① Razmatramo pokretni fluid u idealnoj MHD. Ako je $H dV$ energija elementa fluida koji se kreće, pokazati da važi $\frac{d}{dt}(H dV) \neq 0$ u opštem slučaju.

$$\frac{d}{dt}(H dV) = H \frac{d}{dt}(dV) + \frac{dH}{dt} dV \ominus$$

$$\frac{\partial H}{\partial t} + \nabla \cdot \vec{U} = 0 \quad - \vec{v} \cdot \nabla H + \vec{v} \cdot \nabla H$$

$$\frac{dH}{dt} = - \nabla \cdot \vec{U} + \vec{v} \cdot \nabla H$$

$$\frac{d}{dt}(dV) = (\nabla \cdot \vec{v}) dV$$

$$\ominus H(\nabla \cdot \vec{v}) dV - (\nabla \cdot \vec{U}) dV + (\vec{v} \cdot \nabla H) dV =$$

$$= \nabla \cdot (H \vec{v}) dV - (\nabla \cdot \vec{U}) dV =$$

$$= - \nabla \cdot (\vec{U} - H \vec{v}) dV =$$

$$= - \nabla \cdot \left[\left(\cancel{\frac{1}{2} \rho v^2} + \frac{\gamma_g}{\gamma_g - 1} p \right) \vec{v} + \frac{B^2}{2 \mu_0} \vec{v} - \vec{v} \cdot \frac{1}{\mu_0} \vec{B} \otimes \vec{B} - \left(\cancel{\frac{1}{2} \rho v^2} + \frac{p}{\gamma_g - 1} + \cancel{\frac{B^2}{2 \mu_0}} \right) \vec{v} \right] dV \ominus$$

$$\frac{\gamma_g p}{\gamma_g - 1} - \frac{p}{\gamma_g - 1} = \frac{\gamma_g - 1}{\gamma_g - 1} p = p$$

$$\ominus \; -\nabla \cdot \left[\left(p + \frac{B^2}{2\mu_0} \right) \vec{v} - \frac{1}{\mu_0} \vec{v} \cdot (\vec{B} \otimes \vec{B}) \right] dV =$$

$\vec{v} = \vec{v} \cdot \hat{I}$

$$= -\nabla \cdot \left[\left(p + \frac{B^2}{2\mu_0} \right) \hat{I} - \frac{1}{\mu_0} \vec{B} \otimes \vec{B} \right] \cdot \vec{v} \, dV \neq 0$$

u opštem slučaju.

② Posmatramo $d\vec{\sigma}$. Proventi kako se menja fluks mag. polja kroz $d\vec{\sigma}$ u idealnoj MHD.

$$\Phi = \vec{B} \cdot d\vec{\sigma}$$

$$\frac{d}{dt}(\vec{B} \cdot d\vec{\sigma}) = \vec{B} \cdot \frac{d}{dt}(d\vec{\sigma}) + d\vec{\sigma} \cdot \frac{d\vec{B}}{dt} =$$

$$\frac{d}{dt}(d\vec{\sigma}) = -(\nabla \otimes \vec{v}) \cdot d\vec{\sigma} + d\vec{\sigma} \, (\nabla \cdot \vec{v})$$

$$\frac{d\vec{B}}{dt} = \frac{\partial \vec{B}}{\partial t} + \vec{v} \cdot (\nabla \otimes \vec{B})$$

$$= \nabla \times (\vec{v} \times \vec{B})$$

$$= \vec{B}\cdot\Big(-(\nabla\otimes\vec{v})\cdot d\vec{\sigma} + d\vec{\sigma}\,(\nabla\cdot\vec{v})\Big) +$$

$$+\, d\vec{\sigma}\cdot\Big(\nabla\times(\vec{v}\times\vec{B}) + \vec{v}\cdot(\nabla\otimes\vec{B})\Big) =$$

$$\boxed{\begin{aligned}\nabla\times(\vec{v}\times\vec{B}) &= \vec{B}\cdot(\nabla\otimes\vec{v}) - \vec{B}\,(\nabla\cdot\vec{v}) + \\ &\quad + \vec{v}\,(\nabla\cdot\vec{B}) - \vec{v}\cdot(\nabla\otimes\vec{B})\end{aligned}}$$

$$= \cancel{-\vec{B}\cdot(\nabla\otimes\vec{v})\cdot d\vec{\sigma}} + \cancel{\vec{B}\cdot d\vec{\sigma}\,(\nabla\cdot\vec{v})} +$$

$$+ \cancel{d\vec{\sigma}\cdot\big(\vec{B}\cdot(\nabla\otimes\vec{v})\big)} - \cancel{d\vec{\sigma}\cdot\vec{B}\,(\nabla\cdot\vec{v})} +$$

$$+ \cancel{d\vec{\sigma}\cdot\vec{v}}\,\underbrace{(\nabla\cdot\vec{B})}_{\operatorname{div}\vec{B}=0} - \cancel{d\vec{\sigma}\cdot\big(\vec{v}\cdot(\nabla\otimes\vec{B})\big)} + \cancel{d\vec{\sigma}\cdot\big(\vec{v}\cdot(\nabla\otimes\vec{B})\big)} =$$

$$= 0$$

(3) Neka je $S = p\rho^{-\gamma_g}$. U idealnoj MHD

važi $\frac{d}{dt}\left(p\rho^{-\gamma_g}\right) = 0$

$$\frac{\partial S}{\partial t} + \vec{v}\cdot(\nabla S) = 0$$

posmatraćemo ρs:

$$\frac{\partial}{\partial t}(\rho s) + \nabla\cdot(\rho s\vec{v}) =$$

$$= \frac{\partial \rho}{\partial t} s + \rho\frac{\partial s}{\partial t} + s\,\nabla\cdot(\rho\vec{v}) + \rho\vec{v}\cdot\nabla s =$$

$$= \rho\frac{\partial s}{\partial t} + s\underbrace{\left(\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{v})\right)}_{0} + \rho\hat{v}\cdot\nabla s =$$

$$= \rho\left(\frac{\partial s}{\partial t} + \vec{v}\cdot\nabla s\right) = \rho\frac{ds}{dt} = 0$$

④ Plazmeno β je dato kao:

$\beta = \frac{p_g}{p_m} = \frac{2\mu_0 nkT}{B^2}$. Isto tako dato je:

$v_A^2 = \frac{B^2}{\mu_0\rho}$ – Alfenova brzina

$v_s^2 = \gamma_g\frac{p}{\rho}$ – brzina zvuka.

Izraziti β preko v_A^2 i v_s^2

$$v_s^2 = \frac{\gamma_g nkT}{\rho} \Rightarrow nkT = \frac{v_s^2 \rho}{\gamma_g}$$

$$v_A^2 = \frac{B^2}{\mu_0 \rho} \Rightarrow \frac{B^2}{\mu_0} = v_A^2 \rho$$

$$\beta = 2 \cdot \frac{\mu_0}{B^2} nkT = 2 \cdot \frac{1}{v_A^2 \cancel{\rho}} \cdot \frac{v_s^2 \cancel{\rho}}{\gamma_g}$$

$$\beta = \frac{2}{\gamma_g} \left(\frac{v_s}{v_A}\right)^2$$

5) Da li mag. polje oblika:

$\vec{B} = B_x(z)\vec{e}_x + B_y(z)\vec{e}_y$ ispunjava uslov magnetohidrostatike $(X \neq X(t),\ \vec{v} = \vec{0})$ u idealnoj MHD? Neka i $p = p(z)$ i $\rho = \rho(z)$

$$\nabla p = \vec{j} \times \vec{B} = \frac{1}{\mu_0}(\nabla \times \vec{B}) \times \vec{B}$$

rotgrad $= 0$

$$\nabla \times \nabla p = \nabla \times \left(\frac{1}{\mu_0}(\nabla \times \vec{B}) \times \vec{B}\right) = 0$$

uslov za $\vec{B}$: $\boxed{\nabla \times (\vec{B} \times (\nabla \times \vec{B})) = \vec{0}}$

→ ekvivalentna forma:

$$\nabla \times (\vec{B} \times (\nabla \times \vec{B})) = \nabla \times \left(\nabla (\vec{B} \cdot \vec{B}) - \vec{B}(\vec{B} \cdot \nabla) \right) \ominus$$

$$\nabla (\vec{B} \cdot \vec{B}) = 2 \nabla (\vec{B} \cdot \vec{B})$$

$$\ominus \nabla \times \left(\nabla \left(\frac{B^2}{2} \right) - \vec{B} \cdot (\nabla \otimes \vec{B}) \right) =$$

$$= \underbrace{\nabla \times \nabla \left(\frac{B^2}{2} \right)}_{\text{rotgrad} = 0} - \nabla \times \left(\vec{B} \cdot (\nabla \otimes \vec{B}) \right) =$$

$$= - \nabla \times \left(\vec{B} \cdot (\nabla \otimes \vec{B}) \right)$$

$$\Rightarrow \boxed{\nabla \times \left(\vec{B} \cdot (\nabla \otimes \vec{B}) \right) = \vec{0}}$$

$$\nabla \times \vec{B} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \partial_x & \partial_y & \partial_z \\ B_x(z) & B_y(z) & 0 \end{vmatrix} = \begin{bmatrix} -\partial_z B_y \\ \partial_z B_x \\ 0 \end{bmatrix}$$

$$\vec{B} \times (\nabla \times \vec{B}) = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ B_x & B_y & 0 \\ -\partial_z B_y & \partial_z B_x & 0 \end{vmatrix} = \begin{bmatrix} 0 \\ 0 \\ \underbrace{B_x \partial_z B_x + B_y \partial_z B_y}_{A(z)} \end{bmatrix}$$

$$\nabla \times (\vec{B} \times (\nabla \times \vec{B})) = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \partial_x & \partial_y & \partial_z \\ 0 & 0 & A(z) \end{vmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \blacksquare$$

• naš slučaj, uslov je

$$\nabla p = \vec{j} \times \vec{B}$$

$$\frac{dp}{dz} \vec{e}_z = -\frac{1}{\mu_0} \begin{bmatrix} 0 \\ 0 \\ B_x \partial_z B_x + B_y \partial_z B_y \end{bmatrix}$$

$$\frac{dp}{dz} = -\frac{1}{\mu_0} \vec{B} \cdot \frac{d\vec{B}}{dz} = -\frac{1}{\mu_0} \frac{d}{dz} \left(\frac{B^2}{2} \right)$$

$$\Rightarrow \quad \frac{d}{dz} \left(p(z) + \frac{B^2(z)}{2\mu_0} \right) = 0$$

(6) Dato $\vec{B} = B_0 \left(y\vec{e}_x + \alpha^2 x \vec{e}_y \right)$ $\alpha^2 > 1$

$B_0 = \text{konst.}$

Odrediti jdnu mag. linija. Izračunati gustinu struje i magnetnu sila u modelu idealne MHD.

$$\frac{dx}{B_x} = \frac{dy}{B_y} = \frac{dz}{B_z}$$

$$\frac{dx}{\cancel{B_0} y} = \frac{dy}{\cancel{B_0} \alpha^2 x} \Rightarrow \alpha^2 x\,dx = y\,dy \quad / \int$$

$$\alpha^2 \frac{x^2}{2} - \frac{y^2}{2} = C \quad /:C$$

$$\frac{x^2}{\frac{2C}{\alpha^2}} - \frac{y^2}{2C} = 1$$

$$\frac{x^2}{\left(\frac{\sqrt{2C}}{\alpha}\right)^2} - \frac{y^2}{(\sqrt{2C})^2} = 1 \rightarrow \text{jdna hiperbole}$$

$$\vec{j} = \frac{1}{\mu_0} \nabla \times \vec{B}$$

$$\nabla \times \vec{B} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \partial_x & \partial_y & \partial_z \\ B_0 y & B_0 \alpha^2 x & 0 \end{vmatrix} = \begin{bmatrix} 0 \\ 0 \\ B_0 \alpha^2 - B_0 \end{bmatrix}$$

$$\vec{j} = \frac{1}{\mu_0} B_0 (\alpha^2 - 1) \vec{e}_z$$

$$\vec{F} = \vec{j} \times \vec{B}$$

$$\vec{F} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & j_z \\ B_0 y & B_0 \alpha^2 x & 0 \end{vmatrix} = \begin{bmatrix} -j_z B_0 \alpha^2 x \\ j_z B_0 y \\ 0 \end{bmatrix}$$

$$\vec{F} = \frac{1}{\mu_0} B_0^2 \alpha^2 (\alpha^2 - 1) x \vec{e}_x + \frac{1}{\mu_0} B_0^2 (\alpha^2 - 1) y \vec{e}_y$$

(7) Izvesti Rankin-Igonioove jdne posmatrajući hidrodinamički udarni talas. Sistem reference vezati za V.T., a V.T. smatrati jednodimenzionim duž x-ose. Takođe, V.T. je stacionaran. $\frac{\partial}{\partial t}=\frac{\partial}{\partial y}=\frac{\partial}{\partial z}=0$

1) $\rho v = const$

2) $\rho v^2 + p = const.$

3) $\frac{1}{2}v^2 + \frac{\gamma_g}{\gamma_g - 1}\frac{p}{\rho} = const.$

1) Jdna kontinuiteta:

$! \; v_x \equiv v$

$$\left.\cancel{\frac{\partial \rho}{\partial t}} + \underbrace{\nabla\cdot(\rho\vec{v})}_{\frac{\partial}{\partial x}(\rho v_x)} = 0\right\} \quad \frac{\partial}{\partial x}(\rho v) = 0$$

$$\Rightarrow \boxed{\rho v = const}$$

2) $\cancel{\frac{\partial \vec{\pi}}{\partial t}} + \underbrace{\nabla \cdot}_{\partial/\partial x} \hat{T} = \vec{0}$ $\qquad \hat{T} = \rho \underset{v_x}{\vec{v}} \otimes \underset{v_x}{\vec{v}} + p\hat{I}$

$$\frac{\partial}{\partial x}\left(\rho v_x^2 + p\right) =$$

$$\Rightarrow \boxed{\rho v^2 + p = \text{const}}$$

3) $\cancel{\frac{\partial H}{\partial t}} + \underbrace{\nabla \cdot}_{\partial/\partial x} \vec{U} = 0$ $\qquad \vec{U} = \left(\frac{1}{2}\rho \underset{v_x}{v^2} + \frac{\gamma_g}{\gamma_g - 1} p\right)\vec{v}$

$$\frac{d}{dx}\left[\left(\frac{1}{2}\rho v^2 + \frac{\gamma_g}{\gamma_g - 1} p\right) v\right] = 0$$

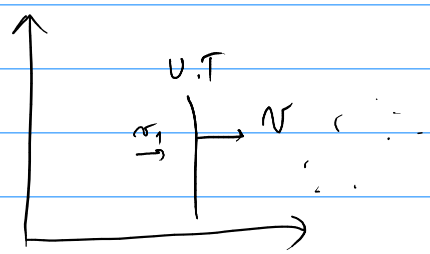
$$\left(\frac{1}{2}\rho v^2 + \frac{\gamma_g}{\gamma_g - 1} p\right) v = \text{const.}$$

$\rho v = \text{const.}$

$$\left(\frac{1}{2} v^2 + \frac{\gamma_g}{\gamma_g - 1} \frac{p}{\rho}\right) \rho v = \text{const.}$$

$$\Rightarrow \boxed{\frac{1}{2}v^2 + \frac{\gamma_g}{\gamma_g - 1}\frac{p}{\rho} = \text{const.}}$$

lab.



S.r. vezan za U.T