ZAD nad. čest. $\vec{B} = B\vec{e}_\varphi$, $B = \text{const.}$

$$m\frac{d\vec{v}}{dt} = q(\vec{v}\times\vec{B}) \qquad \vec{B} = \begin{bmatrix} 0 \\ B \\ 0 \end{bmatrix}$$

cilindr. (ρ, φ, z)

(*) $\vec{r} = \rho\vec{e}_\rho + z\vec{e}_z$

$\dot{\vec{r}} = \dot{\rho}\vec{e}_\rho + \rho\dot{\varphi}\vec{e}_\varphi + \dot{z}\vec{e}_z$

$\ddot{\vec{r}} = (\ddot{\rho} - \rho\dot{\varphi}^2)\vec{e}_\rho + (2\dot{\rho}\dot{\varphi} + \rho\ddot{\varphi})\vec{e}_\varphi + \ddot{z}\vec{e}_z$

$$\vec{v}\times\vec{B} = \begin{vmatrix} \vec{e}_\rho & \vec{e}_\varphi & \vec{e}_z \\ \dot{\rho} & \rho\dot{\varphi} & \dot{z} \\ 0 & B & 0 \end{vmatrix} = \begin{bmatrix} -\dot{z}B \\ 0 \\ \dot{\rho}B \end{bmatrix}$$

$\vec{e}_\rho$: $m(\ddot{\rho} - \rho\dot{\varphi}^2) = -q\dot{z}B$ (I)

$\vec{e}_\varphi$: $m(2\dot{\rho}\dot{\varphi} + \rho\ddot{\varphi}) = 0$ (II) $\qquad \vec{B} = B\vec{e}_\varphi$

$\vec{e}_z$: $m\ddot{z} = q\dot{\rho}B$ (I)

(II) $v_\varphi = v_{\parallel} = \rho\dot{\varphi} \qquad \dot{\varphi} = \dfrac{v_{\parallel}}{\rho}$

miro

$\vec{e}_\rho$

$m\rho\dot{\varphi}^2$ – zakrivljenost linija

$\rho B\dot{z}$ – $\exists$ grad $\perp$ $\vec{B}$

$\rightarrow$ ρ igra ulogu R_c $\qquad \dot{\varphi} = \dfrac{v_\parallel}{R_c}$

$$\vec{F}_{cf} = m\rho\dot{\varphi}^2\vec{e}_\rho = m R_c \frac{v_\parallel^2}{R_c^2}\vec{e}_\rho = \frac{m v_\parallel^2}{R_c}\vec{e}_\rho$$

$(*)$ $\quad \propto \vec{B}\times\underbrace{(\vec{b}\cdot\nabla)\vec{b}}_{\text{zakr. linija}}$

ZAD vodonična plazma n_α

$\vec{B}$ – stac. i slabo nehom. (⊕ zakrivljene linije)

$(\vec{j}_u)_\perp = ?$

$$\vec{j} = n_{p^+}|q_e|\vec{v}_{p^+} + n_{e^-}(-|q_e|)\vec{v}_{e^-}$$

miro

- gustina driftne struje:

$$\vec{j}_D = \sum_\alpha n_\alpha q_\alpha \langle \vec{v}_D \rangle_\alpha$$

1. $\vec{j}_D^{\,m\vec{g}} = \dfrac{\rho\, \vec{g} \times \vec{B}}{B^2}$ $\qquad \rho = \sum_\alpha n_\alpha m_\alpha$

2. $\vec{j}_D^{\,\vec{E}\times\vec{B}} = \vec{0}$

3. gradijentni drift: $\vec{j}_D^{\,\mathrm{grad} B} = \dfrac{W_\perp}{B^3} \underbrace{\vec{B} \times \nabla B}$

4. centrifugalni drift: $\vec{j}_D^{\,cf} = \dfrac{2W_\parallel}{B^2} \underbrace{\vec{B} \times (\vec{b} \cdot \nabla)\vec{b}}$

5. $\underbrace{\vec{j}_M = \nabla \times \vec{M}}_{\exists \text{ nehomogenost } (\vec{B})}$ $\qquad \vec{M} = \sum_\alpha n_\alpha \langle \vec{M} \rangle_\alpha = -\dfrac{W_\perp}{B} \vec{b}$

$$W_\perp = \sum_\alpha n_\alpha \langle W_\perp \rangle_\alpha$$

$$\vec{j}_u = \underbrace{\nabla \times \vec{M}}_{\mathrm{I}} + \underbrace{\vec{j}_D^{\,\mathrm{grad} B}}_{\mathrm{II}} + \underbrace{\vec{j}_D^{\,cf}}_{\mathrm{III}}$$

miro

(I) $$\nabla \times \vec{\mu} = \nabla \times \left(- \frac{W_\perp}{B} \vec{b} \right) = \nabla \left(- \frac{W_\perp}{B} \right) \times \vec{b} + \left(- \frac{W_\perp}{B} \right) (\nabla \times \vec{b}) =$$

$$= - \frac{B \nabla W_\perp - W_\perp \nabla B}{B^2} \times \vec{b} - \frac{W_\perp}{B} (\nabla \times \vec{b}) =$$

$$= - \frac{\nabla W_\perp}{B} \times \vec{b} + \underbrace{\cancel{\frac{W_\perp}{B^2} \nabla B \times \vec{b}}} - \frac{W_\perp}{B} (\nabla \times \vec{b})$$

(II) $$\frac{W_\perp}{B^3} \vec{B} \times \nabla B = \underbrace{\cancel{\frac{W_\perp}{B^2} \vec{b} \times \nabla B}}$$

$\vec{a} \times \vec{b} = - \vec{b} \times \vec{a}$

(III) $$\frac{2W_\parallel}{B^2} \vec{B} \times \underbrace{(\vec{b} \cdot \nabla)}\vec{b} = \frac{2W_\parallel}{B^2} \vec{B} \times \left(\frac{\nabla_\perp B}{B} \right)$$

$\underline{\nabla_\perp B = B (\vec{b} \cdot \nabla) \vec{b}}$

$$\vec{j}_u = - \frac{\nabla W_\perp}{B} \times \vec{b} - \underbrace{\frac{W_\perp}{B} (\nabla \times \vec{b})}_{*} + \frac{2W_\parallel}{B^2} \vec{B} \times \left(\frac{\nabla_\perp B}{B} \right)$$

$\vec{B} = B \vec{b}$,

$$(\vec{j}_u)_\perp = (\vec{b} \times \vec{j}_u) \times \vec{b}$$

$\vec{b} \times$ vektor

$(*)$

$$\underbrace{\left(\vec{b}\times(\nabla\times\vec{b})\right)}_{-(\vec{b}\cdot\nabla)\vec{b}}\times\vec{b} = -\left((\vec{b}\cdot\nabla)\vec{b}\right)\times\vec{b} =$$

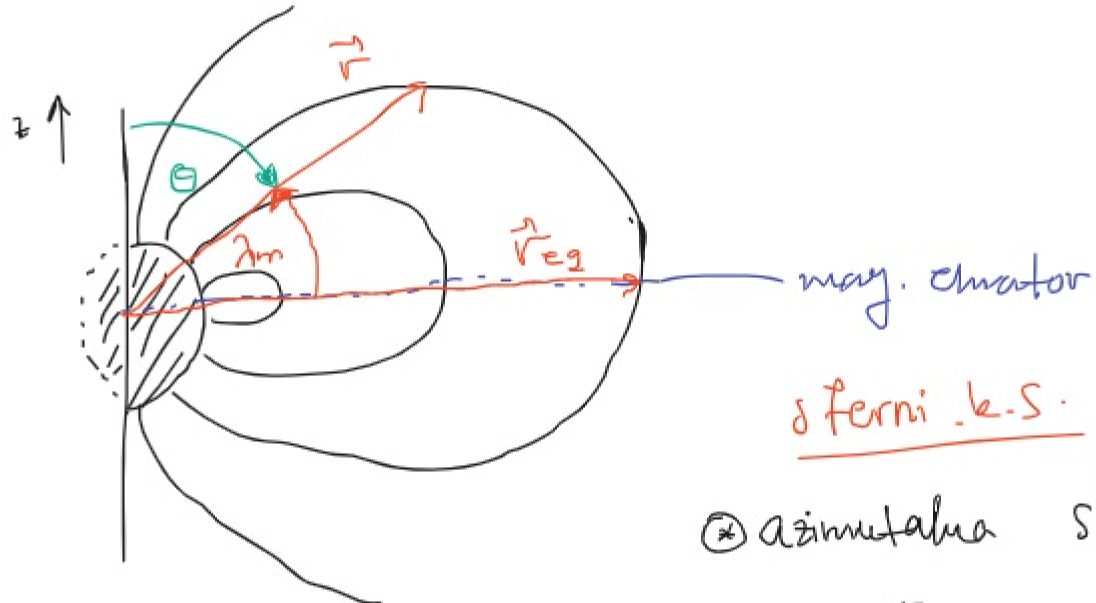
$$= \vec{b}\times\underbrace{(\vec{b}\cdot\nabla)\vec{b}}_{\frac{\vec{\nabla}_\perp B}{B}}$$

$\vec{B} = B\times\vec{b}$

$$(\vec{j}^u)_\perp = \vec{b}\times\frac{\nabla W_\perp}{B} + \frac{2W_\parallel}{B^2}\vec{B}\times\left(\frac{\nabla_\perp B}{B}\right) - \frac{W_\perp}{B^2}\vec{b}\times\nabla_\perp B$$

$$\boxed{(\vec{j}^u)_\perp = \vec{b}\times\frac{\nabla W_\perp}{B} + \left(\frac{2W_\parallel - W_\perp}{B^2}\right)\vec{b}\times\nabla_\perp B}$$

ZAD $\vec{B}(\vec{r}) = ?$ za dipolno mag. polje Zemlje



sferni k.s. $(r, \theta, \varphi) \to (r, \lambda_m, \varphi)$

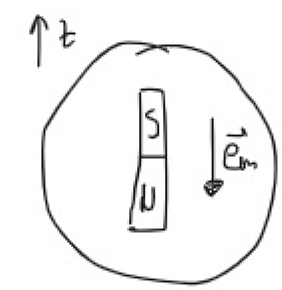
⊛ azimutalna simetrija

⊛ $\theta = \frac{\pi}{2} - \lambda_m$ ←

$\vec{e}_\theta = -\vec{e}_{\lambda_m}$

⊛ $\vec{r}(\lambda_m = 0) = \vec{r}_{eq}$ ←

⊛ $\vec{r} = r\vec{e}_r$



$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{3\vec{r}(\vec{r}\cdot\vec{M}) - r^2\vec{M}}{r^5}$$

$\vec{M} = M\vec{e}_m$

$\vec{e}_m = -\vec{e}_z$

$\vec{e}_z = \cos\theta\,\vec{e}_r - \sin\theta\,\vec{e}_\theta$

$$\vec{e}_z = \cos\left(\frac{\pi}{2} - \lambda_m\right)\vec{e}_r - \sin\left(\frac{\pi}{2} - \lambda_m\right)(-\vec{e}_{\lambda_m})$$

miro

$$\vec{e}_t = \sin\lambda_m \, \vec{e}_r + \cos\lambda_m \, \vec{e}_{\lambda m}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{3r\vec{e}_r\left(r\vec{e}_r \cdot (-M\vec{e}_t)\right) - r^2(-M\vec{e}_t)}{r^5} \qquad \vec{e}_r \cdot \vec{e}_{\lambda m} = \vec{e}_r \cdot (-\vec{e}_\theta) = 0$$

$$\vec{B}(\vec{r}) = \frac{\mu_0 M}{4\pi r^3}\left(-3\vec{e}_r\left(\vec{e}_r \cdot (\sin\lambda_m \vec{e}_r + \cos\lambda_m \vec{e}_{\lambda m})\right) + (\sin\lambda_m \vec{e}_r + \cos\lambda_m \vec{e}_{\lambda m})\right)$$

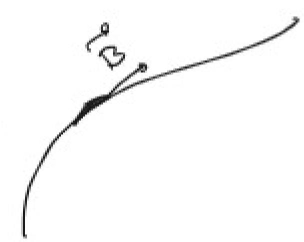
$$\vec{B}(\vec{r}) = \frac{\mu_0 M}{4\pi r^3}\left(-3\vec{e}_r \sin\lambda_m + \sin\lambda_m \vec{e}_r + \cos\lambda_m \vec{e}_{\lambda m}\right)$$

$$\boxed{\vec{B}(\vec{r}) = \frac{\mu_0 M_\oplus}{4\pi r^3}\left(-2\sin\lambda_m \vec{e}_r + \cos\lambda_m \vec{e}_{\lambda m}\right)} \qquad M = M_\oplus$$

ZAD jdne dip. mag. linije ?

dužina mag. linije za r_{eq}?

$\vec{B}$ u $\forall$ tački ima pravac tang. na liniju



ds - luk duž. mag. linije $\rightarrow d\vec{s}$

$d\vec{s} \times \vec{B} = \vec{0}$

⊛ Dekartove koord. $d\vec{s} = dx\,\vec{e}_x + dy\,\vec{e}_y + dz\,\vec{e}_z$, $\vec{B} = (B_x, B_y, B_z)$

$$d\vec{s} \times \vec{B} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ dx & dy & dz \\ B_x & B_y & B_z \end{vmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{dx}{B_x} = \frac{dy}{B_y} = \frac{dz}{B_z}$$ opšti slučaj

⊛ dip. mag. polje Zemlje : k.s (r, λ_m, φ)

sferne: $d\vec{s} = dr\,\vec{e}_r + \underset{h_i}{r}\,d\theta\,\vec{e}_\theta + \underbrace{r\sin\theta}_{h}\,d\varphi\,\vec{e}_\varphi$

$\theta = \frac{\pi}{2} - \lambda_m$

$d\theta = -d\lambda_m$

$\vec{e}_\theta = -\vec{e}_{\lambda_m}$

$d\varphi = 0$

$$\boxed{d\vec{s} = dr\,\vec{e}_r + r\,d\lambda_m\,\vec{e}_{\lambda_m}}$$

$$\vec{B} = B_r\,\vec{e}_r + B_{\lambda_m}\,\vec{e}_{\lambda_m} + B_\varphi\,\vec{e}_\varphi$$

$$\vec{B}(\vec{r}) = \frac{\mu_0 M_\oplus}{4\pi r^3}\left(-2\sin\lambda_m\,\vec{e}_r + \cos\lambda_m\,\vec{e}_{\lambda_m}\right)$$

miro

$$d\vec{s} \times \vec{B} = \begin{vmatrix} \vec{e}_r & \vec{e}_{\lambda_m} & \vec{e}_\varphi \\ dr & r\,d\lambda_m & 0 \\ B_r & B_{\lambda_m} & 0 \end{vmatrix} = \begin{bmatrix} 0 \\ 0 \\ dr\,B_{\lambda_m} - r\,d\lambda_m\,B_r \end{bmatrix} = \vec{0}$$

$$\frac{dr}{B_r} = \frac{r\,d\lambda_m}{B_{\lambda_m}}$$

$$\frac{dr}{-2\sin\lambda_m} = \frac{r\,d\lambda_m}{\cos\lambda_m}$$

$$\frac{dr}{r} = -2\,\frac{\sin(\lambda_m)\,d\lambda_m}{\cos\lambda_m} = 2\,\frac{d(\cos\lambda_m)}{\cos\lambda_m} \quad / \int$$

$$\ln r = 2\ln(\cos\lambda_m) + \ln C$$

$$r = C\cos^2\lambda_m$$

$$C = ? \qquad \left.\begin{array}{l} \lambda_m = 0 \Rightarrow r = C \\ \lambda_m = 0 \Rightarrow r = r_{eq} \end{array}\right\} \; C = r_{eq}$$

miro

$$\boxed{r = r_{eq}\cos^2\lambda_m}$$ jdna $\overbrace{\text{mag.}}^{\text{dipolne}}$ linije

? dužina

$$d\vec{s}\cdot d\vec{s} = (ds)^2 = (dr)^2 + (r\,d\lambda_m)^2 \qquad /:(d\lambda_m)^2$$

$$\left(\frac{ds}{d\lambda_m}\right)^2 = \left(\frac{dr}{d\lambda_m}\right)^2 + r^2$$

$$\frac{ds}{d\lambda_m} = \sqrt{\left(\frac{dr}{d\lambda_m}\right)^2 + r^2}$$

$$\frac{dr}{d\lambda_m} = r_{eq}\,2\cos\lambda_m\,(-\sin\lambda_m) = -2r_{eq}\sin\lambda_m\cos\lambda_m$$

$$\frac{ds}{d\lambda_m} = \sqrt{4r_{eq}^2\sin^2\lambda_m\cos^2\lambda_m + r_{eq}^2\cos^4\lambda_m}$$

miro

$$\frac{ds}{d\lambda_m} = r_{eq} \cos\lambda_m \sqrt{4\sin^2\lambda_m + \cos^2\lambda_m}$$

$$\boxed{\frac{ds}{d\lambda_m} = r_{eq} \cos\lambda_m \sqrt{1+3\sin^2\lambda_m}}$$

ZAD $|\vec{B}|$ i B_{min}

$$\vec{B}(\vec{r}) = \frac{\mu_0 M_\oplus}{4\pi r^3}\left(-2\sin\lambda_m \vec{e}_r + \cos\lambda_m \vec{e}_{\lambda_m}\right)$$

$$B = |\vec{B}| = \frac{\mu_0 M_\oplus}{4\pi r^3}\sqrt{4\sin^2\lambda_m + \cos^2\lambda_m}$$

$$\boxed{B = \frac{\mu_0 M_\oplus}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m}}$$

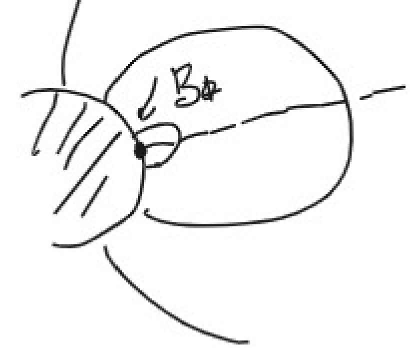
$B = B(\lambda_m)$

miro

uvode se :

1) $L = \dfrac{r_{eq}}{R_\oplus}$

2) $B_\oplus = \dfrac{\mu_0 M_\oplus}{4\pi R_\oplus^3} = 0{,}311\,G$

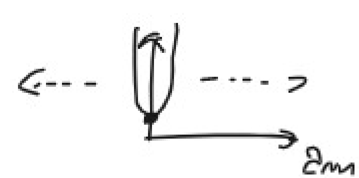


$\to r = L R_\oplus \cos^2 \lambda_m$

$$B = \frac{\mu_0 M_\oplus}{4\pi L^3 R_\oplus^3} \frac{1}{\cos^6 \lambda_m} \sqrt{1 + 3\sin^2 \lambda_m}$$

$$\boxed{B = \frac{B_\oplus}{L^3} \underbrace{\frac{\sqrt{1 + 3\sin^2 \lambda_m}}{\cos^6 \lambda_m}}}$$

$$\underline{B_{min} = \frac{B_\oplus}{L^3}}$$



ZAD vod. centri duž mag. linija } za E ?
u stac. i slabo nehom $\vec{B}$

$\vec{v}$ - brzina čestice

$$\langle \vec{v} \rangle_c = \vec{v}_{gc} = \langle \vec{v}_{\parallel} + \vec{v}_{\perp} \rangle_c = \underbrace{\langle \vec{v}_{\parallel} \rangle_c}_{\vec{V}_{\parallel}} + \underbrace{\langle \vec{v}_{\perp} \rangle_c}_{\vec{V}_D} = \vec{V}_{\parallel} + \vec{V}_D$$

- sistem. vod. centra

$$\tilde{\vec{v}}_{\perp} = \vec{v}_{\perp} - \vec{V}_D$$

$\vec{V}_D = \text{const}$

$$\langle \tilde{\vec{v}}_{\perp} \rangle_c = \langle \vec{v}_{\perp} - \vec{V}_D \rangle_c = \underbrace{\langle \vec{v}_{\perp} \rangle_c} - \vec{V}_D = \vec{V}_D - \vec{V}_D = \vec{0}$$

- $$\langle \tilde{v}_{\perp}^2 \rangle_c = \langle (\vec{v}_{\perp} - \vec{V}_D)^2 \rangle_c = \langle v_{\perp}^2 \rangle_c + V_D^2 - 2 \underbrace{\langle \vec{v}_{\perp} \rangle_c}_{\vec{V}_D} \vec{V}_D$$

$$= \langle v_{\perp}^2 \rangle_c - V_D^2 \Longrightarrow \langle v_{\perp}^2 \rangle_c = \langle \tilde{v}_{\perp}^2 \rangle_c + V_D^2$$

miro

• ukupna energija: $\langle w \rangle_c = W = \frac{m v_\parallel^2}{2} + \frac{1}{2} m \langle v_\perp^2 \rangle_c$

$$W = \frac{m v_\parallel^2}{2} + \frac{1}{2} m \langle \tilde{v}_\perp^2 \rangle_c + \frac{1}{2} m V_D^2 = const$$

• $M = const$ → ciklotronska rotacija

$$M = \frac{W_\perp}{B} = \frac{m}{2} \langle \tilde{v}_\perp^2 \rangle_c \frac{1}{B}$$

$$MB = \frac{1}{2} m \langle \tilde{v}_\perp^2 \rangle_c$$

→ $V_D = const$

$$W = \frac{m v_\parallel^2}{2} + MB + \frac{1}{2} m V_D^2 = const. \quad \Big/ \frac{d}{dt}$$

$$\boxed{\frac{d}{dt}\underbrace{\left(\frac{m}{2} v_\parallel^2\right)}_{w_\parallel} + M \frac{dB}{dt} + B \frac{dM}{dt} = 0} \quad (*)'$$

• na jdn za long. komp (II)

$$m \frac{d\vec{v}_{\parallel}}{dt} \vec{b} = -M \nabla_{\parallel} B \qquad \Big/ \cdot |\vec{v}_{\parallel}| = v_{\parallel} = \frac{ds}{dt}$$

$$m v_{\parallel} \frac{d\vec{v}_{\parallel}}{dt} \vec{b} = -v_{\parallel} M \nabla_{\parallel} B \qquad \nabla_{\parallel} B = \frac{\partial B}{\partial s} \vec{b}$$

$$\frac{d}{dt} \Big(\underbrace{\frac{m}{2} v_{\parallel}^2}_{W_{\parallel}} \Big) \vec{b} = -M \frac{ds}{dt} \frac{\partial B}{\partial s} \vec{b}$$

$$\frac{dW_{\parallel}}{dt} = -M \frac{dB}{dt}$$

$$\boxed{\frac{dW_{\parallel}}{dt} + M \frac{dB}{dt} = 0} \quad (*)^2$$

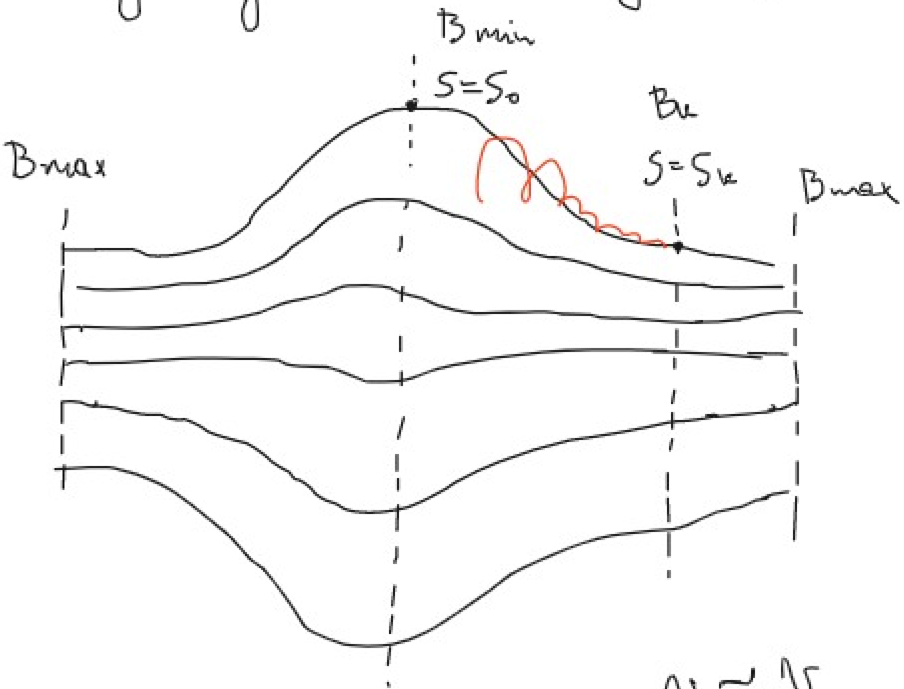
$$(*)^1, (*)^2 \Rightarrow B \frac{dM}{dt} = 0 \Longrightarrow M = \text{const}.$$

$$\boxed{\frac{m v_{\parallel}^2(s)}{2} + M B(s) = \text{const}}$$

miro

ZAD

Magn. ogledalom, mag. klopka



1) $s = s_0$ $\quad B_{min}$, $\tan\alpha = \frac{v_\perp}{v_\parallel}$

$v_\parallel = v_{\parallel max}$

2) $s = s_k$ $\quad B_k$ $\quad v_\parallel \to 0$, $v \to v_\perp$

$\alpha = 90^\circ$

$$v_\perp \approx v_c \quad \begin{cases} v_\perp \uparrow \text{ kako } \uparrow B \\ v_c = r_c \omega_c \propto \frac{1}{\sqrt{B}} B = \sqrt{B} \end{cases}$$

$v_\parallel^2 + v_\perp^2 = \text{const} \qquad v_\parallel \downarrow$

$v_\parallel = 0$

$$m \frac{dv_\parallel}{dt} \hat{b} = -\mu \nabla_\parallel B$$

ZoE: $$\frac{m v_\parallel^2(s)}{2} + \mu B(s) = \underbrace{\frac{m v_\parallel^2(s_k)}{2}}_{v_\parallel = 0} + \mu \underbrace{B(s_k)}_{B_k}$$

$$v_\parallel(s) = \sqrt{\frac{2}{m} \mu \left(B_k - B(s)\right)}$$

miro

? α ?

$$\operatorname{tg}\alpha = \frac{v_\perp}{v_\parallel} = \frac{\sin\alpha}{\cos\alpha} = \frac{v_\perp}{\underbrace{\sqrt{\frac{2}{m}\mu B(s)}}_{\simeq 1}} \quad \frac{1}{\sqrt{\frac{B_k}{B(s)} - 1}}$$

$$\mu B \simeq \frac{1}{2} m \langle \tilde{v}_\perp^2 \rangle_c = \frac{1}{2} m v_c^2$$

$$\sin^2\alpha = \frac{B(s)}{B_k} = \frac{\sin^2\alpha}{\sin^2 90^\circ}$$

$$\boxed{\frac{\sin^2\alpha}{B(s)} = \frac{\sin^2\alpha_0}{B_{min}}}$$

miro