30. mart

$$(1) \quad m\left(\underline{\underline{v_{\parallel}^2 \frac{\vec{e}_n}{R_c}}} + \left(\frac{d\vec{V}_D}{dt}\right)_\perp\right) = q\,\vec{V}_D \times \vec{B} - M\nabla_\perp B$$

$$\to m\left(\frac{d\vec{V}_D}{dt}\right)_\perp = \underbrace{q\vec{V}_D\times\vec{B} - M\nabla_\perp B - \boxed{m v_\parallel^2 \frac{\vec{e}_n}{R_c}}}_{\langle \vec{F}_\perp\rangle} \quad / \times \vec{B}$$

$$\frac{2W_\parallel}{R_c}\vec{e}_n$$

~ krivina mag. linija ~

$$\Rightarrow m\left(\frac{d\vec{V}_D}{dt}\right)_\perp \times \vec{B} = q\underbrace{\left(\vec{V}_D\times\vec{B}\right)\times\vec{B}} + \langle\vec{F}_\perp\rangle \times \vec{B}$$

$$\vec{B}\,(\underbrace{\vec{V}_D\cdot\vec{B}}_{\perp \Rightarrow 0}) - \underbrace{\vec{V}_D\,(\vec{B}\cdot\vec{B})}_{B^2\vec{V}_D} = -B^2\vec{V}_D$$

$$m\left(\frac{d\vec{V}_D}{dt}\right)_\perp \times \vec{B} - \langle \vec{F}_\perp \rangle \times \vec{B} = -qB^2 \vec{V}_D$$

$$\boxed{\vec{V}_D = \frac{\left(\langle \vec{F}_\perp \rangle - m\left(\frac{d\vec{V}_D}{dt}\right)_\perp\right) \times \vec{B}}{qB^2}}$$

$$\vec{V}_D = \frac{1}{q}\frac{\vec{\mathbb{F}}_\perp \times \vec{B}}{B^2}$$

$$\vec{\mathbb{F}}_\perp = \langle \vec{F}_\perp \rangle - m\left(\frac{d\vec{V}_D}{dt}\right)_\perp$$

⊛ REŠAVANJE BRZINE DRIFTA:

pps. $\vec{V}_D(t) \approx const.$ $\longrightarrow$ ITERATIVNO $\vec{V}_D = \frac{1}{q}\frac{\vec{\mathbb{F}}_\perp \times \vec{B}}{B^2}$

1. red: pps: nema promene $\vec{V}_D \Rightarrow \frac{d\vec{V}_D}{dt} = \vec{0}$

$$\vec{V}_D^{(I)} = \frac{\langle \vec{F}_\perp \rangle \times \vec{B}}{qB^2}$$

miro

$$\vec{V}_D^{(I)} = \frac{1}{qB^2}\left(-M\nabla_\perp B - \frac{2W_\parallel}{R_C}\vec{e}_n\right)\times\vec{B}$$

$$\vec{V}_D^{(I)} = -\frac{M}{qB^2}\left(\nabla_\perp B\times\vec{B}\right) - \frac{2W_\parallel}{qB^2}\left(\frac{\vec{e}_n}{R_C}\times\vec{B}\right)$$

$$\frac{\vec{e}_n}{R_C} \downarrow (\vec{b}\cdot\nabla)\vec{b}$$

$$! \; \nabla_\parallel B\times\vec{B} = \vec{0}$$

$$\nabla B = \underline{\underline{\nabla_\parallel}} B + \nabla_\perp B$$

$$\boxed{\vec{V}_D^{(I)} = \underbrace{\frac{M}{qB^2}\left(\vec{B}\times\underline{\underline{\nabla B}}\right)}_{\text{GRADIJENTNI DRIFT VOD. CENTRA}} + \underbrace{\frac{2W_\parallel}{qB^2}\left(\vec{B}\times(\vec{b}\cdot\nabla)\vec{b}\right)}_{\text{CENTRIFUGALNI DRIFT VOD. CENTRA}}}$$

\# <u>diskusija</u> $\vec{B} \to$ Laplasovo polje $(\nabla\cdot\vec{a} = 0, \; \nabla\times\vec{a} = \vec{0})$

$$\nabla\times\vec{B} \simeq \vec{0}$$

miro

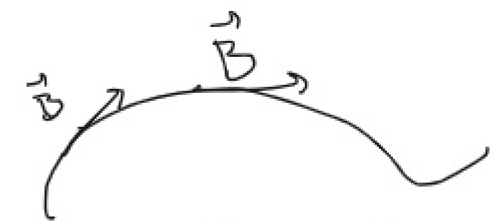
$$\rightarrow \quad \nabla(B^2) = \nabla(B \cdot B) = B\nabla B + B\nabla B = 2B\nabla B$$

$$\leftarrow \quad \nabla(\vec{B}\cdot\vec{B}) = 2\vec{B}\times(\nabla\times\vec{B}) + 2(\vec{B}\cdot\nabla)\vec{B} = 2(\vec{B}\cdot\nabla)\vec{B}$$

$$\rightarrow \quad \nabla(\vec{A}\cdot\vec{B}) = \underset{=}{\vec{A}}\times(\nabla\times\vec{B}) + \vec{B}\times(\nabla\times\underset{=}{\vec{A}}) + (\vec{A}\cdot\nabla)\vec{B} + (\vec{B}\cdot\nabla)\vec{A}$$

$$\Rightarrow \quad B\nabla B = (\vec{B}\cdot\nabla)\vec{B}$$

$$\boxed{\nabla B = (\vec{b}\cdot\nabla)\vec{B}}$$



$$\vec{B} = B\,\vec{b}$$

$\nabla_\perp B = ?$

$$\nabla\times\vec{B} = \vec{0}$$

$$\nabla\times(\underset{\overset{\uparrow}{f}}{B}\,\underset{\overset{\uparrow}{\vec{A}}}{\vec{b}}) = \nabla B\times\vec{b} + B(\nabla\times\vec{b})$$

$$\nabla B\times\vec{b} = -B(\nabla\times\vec{b})$$

$$\vec{b}\times\nabla B = B(\nabla\times\vec{b})$$

$$|\vec{b}| = 1$$

$$\nabla_\perp B = (\vec{b} \times \nabla B) \times \vec{b} = B(\nabla \times \vec{b}) \times \vec{b} = -B\,\vec{b} \times (\nabla \times \vec{b})$$

$$\nabla(\vec{b} \cdot \vec{b}) = 0$$

$$\nabla(\vec{b} \cdot \vec{b}) = 2\,\vec{b} \times (\nabla \times \vec{b}) + 2(\vec{b} \cdot \nabla)\vec{b}$$

$$-\vec{b} \times (\nabla \times \vec{b}) = (\vec{b} \cdot \nabla)\vec{b}$$

$$\nabla_\perp B = B\,\underbrace{(\vec{b} \cdot \nabla)}\vec{b} = B\,\frac{\vec{e_n}}{R_c}$$

$$\boxed{\frac{\nabla_\perp B}{B} = \frac{\vec{e_n}}{R_c}}$$

$$\left| r_c \, \frac{\|\nabla_\perp B\|}{B} \right| \ll 1$$

ZAD Mag. polje: $\vec{B} \approx \left(B_0 + \left(\frac{\partial B}{\partial x}\right)_0 x \right) \vec{e}_z$

$B_0 = B(\vec{r}_{gc}(0))$ $\qquad \vec{r}_c = \vec{r} - \vec{r}_{gc}$

pps. linije polja skoro prave

$\left(\frac{\partial B}{\partial x}\right)_0$ – mala veličina

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* $m \frac{d\vec{v}_\perp}{dt} = q \vec{v}_\perp \times \vec{B}$

$$\vec{v}_\perp = \begin{bmatrix} \dot{x} \\ \dot{y} \\ 0 \end{bmatrix} \qquad \dot{\vec{v}}_\perp = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ 0 \end{bmatrix} \qquad \vec{B} = \begin{bmatrix} 0 \\ 0 \\ B_0 + \left(\frac{\partial B}{\partial x}\right)_0 x \end{bmatrix}$$

$$\vec{v}_\perp \times \vec{B} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \dot{x} & \dot{y} & 0 \\ 0 & 0 & \underbrace{B_0 + \left(\frac{\partial B}{\partial x}\right)_0 x}_{\gamma} \end{vmatrix} = \begin{bmatrix} \gamma \dot{y} \\ -\gamma \dot{x} \\ 0 \end{bmatrix}$$

$$m\ddot{x} = q\left(B_0 + \left(\frac{\partial B}{\partial x}\right)_0 x\right)\dot{y}$$

$$m\ddot{y} = -q\left(B_0 + \left(\frac{\partial B}{\partial x}\right)_0 x\right)\dot{x}$$

$$\ddot{x} = \frac{q}{m}B_0\dot{y} + \frac{q}{m}\left(\frac{\partial B}{\partial x}\right)_0 x\dot{y}$$

$$\ddot{y} = -\frac{q}{m}B_0\dot{x} - \frac{q}{m}\left(\frac{\partial B}{\partial x}\right)_0 x\dot{x}$$

$$\frac{q}{m}\left(\frac{\partial B}{\partial x}\right)_0 = \eta \text{ - malo!}$$

$$\frac{q}{m}B_0 = \omega_c$$

$$\boxed{\begin{aligned} \ddot{x} &= \omega_c\dot{y} + \eta x\dot{y} = (\omega_c + \eta x)\dot{y} \\ \ddot{y} &= -(\omega_c + \eta x)\dot{x} \end{aligned}} \quad (*)$$

• RAZVOJ F-je po MALOM PARAMETRU

$$x(t) = \sum_{k=0}^{\infty} \eta^k \psi^{(k)}(t) \; ; \qquad y(t) = \sum_{k=0}^{\infty} \eta^k \varphi^{(k)}(t)$$

$$\xi \qquad\qquad \zeta$$

nulti član: (uz η^0)

$$x(t) = \psi^{(0)} \qquad y(t) = \varphi^{(0)}$$

$$\ddot{x}(t) = \ddot{\psi}^{(0)} \qquad \ddot{y}(t) = \ddot{\varphi}^{(0)}$$

$$\left.\boxed{\begin{aligned} \ddot{x} &= \omega_c \dot{y} \\ \ddot{y} &= -\omega_c \dot{x} \end{aligned}}\right\} \text{ rešili kod hom. i stac. } \vec{B}$$

$$\boxed{\begin{aligned} \psi^{(0)} &= r_c (1 - \cos\omega_c t) \\ \varphi^{(0)} &= r_c \sin\omega_c t \end{aligned}} \qquad \begin{aligned} \dot{\psi}^{(0)} &= r_c \omega_c \sin\omega_c t \\ \dot{\varphi}^{(0)} &= r_c \omega_c \cos\omega_c t \end{aligned}$$

prvi red (η^1)

$$x(t) = \psi^{(0)} + \eta \psi^{(1)} \qquad y(t) = \varphi^{(0)} + \eta \varphi^{(1)}$$

$$\ddot{x}(t) = \ddot{\psi}^{(0)} + \eta \ddot{\psi}^{(1)} \qquad \ddot{y}(t) = \ddot{\varphi}^{(0)} + \eta \ddot{\varphi}^{(1)}$$

$\rightarrow$ u (*):

$$\ddot{\psi}^{(0)} + \eta \ddot{\psi}^{(1)} = \left[\omega_c + \eta\left(\psi^{(0)} + \eta \psi^{(1)}\right)\right]\left(\dot{\varphi}^{(0)} + \eta \dot{\varphi}^{(1)}\right)$$

$$\omega_c \cancel{\dot\varphi^{(0)}} + \eta\ddot\psi^{(1)} = \omega_c \cancel{\dot\varphi^{(0)}} + \omega_c\eta\dot\varphi^{(1)} + \eta\psi^{(0)}\dot\varphi^{(0)} + \cancel{\eta^2\psi^{(0)}\dot\varphi^{(1)}} +$$

$$+ \cancel{\eta^2\psi^{(1)}\dot\varphi^{(0)}} + \cancel{\eta^3\psi^{(1)}\dot\varphi^{(1)}}$$

η – malo , samo η

$$\boxed{\ddot\psi^{(1)} = \omega_c\dot\varphi^{(1)} + \psi^{(0)}\dot\varphi^{(0)}}$$

$$\ddot\varphi^{(0)} + \eta\ddot\varphi^{(1)} = -\left(\omega_c + \eta\left(\psi^{(0)} + \eta\psi^{(1)}\right)\right)\left(\dot\psi^{(0)} + \eta\dot\psi^{(1)}\right)$$

$$\boxed{\ddot\varphi^{(1)} = -\omega_c\dot\psi^{(1)} - \psi^{(0)}\dot\psi^{(0)}}$$

$$\ddot\psi^{(1)} = \omega_c\dot\varphi^{(1)} + r_c(1-\cos\omega_c t)\, r_c\omega_c\cos\omega_c t$$

$$\ddot\varphi^{(1)} = -\omega_c\dot\psi^{(1)} - r_c(1-\cos\omega_c t)\, r_c\omega_c\sin\omega_c t$$

$$r_c\omega_c = v_\perp^{0}$$

smena: $\chi = \psi^{(1)} + i\varphi^{(1)}$

$$\dot{\chi} = \dot{\psi}^{(1)} + i\dot{\varphi}^{(1)}$$

$$\rightarrow \ddot{\chi} = \ddot{\psi}^{(1)} + i\ddot{\varphi}^{(1)}$$

$$\ddot{\psi}^{(1)} + i\ddot{\varphi}^{(1)} = \omega_c \dot{\varphi}^{(1)} + r_c(1-\cos\omega_c t) r_c\omega_c \cos\omega_c t - i\omega_c \dot{\psi}^{(1)} - i r_c(1-\cos\omega_c t) r_c\omega_c \sin\omega_c t$$

$$\ddot{\psi}^{(1)} + i\ddot{\varphi}^{(1)} = \omega_c \frac{1}{i}\frac{i}{i}\left(i\dot{\varphi}^{(1)} - i^2\dot{\psi}^{(1)}\right) + r_c v_\perp^0 (1-\cos\omega_c t)\left[\cos\omega_c t - i\sin\omega_c t\right]$$

$$\ddot{\chi} = -i\omega_c \dot{\chi} + r_c v_\perp^0 (1-\cos\omega_c t)\, e^{-i\omega_c t}$$

smena: $\xi = \dot{\chi} \qquad \dot{\xi} = \ddot{\chi}$

$$\dot{\xi} = -i\omega_c \xi + r_c v_\perp^0 (1-\cos\omega_c t)\, e^{-i\omega_c t}$$

$$y' + P(x)y = Q(x) \quad ; \quad C \in \mathbb{C}$$

$$y = e^{-\int p(x)dx}\left[C + \int Q(x)\, e^{\int p(x)dx} dx\right]$$

$$p(x) = i\omega_c$$
$$Q(x) = r_c v_\perp^\circ (1-\cos\omega_c t)\, e^{-i\omega_c t}$$

$$\int i\omega_c \, dt = i\omega_c t$$

$$\int r_c v_\perp^\circ (1-\cos\omega_c t)\, \cancel{e^{-i\omega_c t}}\, \cancel{e^{i\omega_c t}}\, dt =$$

$$= r_c v_\perp^\circ \left(t - \frac{1}{\omega_c}\sin\omega_c t\right)$$

$$\xi(t) = e^{-i\omega_c t}\left[C + r_c^2\left(\omega_c t - \sin\omega_c t\right)\right]$$

$$\dot{\psi}^{(1)} + i\,\dot{p}^{(1)} = (C_1 + iC_2)\underbrace{e^{-i\omega_c t}}_{\cos\omega_c t - i\sin\omega_c t} + r_c^2(\omega_c t - \sin\omega_c t)(\cos\omega_c t - i\sin\omega_c t)$$

(R) $\dot{\psi}^{(1)} = C_1 \cos\omega_c t + C_2 \sin\omega_c t + r_c^2 \omega_c t \cos\omega_c t - r_c^2 \sin\omega_c t \cos\omega_c t$

(C) $\dot{\varphi}^{(1)} = C_2 \cos\omega_c t - C_1 \sin\omega_c t - r_c^2 \omega_c t \sin\omega_c t + r_c^2 \sin^2\omega_c t$

<u>početni uslovi:</u> $t=0$

$\dot{x}(0) = 0$ $\quad\quad$ $\dot{\psi}^{(0)} = 0$

$\dot{y}(0) = v_\perp^0$ $\quad\quad$ $\dot{\varphi}^{(0)} = v_\perp^0$

$$\dot{x} = \dot{\psi}^{(0)} + \eta \dot{\psi}^{(1)}$$

$$\dot{y} = \ldots$$

$$\dot{\psi}^{(1)} = \dot{\varphi}^{(1)} = 0$$

iz R, C: $\quad \dot{\psi}^{(1)}(0) = C_1 = 0$

$\dot{\varphi}^{(1)}(0) = C_2 = 0$

$$\dot{\psi}^{(1)} = r_c^2 \omega_c t \cos\omega_c t - r_c^2 \sin\omega_c t \cos\omega_c t$$

$$\dot{\varphi}^{(1)} = - r_c^2 \omega_c t \sin\omega_c t + r_c^2 \sin^2\omega_c t$$

$$\dot{x}(t) = r_c \omega_c \sin\omega_c t + \eta r_c^2 (\omega_c t - \sin\omega_c t)\cos\omega_c t$$

$$\dot{y}(t) = \underbrace{r_c \omega_c}_{v_\perp^0} \cos\omega_c t - \eta r_c^2 (\omega_c t - \sin\omega_c t)\sin\omega_c t$$

- Drift?

$$\langle \dots \rangle_c = \frac{1}{\tau_c}\int_0^{\tau_c} (\dots)\, dt$$

$$\langle \dot{x}(t)\rangle_c = ? \qquad \langle \dot{y}(t)\rangle_c = ?$$

$$\langle \sin\omega_c t\rangle_c = \frac{\omega_c}{2\pi}\int_0^{2\pi/\omega_c} \sin\omega_c t\, dt = 0$$

$$\langle \cos\omega_c t\rangle_c = \langle \sin\omega_c t \cos\omega_c t\rangle_c = 0$$

$$\langle \sin^2\omega_c t\rangle_c = \langle \cos^2\omega_c t\rangle_c = 1/2$$

$$\langle \dot{x}(t)\rangle_c = \frac{1}{t_c}\int_0^{t_c}\left(r_c\omega_c \cancel{\sin\omega_c t} + \eta r_c^2(\omega_c t - \cancel{\sin\omega_c t})\cos\omega_c t\right)dt =$$

$$= \frac{\omega_c}{2\pi}\int_0^{2\pi/\omega_c} \eta r_c^2 \omega_c t\cos\omega_c t\, dt = 0$$

$u = t$

$dv = \cos\omega_c t\, dt$

$$\langle \dot{y}(t)\rangle_c = \frac{1}{t_c}\int_0^{t_c}\left(r_c\omega_c \cancel{\cos\omega_c t} - r_c^2\eta(\cancel{\omega_c t} - \sin\omega_c t)\sin\omega_c t\right)dt$$

$$= \frac{\omega_c}{2\pi}\int_0^{2\pi/\omega_c} + r_c^2\eta\sin^2\omega_c t = r_c^2\eta\langle\sin^2\omega_c t\rangle_c = \frac{1}{2}\eta r_c^2$$

postoji drift duž y-ose!

⊞ $\propto \vec{B}\times\nabla B \to$ grad. drift