}{\partial t}\left(\frac{3}{2} p_\alpha\right) + \boxed{\nabla \cdot \left(\frac{1}{2}\rho_\alpha u_\alpha^2 \vec{u}_\alpha\right)} + \nabla \cdot \vec{q}_\alpha +$$

$$+ \frac{3}{2} p_\alpha \nabla \cdot \vec{u}_\alpha + \frac{3}{2} \nabla p_\alpha \cdot \vec{u}_\alpha + \nabla \cdot \left(\hat{p}_\alpha \cdot \vec{u}_\alpha\right) =$$

$$= \vec{j}_\alpha \cdot \vec{E} + C_{3,\alpha} \qquad (*)$$

$$I_{51} = \frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha u_\alpha^2\right) + \nabla \cdot \left(\frac{1}{2}\rho_\alpha u_\alpha^2 \vec{u}_\alpha\right) =$$

$$= \frac{1}{2} u_\alpha^2 \frac{\partial \rho_\alpha}{\partial t} + \frac{1}{2}\rho_\alpha \underbrace{\frac{\partial}{\partial t}\left(\vec{u}_\alpha \cdot \vec{u}_\alpha\right)} +$$

$$+ \rho_\alpha \vec{u}_\alpha \cdot \nabla \left(\frac{1}{2} u_\alpha^2 \right) + \frac{1}{2} u_\alpha^2 \, \nabla \cdot \left(\rho_\alpha \vec{u}_\alpha \right) =$$

$$= \frac{1}{2} u_\alpha^2 \left(\frac{\partial \rho_\alpha}{\partial t} + \nabla \cdot \left(\rho_\alpha \vec{u}_\alpha \right) \right) + \cancel{2} \cdot \frac{1}{\cancel{2}} \rho_\alpha \, \vec{u}_\alpha \cdot \frac{\partial \vec{u}_\alpha}{\partial t} +$$

$$+ \rho_\alpha \vec{u}_\alpha \cdot \nabla \left(\frac{1}{2} \vec{u}_\alpha \cdot \vec{u}_\alpha \right) \overset{**}{=}$$

$$\nabla (\vec{A} \cdot \vec{B}) = \vec{A} \times (\nabla \times \vec{B}) + \vec{B} \times (\nabla \times \vec{A}) +$$

$$+ (\vec{A} \cdot \nabla) \vec{B} + (\vec{B} \cdot \nabla) \vec{A}$$

$$\Rightarrow \quad \nabla \left(\frac{1}{2} \vec{u}_\alpha \cdot \vec{u}_\alpha \right) = \frac{1}{\cancel{2}} \Big(\cancel{2} \vec{u}_\alpha \times (\nabla \times \vec{u}_\alpha) +$$

$$+ \cancel{2} (\vec{u}_\alpha \cdot \nabla) \vec{u}_\alpha \Big)$$

$$\nabla \left(\frac{1}{2} u_\alpha^2 \right) = \vec{u}_\alpha \times (\nabla \times \vec{u}_\alpha) + (\vec{u}_\alpha \cdot \nabla) \vec{u}_\alpha$$

$$\vec{u}_\alpha \cdot \nabla \left(\frac{1}{2} u_\alpha^2 \right) = \underbrace{\vec{u}_\alpha \cdot \left(\vec{u}_\alpha \times (\nabla \times \vec{u}_\alpha) \right)}_{0} + \vec{u}_\alpha \left(\vec{u}_\alpha \cdot \nabla \right) \vec{u}_\alpha$$

$$\overset{**}{=} \frac{1}{2} u_\alpha^2 \left(\frac{\partial \rho_\alpha}{\partial t} + \nabla \cdot (\rho_\alpha \vec{u}_\alpha) \right) + \rho_\alpha \vec{u}_\alpha \cdot \frac{\partial \vec{u}_\alpha}{\partial t} +$$

$$+ \rho_\alpha \vec{u}_\alpha (\vec{u}_\alpha \cdot \nabla) \vec{u}_\alpha =$$

$$= \frac{1}{2} u_\alpha^2 \left(\frac{\partial \rho_\alpha}{\partial t} + \nabla \cdot (\rho_\alpha \vec{u}_\alpha) \right) +$$

jdna kont.

$$+ \rho_\alpha u_\alpha \cdot \left(\frac{\partial \vec{u}_\alpha}{\partial t} + (\vec{u}_\alpha \cdot \nabla) \vec{u}_\alpha \right)$$

jdna hetanja

d/dt

$$\frac{3}{2}\left(\frac{\partial}{\partial t} + \vec{u}_\alpha \cdot \nabla \right) p_\alpha + \frac{1}{2} u_\alpha^2 \Big(\underbrace{\frac{\partial \rho_\alpha}{\partial t} + \nabla \cdot (\rho_\alpha \vec{u}_\alpha)}_{C_{1,\alpha}} \Big)$$

$$+ \rho_\alpha \vec{u}_\alpha \cdot \left(\frac{\partial \vec{u}_\alpha}{\partial t} + (\vec{u}_\alpha \cdot \nabla) \vec{u}_\alpha \right) + \frac{3}{2} p_\alpha \nabla \cdot \vec{u}_\alpha +$$

$$+ \nabla \cdot \vec{q}_\alpha + \nabla \cdot (\hat{p}_\alpha \cdot \vec{u}_\alpha) = \vec{j}_\alpha \cdot \vec{E} + C_{3,\alpha}$$

$$\rho_\alpha \frac{\partial \vec{u}_\alpha}{\partial t} + \rho_\alpha \vec{u}_\alpha \cdot (\nabla \otimes \vec{u}_\alpha) = \rho_\alpha \frac{d\vec{u}_\alpha}{dt}$$

$$\rho_\alpha \frac{d\vec{u}_\alpha}{dt} = -\nabla \cdot \hat{P}_\alpha + \rho_\alpha^{el} \vec{E} + \vec{j}_\alpha \times \vec{B} + \vec{C}_{2,\alpha} -$$

$$- \vec{u}_\alpha C_{1,\alpha}$$

$$\Rightarrow \frac{3}{2} \frac{dp_\alpha}{dt} + \frac{1}{2} u_\alpha^2 C_{1\alpha} + \vec{u}_\alpha \cdot \boxed{\rho_\alpha \frac{d\vec{u}_\alpha}{dt}} +$$

$$+ \frac{3}{2} p_\alpha \nabla\cdot\vec{u}_\alpha + \nabla\cdot\vec{q}_\alpha + \nabla\cdot\left(\hat{P}_\alpha\cdot\vec{u}_\alpha\right) = \vec{j}_\alpha\cdot\vec{E} + C_{3\alpha}$$

$$\nabla\cdot\left(\hat{T}\cdot\vec{A}\right) = \vec{A}\cdot\left(\nabla\cdot\hat{T}\right) + \left(\hat{T}\cdot\nabla\right)\cdot\vec{A}$$

$$\frac{3}{2}\frac{dp_\alpha}{dt} + \frac{1}{2}u_\alpha^2 C_{1\alpha} + \vec{u}_\alpha\cdot\left(\cancel{-\nabla\cdot\hat{P}_\alpha} + \cancel{\rho_\alpha^{el}\vec{E}} + \cancel{\vec{j}_\alpha\times\vec{B}} + \vec{C}_{2\alpha} - \vec{u}_\alpha C_{1\alpha}\right) + \frac{3}{2}p_\alpha\nabla\cdot\vec{u}_\alpha + \nabla\cdot\vec{q}_\alpha + \cancel{\vec{u}_\alpha\cdot\left(\nabla\cdot\hat{P}_\alpha\right)} +$$

$$+ \left(\hat{P}_\alpha\cdot\nabla\right)\cdot\vec{u}_\alpha = \cancel{\vec{j}_\alpha\cdot\vec{E}} + C_{3,\alpha}$$

$$\frac{3}{2}\frac{dp_\alpha}{dt} - \frac{1}{2}u_\alpha^2 C_{1,\alpha} + \vec{u}_\alpha\cdot\vec{C}_{2,\alpha} + \frac{3}{2}p_\alpha\nabla\cdot\vec{u}_\alpha + \nabla\cdot\vec{q}_\alpha + \left(\hat{P}_\alpha\cdot\nabla\right)\cdot\vec{u}_\alpha = C_{3,\alpha}$$

$$\boxed{\frac{3}{2}\frac{dp_\alpha}{dt} + \frac{3}{2}p_\alpha\nabla\cdot\vec{u}_\alpha + \nabla\cdot\vec{q}_\alpha + \left(\hat{P}_\alpha\cdot\nabla\right)\cdot\vec{u}_\alpha = \frac{1}{2}u_\alpha^2 C_{1\alpha} - \vec{u}_\alpha\cdot\vec{C}_{2\alpha} + C_{3,\alpha}}$$

- zanemarujemo elastične sudare:

$$C_{1,\alpha} = 0, \quad \forall\alpha$$

$$e = \frac{1}{\gamma_g - 1}\frac{p}{\rho} \qquad \gamma_g = \frac{c_p}{c_v}$$

$$\rho_\alpha e_\alpha = \frac{3}{2} p_\alpha \quad , \gamma_g = \frac{5}{3} \quad | \text{3. step. slob.}$$

12 ⊛ ⇒

$$\frac{3}{2}\frac{\partial p_\alpha}{\partial t} + \frac{3}{2}\vec{u}_\alpha \cdot \nabla p_\alpha + \nabla\cdot\left(\frac{1}{2}\rho_\alpha u_\alpha^2 \vec{u}_\alpha\right) +$$

$$+ \frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha u_\alpha^2\right) + \nabla\cdot\vec{q}_\alpha + \frac{3}{2} p_\alpha \nabla\cdot\vec{u}_\alpha +$$

$$+ \nabla\cdot\left(\hat{p}_\alpha \cdot \vec{u}_\alpha\right) = \vec{j}_\alpha \cdot \vec{E} + C_{3,\alpha}$$

$$\Rightarrow \frac{\partial}{\partial t}(\rho_\alpha e_\alpha) + \vec{u}_\alpha \cdot \nabla(\rho_\alpha e_\alpha) + \rho_\alpha e_\alpha \nabla\cdot\vec{u}_\alpha +$$

$$+ \nabla\cdot\left(\frac{1}{2}\rho_\alpha u_\alpha^2 \vec{u}_\alpha\right) + \frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha u_\alpha^2\right) + \nabla\cdot\vec{q}_\alpha$$

$$+ \nabla\cdot\left(\hat{p}_\alpha \cdot \vec{u}_\alpha\right) = \vec{j}_\alpha \cdot \vec{E} + C_{3,\alpha}$$

$$\nabla\cdot\left(\rho_\alpha e_\alpha \vec{u}_\alpha\right)$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha u_\alpha^2 + \rho_\alpha e_\alpha\right) + \nabla\cdot\left[\left(\frac{1}{2}\rho_\alpha u_\alpha^2 + \rho_\alpha e_\alpha\right)\vec{u}_\alpha + \vec{q}_\alpha + \right.$$

$$\left. + \hat{p}_\alpha\cdot\vec{u}_\alpha\right) = \vec{j}_\alpha\cdot\vec{E} + C_{3,\alpha}$$

$$\vec{q} = \frac{1}{2}\sum_\alpha{}' \rho_\alpha \langle(\vec{v}-\vec{u})^2(\vec{v}-\vec{u})\rangle_\alpha$$

$$\vec{q}_\alpha = \frac{1}{2}\rho_\alpha\langle w_\alpha^2 \vec{w}_\alpha\rangle_\alpha$$

$$\vec{q} = \sum_\alpha{}' \frac{1}{2}\rho_\alpha u_{diff_\alpha}^2 \vec{u}_{diff_\alpha} + \sum_\alpha{}' \vec{q}_\alpha + \sum_\alpha{}' \frac{3}{2}p_\alpha \vec{u}_{diff,\alpha}$$

$$+ \sum_\alpha{}' \hat{p}_\alpha\cdot\vec{u}_{diff,\alpha}$$

$$\rho_\alpha\langle v^2\rangle_\alpha = \rho_\alpha u_\alpha^2 + 3p_\alpha$$

$$\sum_\alpha{}' \frac{1}{2}\rho_\alpha\langle v^2\rangle_\alpha = \sum_\alpha{}' \frac{1}{2}\rho_\alpha u_\alpha^2 + \frac{3}{2}\sum_\alpha{}' p_\alpha$$

- ukupni skal. pritisak:

$$p = \sum_\alpha{}' p_\alpha + \frac{1}{3}\sum_\alpha{}' \rho_\alpha u_{diff_\alpha}^2$$

$$\sum_\alpha{}' p_\alpha = p - \frac{1}{3}\sum_\alpha{}' \rho_\alpha u_{diff_\alpha}^2$$

$$\vec{\ }$$

$$\sum_\alpha{}' \frac{1}{2}\rho_\alpha \langle v^2\rangle_\alpha = \sum_\alpha{}' \frac{1}{2}\rho_\alpha u_\alpha^2 + \frac{3}{2}\sum_\alpha{}' p_\alpha$$

$$= \sum_\alpha{}' \frac{1}{2}\rho_\alpha u_\alpha^2 + \frac{3}{2}p - \frac{1}{2}\sum_\alpha{}' \rho_\alpha u_{diff\,\alpha}^2 =$$

$$\vec{u}_{diff,\alpha} = \vec{u}_\alpha - \vec{u}$$

$$= \sum_\alpha{}' \frac{1}{2}\rho_\alpha u_\alpha^2 + \frac{3}{2}p - \frac{1}{2}\sum_\alpha{}' \rho_\alpha (\vec{u}_\alpha - \vec{u})\cdot(\vec{u}_\alpha - \vec{u}) =$$

$$= \cancel{\sum_\alpha{}' \frac{1}{2}\rho_\alpha u_\alpha^2} + \frac{3}{2}p - \cancel{\frac{1}{2}\sum_\alpha{}' \rho_\alpha u_\alpha^2} - \frac{1}{2}\underbrace{\sum_\alpha{}' \rho_\alpha}_{\rho} u^2 +$$

$$+ \cancel{2}\cdot\cancel{\frac{1}{2}}\sum_\alpha{}' \underbrace{\rho_\alpha \vec{u}_\alpha}_{\rho\vec{u}}\cdot\vec{u} =$$

$$= \frac{3}{2}p - \frac{1}{2}\rho u^2 + \rho u^2$$

$$= \frac{3}{2}p + \frac{1}{2}\rho u^2$$

$$p + \frac{1}{3}\rho u^2 = \sum_\alpha{}' p_\alpha + \frac{1}{3}\sum_\alpha{}' \rho_\alpha u_{diff,\alpha}^2 + \frac{1}{3}\rho u^2$$

$$= \sum_\alpha{}' p_\alpha + \frac{1}{3}\rho u^2 + \frac{1}{3}\sum_\alpha{}' \rho_\alpha (\vec{u}_\alpha - \vec{u})\cdot(\vec{u}_\alpha - \vec{u}) =$$

$$= \sum_\alpha{}' p_\alpha + \frac{1}{3}\rho u^2 + \frac{1}{3}\sum_\alpha{}' \rho_\alpha u_\alpha^2 + \frac{1}{3}\sum_\alpha{}' \rho_\alpha u^2 -$$

$$- \frac{2}{3}\sum_\alpha{}' \rho_\alpha \vec{u}_\alpha \cdot \vec{u} =$$

$$= \sum_\alpha{}' p_\alpha + \cancel{\frac{1}{3}\rho u^2} + \frac{1}{3}\sum_\alpha{}' \rho_\alpha u_\alpha^2 + \cancel{\frac{1}{3}\rho u^2} -$$

$$- \cancel{\frac{2}{3}\rho u^2} =$$

$$p + \frac{1}{3}\rho u^2 = \sum_\alpha{}' p_\alpha + \frac{1}{3}\sum_\alpha{}' \rho_\alpha u_\alpha^2$$

• 12 ⊛

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha \langle v^2\rangle_\alpha\right) + \nabla\cdot\left(\frac{1}{2}\rho_\alpha \langle v^2 \vec{v}\rangle_\alpha\right) = \vec{j}_\alpha \cdot \vec{E} + C_{3,\alpha}$$

$$/\sum_\alpha{}' \quad , \quad \sum_\alpha{}' C_{3,\alpha} = 0$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\sum_\alpha{}' \rho_\alpha \langle v^2\rangle_\alpha\right) + \nabla\cdot\left(\frac{1}{2}\sum_\alpha{}' \rho_\alpha \langle v^2 \vec{v}\rangle_\alpha\right) =$$

$$= \sum_\alpha{}' \vec{j}_\alpha \cdot \vec{E} \quad ⊛$$

$$\frac{1}{2}\rho_\alpha\langle v^2\vec{v}\rangle_\alpha = \frac{1}{2}\rho_\alpha\Big(u_\alpha^2\vec{u}_\alpha + \langle w_\alpha^2\vec{w}_\alpha\rangle_\alpha + \langle w_\alpha^2\rangle_\alpha\vec{u}_\alpha +$$

$$\vec{u}_\alpha = \vec{u}_{diff,\alpha} + \vec{u} \qquad p_\alpha = \frac{1}{3}\rho_\alpha\langle w_\alpha^2\rangle_\alpha$$

$$+ 2\langle\vec{w}_\alpha\otimes\vec{w}_\alpha\rangle_\alpha\cdot\vec{u}_\alpha =$$

$$= \frac{1}{2}\rho_\alpha\Big(\big(u_{diff,\alpha}^2 + 2\vec{u}\cdot\vec{u}_{diff,\alpha} + u^2\big)\big(\vec{u}_{diff,\alpha}+\vec{u}\big)\Big) +$$

$$+ \underbrace{\frac{1}{2}\rho_\alpha\langle w_\alpha^2\vec{w}_\alpha\rangle_\alpha}_{\vec{q}_\alpha} + \frac{1}{2}\underbrace{\rho_\alpha\langle w_\alpha^2\rangle_\alpha}_{3p_\alpha}\vec{u}_\alpha +$$

$$+ \cancel{\frac{1}{2}}\cdot\cancel{2}\underbrace{\rho_\alpha\langle\vec{w}_\alpha\otimes\vec{w}_\alpha\rangle_\alpha}_{\hat{p}_\alpha}\cdot\vec{u}_\alpha =$$

$$= \frac{1}{2}\rho_\alpha\big(u_{diff_\alpha}^2 + 2\vec{u}\cdot\vec{u}_{diff_\alpha} + u^2\big)\big(\vec{u}_{diff_\alpha}+\vec{u}\big) +$$

$$+ \vec{q}_\alpha + \frac{3}{2}p_\alpha + \hat{p}_\alpha\cdot\vec{u}_\alpha$$

$$\sum_\alpha{}' \frac{1}{2}\rho_\alpha\langle v^2\vec{v}\rangle_\alpha =$$

$$= \sum_\alpha{}' \frac{1}{2}\rho_\alpha u_{diff,\alpha}^2\vec{u}_{diff,\alpha} + \sum_\alpha{}' \frac{1}{2}\rho_\alpha u_{diff_\alpha}^2\vec{u} +$$

$$+\sum_\alpha{}' \frac{1}{2}\rho_\alpha \cdot 2(\vec{u}\cdot\vec{u}_{diff\,\alpha})\vec{u}_{diff\,\alpha} + \cancel{\sum_\alpha{}' \frac{1}{2}\rho_\alpha\, 2(\vec{u}\cdot\vec{u}_{diff\,\alpha})\vec{u}} +$$

$$+\cancel{\sum_\alpha{}' \frac{1}{2}\rho_\alpha u^2 \vec{u}_{diff\,\alpha}} + \sum_\alpha{}' \frac{1}{2}\rho_\alpha u^2 \vec{u} + \sum_\alpha{}' \vec{q}_\alpha + \sum_\alpha{}' \frac{3}{2} p_\alpha \vec{u}_\alpha$$

$$+\sum_\alpha{}' \hat{P}_\alpha \cdot \vec{u}_{diff\,\alpha} + \sum_\alpha{}' \hat{P}_\alpha \cdot \vec{u}$$

$$\sum_\alpha{}' \rho_\alpha \vec{u}_{diff} = 0$$

$$\sum_\alpha{}' \frac{1}{2}\rho_\alpha \langle v^2\vec{v}\rangle_\alpha = \sum_\alpha{}' \frac{1}{2}\rho_\alpha u^2_{diff\,\alpha}\vec{u}_{diff\,\alpha} + \sum_\alpha{}' \frac{1}{2}\rho_\alpha u^2_{diff\,\alpha}\vec{u}$$

$$+\sum_\alpha{}' \frac{1}{2}\rho_\alpha u^2 \vec{u} + \sum_\alpha{}' \rho_\alpha (\vec{u}\cdot\vec{u}_{diff\,\alpha})\vec{u}_{diff\,\alpha} +$$

$$+\sum_\alpha{}' \vec{q}_\alpha + \sum_\alpha{}' \frac{3}{2} p_\alpha \vec{u}_{diff\,\alpha} + \sum_\alpha{}' \frac{3}{2} p_\alpha \vec{u} +$$

$$+\sum_\alpha{}' \hat{P}_\alpha \cdot \vec{u}_{diff\,\alpha} + \sum_\alpha{}' \hat{P}_\alpha \cdot \vec{u}$$

$\vec{u}\cdot\hat{P} = \hat{P}\cdot\vec{u}$

$\hat{P}$ - simetričan!

$$\sum_\alpha{}' \frac{1}{2}\rho_\alpha \langle v^2\vec{v}\rangle_\alpha = \vec{q} + \vec{u}\cdot\hat{P} +$$

$$+\sum_\alpha{}' \frac{1}{2}\rho_\alpha \left(u^2_{diff\,\alpha} + u^2\right)\vec{u} + \sum_\alpha{}' \frac{3}{2} p_\alpha \vec{u}$$

nastavak ⊛:

$$\frac{\partial}{\partial t}\left(\frac{3}{2}p+\frac{1}{2}\rho u^2\right)+\nabla\cdot\left(\vec{q}+\vec{u}\cdot\hat{P}+\sum_\alpha\frac{1}{2}\rho_\alpha\left(u^2_{diff\,\alpha}+u^2\right)\vec{u}+\right.$$

$$\left.+\sum_\alpha\frac{3}{2}p_\alpha\vec{u}\right)=\vec{j}\cdot\vec{E}$$

$$\frac{1}{2}\sum_\alpha\rho_\alpha u^2_{diff\,\alpha}\vec{u}=\frac{3}{2}p\vec{u}-\frac{3}{2}\sum_\alpha p_\alpha\vec{u}$$

jer $\quad p=\sum_\alpha p_\alpha+\frac{1}{3}\sum_\alpha\rho_\alpha u^2_{diff\,\alpha}$

$$\frac{\partial}{\partial t}\left(\frac{3p}{2}+\frac{1}{2}\rho u^2\right)+\nabla\cdot\left(\vec{q}+\vec{u}\cdot\hat{P}+\frac{3}{2}p\vec{u}-\cancel{\frac{3}{2}\sum_\alpha p_\alpha\vec{u}}\right.$$

$$\left.+\underbrace{\sum_\alpha\frac{1}{2}\rho_\alpha}_{\rho}u^2\vec{u}+\cancel{\sum_\alpha\frac{3}{2}p_\alpha\vec{u}}\right)=\vec{j}\cdot\vec{E}$$

$$\boxed{\frac{\partial}{\partial t}\left(\frac{3p}{2}+\frac{1}{2}\rho u^2\right)+\nabla\cdot\left(\left(\frac{3}{2}p+\frac{1}{2}\rho u^2\right)\vec{u}+\vec{q}+\vec{u}\cdot\hat{P}\right)=\vec{j}\cdot\vec{E}}$$

jdba

$$\frac{3p}{2}=\rho e$$

$$\boxed{\frac{\partial}{\partial t}\left(\rho e + \frac{1}{2}\rho u^2\right) + \nabla\cdot\left(\left(\rho e + \frac{1}{2}\rho u^2\right)\vec{u} + \vec{u}\cdot\hat{P} + \vec{q}\right) = \vec{j}\cdot\vec{E}}$$