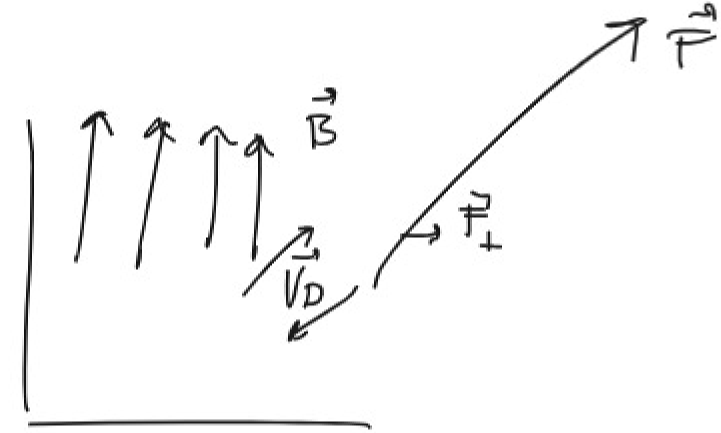
$$\vec{V}_D = \frac{1}{q}\frac{\vec{F}_\perp \times \vec{B}}{B^2}$$

$\vec{E}, \vec{g}$

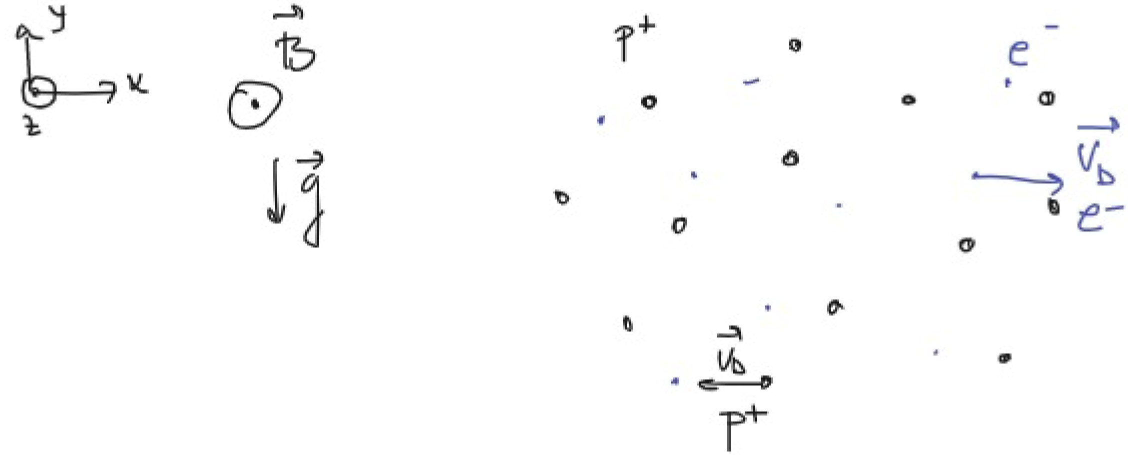


① Opisati kretanje p^+ i e^- u vodoničnoj plazmi koja se nalazi u ogr. delu prostora na nekoj h u $\vec{g} = -g\vec{e}_y$ ⊕ $\vec{B} = B\vec{e}_z$. Odrediti kretanje čestice

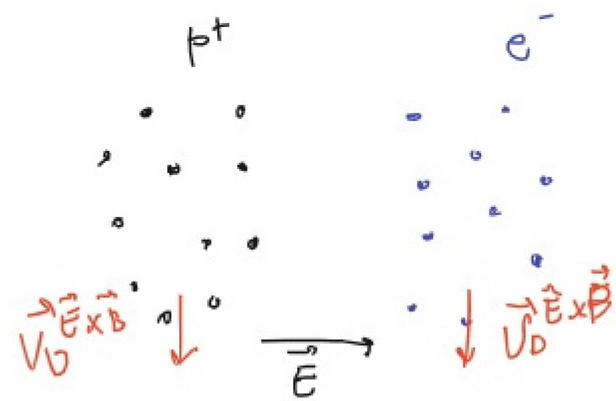
$$\vec{V}_D^{m\vec{g}} = \frac{1}{q}\frac{m\vec{g}\times\vec{B}}{B^2} = \frac{mg}{qB^{\cancel{2}}}\cancel{B}\left(-\vec{e}_y\times\vec{e}_z\right) = -\frac{mg}{qB}\vec{e}_x$$

1) p^+, $q>0$, $m=m_p$ $\quad \boxed{\vec{V}_D^{m\vec{g}} = -\frac{m_p g}{|q|B}\vec{e}_x}$

2) e^-, $q<0$, $m=m_e$ $\quad \boxed{\vec{V}_D^{m\vec{g}} = \frac{m_e g}{|q|B}\vec{e}_x}$



→ RAZDVAJANJE NAEL.



$$\vec{E} = E\vec{e}_x \quad , \quad \vec{F}_E = q\vec{E}$$

$$\vec{v}_D^{\vec{E}\times\vec{B}} = \frac{1}{\cancel{q}}\,\frac{\cancel{q}\,\vec{E}\times\vec{B}}{B^2} = \frac{1}{B^{\cancel{2}}}\left(E\vec{e}_x \times \cancel{B}\vec{e}_z\right)$$

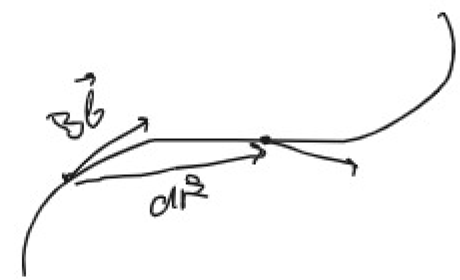
$$\vec{v}_D^{\vec{E}\times\vec{B}} = -\frac{E}{B}\,\vec{e}_y$$

p^+ n e^- → smer $\vec{v}_D \Rightarrow -\vec{e}_y$

② Šta je indikator zakrivljenosti $\vec{B}$?

$$\vec{B} = B\vec{b}$$

$B = const.$



$\vec{b}$ – ort tangente

ds – element luka

$d\vec{r}$ – promena vek. položaja

$$\vec{b} = \frac{d\vec{r}}{ds} \qquad ?$$

važi uvek $|\vec{b}| = 1$

$d\vec{A}$ – prostorni deo

$$d\vec{A} = (d\vec{r}\cdot\nabla)\vec{A}$$

$$d(B\vec{b}) = (d\vec{r}\cdot\nabla)B\vec{b}$$

$$B\,d\vec{b} = (d\vec{r}\cdot\nabla)B\vec{b} \qquad / : ds$$

$$\frac{d\vec{b}}{ds} = \left(\frac{d\vec{r}}{ds} \cdot \nabla\right)\vec{b}$$

$$\boxed{\frac{d\vec{b}}{ds} = (\vec{b} \cdot \nabla)\vec{b}} \quad (1).$$

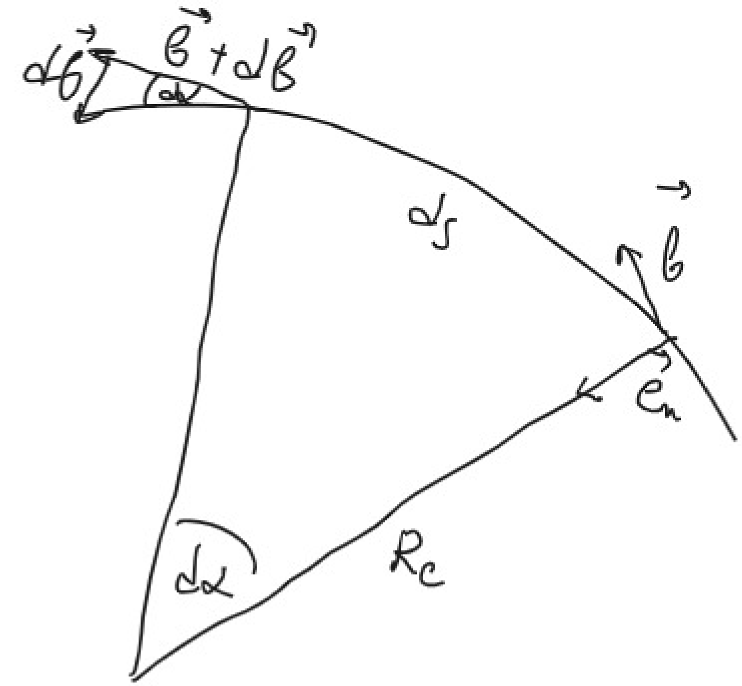
$\frac{d\vec{b}}{ds} \neq 0 \Rightarrow$ promena pravca

$\Rightarrow$ zakrivljeno $\vec{B}$

$d\vec{b} = d\alpha\, \vec{e_n}$

$ds = d\alpha\, R_c$

$$\boxed{\frac{d\vec{b}}{ds} = \frac{\vec{e_n}}{R_c}}$$



③ Indikator nehomogenosti $\vec{B}$?

* lokalni tenzor nehog $\vec{A}$

$$\nabla \otimes \vec{B} = \begin{bmatrix} \boxed{\partial_x B_x} & \boxed{\partial_x B_y} & \boxed{\partial_x B_z} \\ \boxed{\partial_y B_x} & \boxed{\partial_y B_y} & \boxed{\partial_y B_z} \\ \boxed{\partial_z B_x} & \boxed{\partial_z B_y} & \boxed{\partial_z B_z} \end{bmatrix}$$

1) članovi divergencije ($\nabla \cdot \vec{B} = 0$)

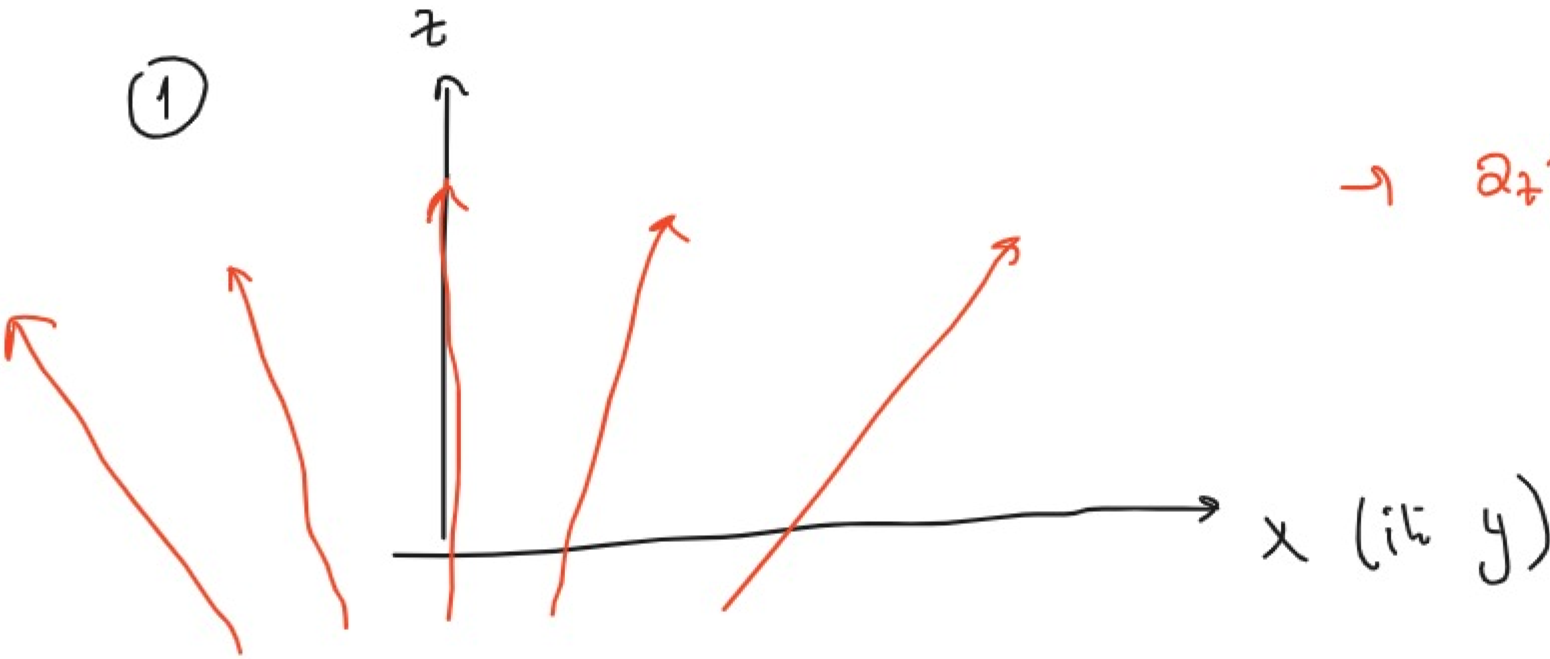
2) gradijentalni članovi

3) članove smicanja (uvijanje mag. linija)

4) krivinski članovi

miro

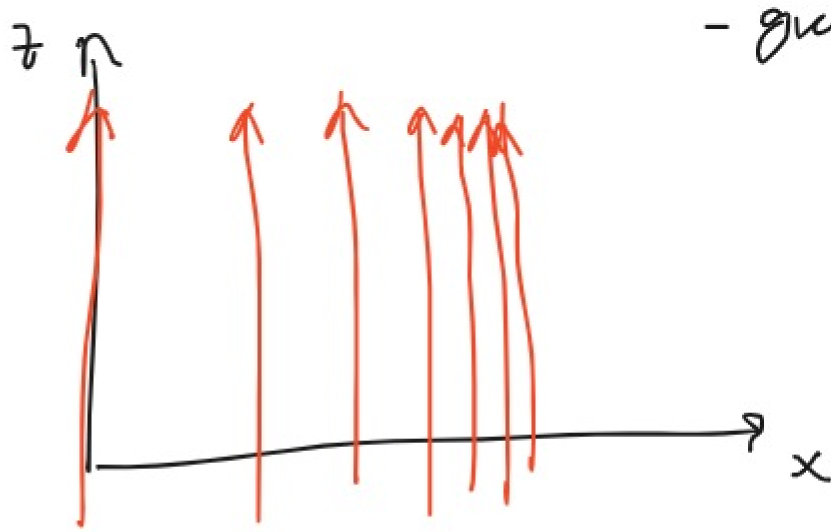
(1)



$\rightarrow \; \partial_z B_z \sim \partial_x B_x$

$\left(\nabla \cdot \vec{B} = 0 \; (w) \right)$

(2)

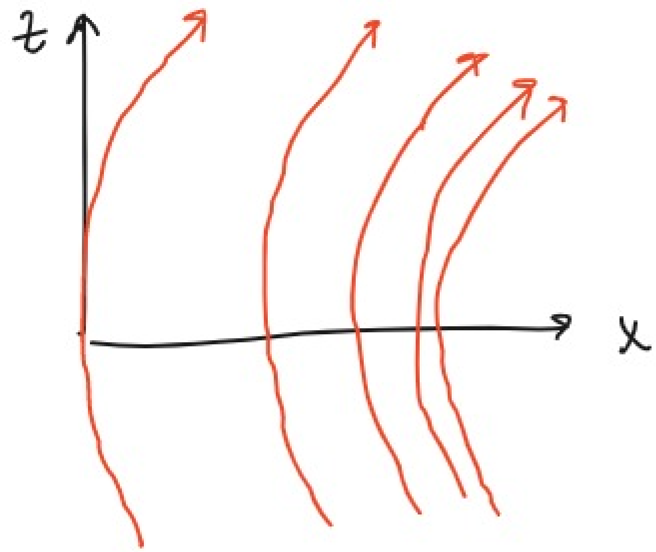


- grad. duž x-ose

$$\vec{B} = B_0 (1 + \alpha x) \vec{e}_z$$

$$\nabla \otimes \vec{B} = \begin{bmatrix} 0 & 0 & B_0\alpha \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(3)

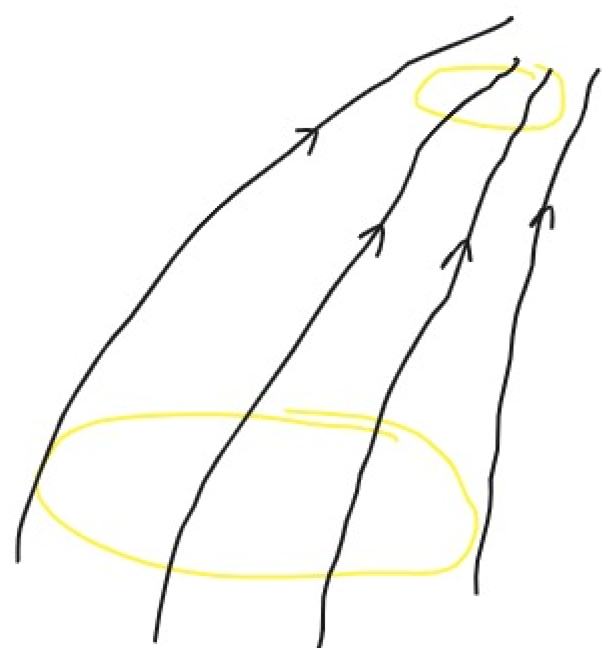


grad ⊕ krivinski član

$$\vec{B} = B_0 \left(\alpha z \vec{e_x} + (1+\alpha) x \vec{e_z} \right)$$

$$\nabla \otimes \vec{B} = \begin{bmatrix} 0 & 0 & B_0(1+\alpha) \\ 0 & 0 & 0 \\ B_0\alpha & 0 & 0 \end{bmatrix}$$

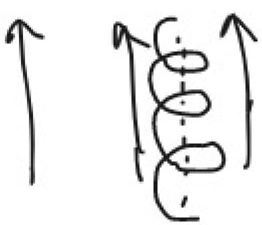
(4) grad ⊕ krivinski ⊕ diverg.



$\vec{B}$

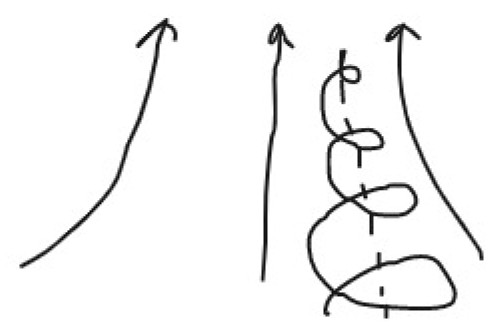
(4) Izvesti usrednjenu jdnu kretanja čestice u blizini 1 proizvoljne mag. lin. stacionarnog i slabo nehomogenog $\vec{B}$ (uz pps. approx vodećeg centra).

* homogenog $\vec{B}$



(√) osa zavojnice → linija $\vec{B}$

(√) const. hod i r_c



(x) osa zavojnice – nije linija $\vec{B}$
→ klizi sa lin. na lin.

(x) ≠ const. hod i r_c

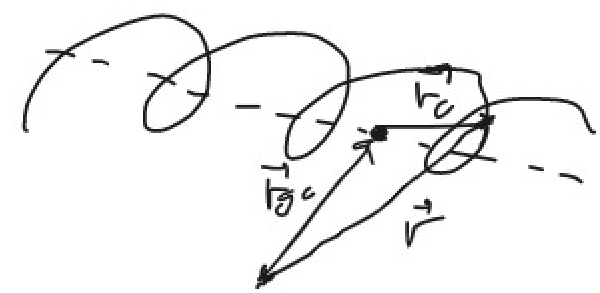
SLABO

* uslovi:

- slaba $\Delta\vec{B}$ na rast. reda r_c i perioda τ_c → $r_c \ll L$
- posm. česticu $t \gg \tau_c$
- ω_c velika

$$\left| r_c \frac{\|\nabla_\perp B\|}{B} \right| \ll 1 \qquad \left| \frac{v_\parallel}{\omega_c} \frac{\|\nabla_\parallel B\|}{B} \right| \ll 1$$

⊛ ref. sistem → VODEĆEG CENTRA



$$m\dot{\vec{v}} = q\vec{v} \times \vec{B}$$

- usrednjimo po τ_c: $\langle \dots \rangle_c = \frac{1}{\tau_c}\int_0^{\tau_c} (\dots)\, d\tau$

$$\langle \sin \omega_c t \rangle_c = \langle \cos \omega_c t \rangle_c = \langle \sin\omega_c t \cdot \cos \omega_c t \rangle_c = 0$$

$$\langle \sin^2 \omega_c t \rangle_c = \langle \cos^2 \omega_c t \rangle_c = \frac{1}{2}$$

$$\langle m\dot{\vec v}\rangle_c = \langle q\,\vec v \times \vec B\rangle_c$$

$$\rightarrow \langle \dot{\vec v}\rangle_c = \left\langle \frac{d\vec v}{dt}\right\rangle_c = \underbrace{\left\langle \frac{d\vec v}{dt_c}\right\rangle_c}_{\to\, 0} + \left\langle \frac{d\vec v}{dt_{sek}}\right\rangle_c = \frac{d}{dt_{sek}}\underbrace{\langle \vec v\rangle_c}_{\vec v_{gc}} = 0$$

$t_{sek} >> t_c$

$$\langle \vec v\rangle_c = \vec v_{gc}$$

PERTURBACIJE (1. red)

1) $\underbrace{\vec v = \vec v_0}_{\text{neperturb.}} + \vec v_1$ → mala popravka $\|\vec v_1\| \ll \|\vec v_0\|$, $\vec v_0 = \vec v_\parallel + \vec v_c$

2) $\vec B = \vec B_0 + \vec B_1$ $\quad \|\vec B_1\| \ll \|\vec B_0\|$

$\rightarrow \vec B_1 = \vec r_c \cdot (\nabla_0 \otimes \vec B)$ $\quad (\vec r_c = \vec r - \vec r_{gc})$

$\dot{\vec v} \ldots q\langle (\vec v_0 + \vec v_1) \times (\vec B_0 + \vec B_1)\rangle_c$

$$m\dot{\vec{v}}_{gc} = q\langle(\vec{v}_0 + \vec{v}_1)\times(\vec{B}_0 + \vec{B}_1)\rangle_c$$

$$= q\Big[\langle\vec{v}_0\times\underline{\vec{B}_0}\rangle_c + \langle\vec{v}_1\times\underline{\vec{B}_0}\rangle_c + \langle\vec{v}_0\times\vec{B}_1\rangle_c + \langle\vec{v}_1\times\vec{B}_1\rangle_c\Big] \rightarrow 0$$

$$= q\Big[\langle\vec{v}\times\vec{B}_0\rangle_c + \langle\vec{v}_0\times(\vec{r}_c\cdot(\nabla_0\otimes\vec{B}))\rangle_c\Big]$$

$$= q\Big[\underbrace{\langle\vec{v}\rangle_c}_{\vec{v}_{gc}}\times\vec{B}_0 + \langle\underbrace{(\vec{v}_\parallel + \vec{v}_c)\times(\vec{r}_c\cdot(\nabla_0\otimes\vec{B}))}\rangle_c\Big]$$

- u sistemu vod. centra $\rightarrow$ ciklotronska rotacija

$$\vec{r}_c(t) = r_c\sin\omega_c t\,\vec{e}_x + r_c\cos\omega_c t\,\vec{e}_y$$

$$\vec{v}_c(t) = \underbrace{r_c\omega_c}_{v_c}\cos\omega_c t\,\vec{e}_x - r_c\omega_c\sin\omega_c t\,\vec{e}_y$$

$\oplus$ poč. uslove: $\vec{r}_c(t=0) = r_c\vec{e}_y$

$\vec{v}_c(t=0) = v_c\vec{e}_x$

$$\langle\vec{v}_c\rangle_c = v_c\langle\underline{\cos\omega_c t}\rangle_c\vec{e}_x - v_c\langle\underline{\sin\omega_c t}\rangle_c\vec{e}_y = 0$$

$\vec{v}_{\parallel} \times (\vec{r}_c \cdot (\nabla_0 \otimes \vec{B}))$

$$\vec{r}_c \cdot (\nabla_0 \otimes \vec{B}) = [r_c \sin\omega_c t \quad r_c \cos\omega_c t \quad 0] \begin{bmatrix} \partial_x B_x & \partial_x B_y & \partial_x B_z \\ \partial_y B_x & \partial_y B_y & \partial_y B_z \\ \partial_z B_x & \partial_z B_y & \partial_z B_z \end{bmatrix} =$$

$[\quad]$ $[\quad]$

$$= [r_c \sin\omega_c t\, \partial_x B_x + r_c \cos\omega_c t\, \partial_y B_x\, j \rightarrow \alpha$$
$$r_c \sin\omega_c t\, \partial_x B_y + r_c \cos\omega_c t\, \partial_y B_y\, j \rightarrow \beta$$
$$r_c \sin\omega_c t\, \partial_x B_z + r_c \cos\omega_c t\, \partial_y B_z] \rightarrow \gamma$$

$\vec{v}_{\parallel} = v_{\parallel} \vec{e}_z$

$$\vec{v}_{\parallel} \times (\vec{r}_c \cdot (\nabla_0 \otimes \vec{B})) = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & v_{\parallel} \\ \alpha & \beta & \gamma \end{vmatrix} = \begin{bmatrix} -v_{\parallel}\beta \\ v_{\parallel}\alpha \\ 0 \end{bmatrix}$$

$$\langle \vec{v}_{\parallel} \times (\vec{r}_c \cdot (\nabla_0 \otimes \vec{B})) \rangle_c = \langle \underbrace{\begin{bmatrix} -v_{\parallel}(r_c \underline{\sin\omega_c t}\, \partial_x B_y + r_c \underline{\cos\omega_c t}\, \partial_y B_y) \\ + v_{\parallel}(r_c \underline{\sin\omega_c t}\, \partial_x B_x + r_c \underline{\cos\omega_c t}\, \partial_y B_x) \\ 0 \end{bmatrix}}_{\vec{0}} \rangle_c$$

$$\vec{v}_c \times (\vec{r}_c \cdot (\nabla_0 \otimes \vec{B})) = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ v_c \cos\omega_c t & -v_c \sin\omega_c t & 0 \\ \alpha & \beta & \gamma \end{vmatrix} = \begin{bmatrix} -v_c \sin\omega_c t\, \gamma \\ -v_c \cos\omega_c t\, \gamma \\ v_c \cos\omega_c t\, \beta + v_c \sin\omega_c t\, \alpha \end{bmatrix}$$

$$= \begin{bmatrix} -v_c \underline{\sin\omega_c t}\,(r_c \underset{1/2}{\underline{\sin\omega_c t}}\, \partial_x B_z + \cancel{r_c \cos\omega_c t}\, \partial_y B_z) \\ -v_c \underline{\cos\omega_c t}\,(\cancel{r_c \sin\omega_c t}\, \partial_x B_z + r_c \underset{1/2}{\underline{\cos\omega_c t}}\, \partial_y B_z) \\ v_c \underline{\cos\omega_c t}\,(r_c \sin\omega_c t\, \partial_x B_y + r_c \underset{1/2}{\underline{\cos\omega_c t}}\, \partial_y B_y) + v_c \underline{\sin\omega_c t}\,(r_c \underset{1/2}{\underline{\sin\omega_c t}}\, \partial_x B_x + \cancel{r_c \cos\omega_c t}\, \partial_y B_x) \end{bmatrix}$$

$$\langle \vec{r}_c \times (\vec{r}_c \cdot (\nabla_0 \otimes \vec{B})) \rangle_c = \begin{bmatrix} -v_c r_c \frac{1}{2} \partial_x B_z \\ -v_c r_c \frac{1}{2} \partial_y B_z \\ v_c r_c \frac{1}{2} \partial_y B_y + v_c r_c \frac{1}{2} \partial_x B_x \end{bmatrix} = -\frac{m v_c^2}{2 q B_0} \begin{bmatrix} \partial_x B_z \\ \partial_y B_z \\ \partial_z B_z \end{bmatrix} = -\frac{m v_c^2}{2 q B_0} \nabla B$$

B_z

$$\mu = \frac{m v_\perp^2}{2B}$$

$$r_c = \frac{v_c}{\omega_c} = \frac{v_c}{\frac{q B_0}{m}} = \frac{m v_c}{q B_0}$$

$$\vec{B} \approx B_z \vec{e}_z$$

miro

$$\oplus \quad m\dot{\vec{v}}_{gc} = q\vec{v}_{gc}\times\vec{B} - \mu\nabla B$$

$$\vec{v}_{gc} = \vec{V}_{\parallel} + \vec{V}_{\perp} = \vec{v}_{\parallel} + \vec{V}_D$$

$$\vec{v}_{\parallel} = v_{\parallel}\vec{b}$$

$$\frac{d\vec{v}_{gc}}{dt} = \frac{d}{dt}\left(v_{\parallel}\vec{b}\right) + \frac{d\vec{V}_D}{dt} = \frac{dv_{\parallel}}{dt}\vec{b} + v_{\parallel}\frac{d\vec{b}}{dt} + \frac{d\vec{V}_D}{dt}$$

$$\frac{d\vec{b}}{dt} = \underbrace{\frac{\partial\vec{b}}{\partial t}}_{\to 0} + \underbrace{v_{\parallel}\frac{\vec{e}_n}{R_c}}_{\text{duž luka linije sil. }\vec{B}} + \underbrace{(\vec{V}_D\cdot\nabla)\vec{b}}_{\to 0}$$

$$\frac{d\vec{v}_{gc}}{dt} = \frac{dv_{\parallel}}{dt}\vec{b} + v_{\parallel}^2\frac{\vec{e}_n}{R_c} + \frac{d\vec{V}_D}{dt}$$

$$\boxed{m\left(\underset{\parallel}{\underline{\frac{dv_{\parallel}}{dt}\vec{b}}} + \underset{\perp}{\underline{v_{\parallel}^2\frac{\vec{e}_n}{R_c}}} + \underset{i\perp\; i\parallel}{\frac{d\vec{V}_D}{dt}}\right) = q\left(\vec{v}_{\parallel} + \vec{V}_D\right)\overset{\perp}{\times}\vec{B} - \underset{\parallel\quad\perp}{\mu\nabla B}}$$

$$\vec{v}_{\parallel}\times\vec{B} = \vec{0}$$

$\perp$ i $\parallel$ $(\vec{B})$ напр.

$$\vec{b} \to \vec{b}_{\perp \vec{a}} \text{ i } \vec{b}_{\parallel \vec{a}}$$

$$\vec{b} = \vec{b}_\perp + \vec{b}_\parallel \quad / \cdot \vec{a}$$

$$\vec{b} \cdot \vec{a} = \underbrace{\vec{b}_\perp \cdot \vec{a}}_{0} + \underbrace{\vec{b}_\parallel \cdot \vec{a}} = \vec{b}_\parallel \cdot \vec{a} = |\vec{b}_\parallel||\vec{a}| = b_\parallel a$$

$$b_\parallel = \frac{\vec{b} \cdot \vec{a}}{a}$$

$$\vec{b}_\parallel = b_\parallel \frac{\vec{a}}{a} = \frac{(\vec{b} \cdot \vec{a})\vec{a}}{a^2} \qquad *1$$

$$\vec{b}_\perp = \vec{b} - \vec{b}_\parallel = \vec{b} - \frac{(\vec{b} \cdot \vec{a})\vec{a}}{a^2} = \frac{(\vec{a} \cdot \vec{a})\vec{b} - (\vec{b} \cdot \vec{a})\vec{a}}{a^2}$$

$$\left[(\vec{A} \times \vec{B}) \times \vec{C} = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{A}(\vec{B} \cdot \vec{C})\right]$$

$$\vec{b}_\perp = \frac{(\vec{a} \times \vec{b}) \times \vec{a}}{a^2} \qquad *2$$

miro

(11)

$$m\left(\frac{dv_\parallel}{dt}\vec{b} + \left(\frac{d\vec{V}_D}{dt}\right)_\parallel\right) = -\mu \nabla_\parallel B \qquad (\nabla_\parallel B = (\nabla B)_\parallel)$$

$$m\frac{dv_\parallel}{dt}\vec{b} = -\mu\nabla_\parallel B - m\left(\frac{d\vec{V}_D}{dt}\right)_\parallel$$

$$\left(\frac{d\vec{V}_D}{dt}\right)_\parallel = \left(\frac{d\vec{V}_D}{dt}\cdot\vec{b}\right)\vec{b} = -v_\parallel\left(\vec{V}_D\cdot\frac{\vec{e}_n}{R_c}\right)\vec{b} = -\vec{v}_\parallel\left(\vec{V}_D\cdot\frac{\vec{e}_n}{R_c}\right)$$

$$\vec{V}_D\cdot\vec{b} = 0$$

$$0 = \frac{d}{dt}(\vec{V}_D\cdot\vec{b}) = \vec{V}_D\cdot\boxed{\frac{d\vec{b}}{dt}} + \vec{b}\cdot\frac{d\vec{v}}{dt} = \underbrace{\vec{V}_D\cdot v_\parallel\frac{\vec{e}_n}{R_c} + \vec{b}\cdot\frac{d\vec{v}}{dt}}$$

$$\frac{d\vec{V}_D}{dt}\cdot\vec{b} = -v_\parallel\,\vec{V}_D\cdot\frac{\vec{e}_n}{R_c}$$

$$m\frac{dv_\parallel}{dt}\vec{b} = -\mu\underline{\nabla_\parallel} B + m\vec{v}_\parallel\left(\vec{V}_D\cdot\frac{\vec{e}_n}{R_c}\right)$$

$$\nabla_\parallel B \qquad \nabla\cdot\vec{B} = 0$$

miro

$\nabla_{\parallel} B \qquad \nabla \cdot \vec{B} = 0$

$\rightarrow \quad \vec{\nabla}(B\vec{b}) = \nabla B \cdot \vec{b} + B(\nabla \cdot \vec{b}) = 0$

$\nabla B \cdot \vec{b} = -B(\nabla \cdot \vec{b})$

$\leftarrow \quad (\nabla B)_{\parallel} = (\nabla B \cdot \vec{b})\vec{b} = -B(\nabla \cdot \vec{b})\vec{b}$

$$\boxed{m \frac{dv_{\parallel}}{dt}\vec{b} = \mu_B (\nabla \cdot \vec{b})\vec{b} + m\vec{v}_{\parallel}\left(\vec{V}_D \cdot \frac{\vec{e}_n}{R_C}\right)} \quad \parallel$$

longitudinalna