jdna prenosa:

$$(*)\quad \frac{\partial}{\partial t}\left(\frac{1}{2}\rho u^2+\rho e\right)+\nabla\cdot\left(\left(\frac{1}{2}\rho u^2+\rho e\right)\vec{u}+\vec{n}\cdot\hat{P}+\vec{q}\right)=\vec{j}\cdot\vec{E}$$

[...]

dodatak na izvođenje jdne prenosa:

$$\mu_0\vec{j}=\nabla\times\vec{B}-\frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}\qquad \frac{1}{c^2}=\varepsilon_0\mu_0$$

$$\nabla\cdot\left(\vec{E}\times\vec{B}\right)=\vec{B}\cdot\left(\nabla\times\vec{E}\right)-\vec{E}\cdot\left(\nabla\times\vec{B}\right)$$

$$\Rightarrow \vec{j}\cdot\vec{E}=\frac{1}{\mu_0}\left(\nabla\times\vec{B}\right)\cdot\vec{E}-\frac{1}{c^2\mu_0}\frac{\partial\vec{E}}{\partial t}\cdot\vec{E}=$$

$$=\frac{1}{\mu_0}\vec{B}\cdot\underbrace{\left(\nabla\times\vec{E}\right)}_{-\frac{\partial\vec{B}}{\partial t}}-\frac{1}{\mu_0}\nabla\cdot\left(\vec{E}\times\vec{B}\right)-\underbrace{\varepsilon_0\frac{\partial\vec{E}}{\partial t}\cdot\vec{E}}$$

$$\frac{1}{2}\frac{\partial}{\partial t}\left(\vec{E}\cdot\vec{E}\right)=\frac{\partial}{\partial t}\left(\frac{E^2}{2}\right)$$

$$=-\frac{1}{\mu_0}\vec{B}\cdot\frac{\partial\vec{B}}{\partial t}-\frac{1}{\mu_0}\nabla\cdot\left(\vec{E}\times\vec{B}\right)-\varepsilon_0\frac{\partial}{\partial t}\left(\frac{E^2}{2}\right)$$

$$=-\frac{1}{\mu_0}\nabla\cdot\left(\vec{E}\times\vec{B}\right)-\frac{1}{\mu_0}\frac{\partial}{\partial t}\left(\frac{B^2}{2}\right)-\varepsilon_0\frac{\partial}{\partial t}\left(\frac{E^2}{2}\right)$$

$$=-\frac{1}{\mu_0}\nabla\cdot\left(\vec{E}\times\vec{B}\right)-\frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon_0E^2+\frac{1}{2\mu_0}B^2\right)$$

Pointingov vektor $\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$

$$\Rightarrow \underbrace{\vec{j}\cdot\vec{E} = -\nabla\cdot\vec{S} - \frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}B^2\right)}_{\text{u jdnu kontinuiteta } (*)}$$

$$\Rightarrow \frac{\partial}{\partial t}\left(\frac{1}{2}\rho u^2 + \rho e + \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}B^2\right) +$$

$$+ \nabla\cdot\left(\left(\frac{1}{2}\rho u^2 + \rho e\right)\vec{u} + \vec{u}\cdot\hat{P} + \vec{q} + \vec{S}\right) = 0$$

jdna kretanja: $\rho\frac{d\vec{u}}{dt} = -\nabla\cdot\hat{P} + \vec{j}\times\vec{B}$

$$\frac{\partial}{\partial t}(\rho\vec{u}) + \nabla\cdot(\rho\vec{u}\otimes\vec{u}) = -\nabla\cdot\hat{P} + \vec{j}\times\vec{B}$$

$\rho e \rightarrow \frac{3}{2}p$ (raspišemo jdnu prenosa energije $(*)$)

$$\boxed{\frac{\partial}{\partial t}\left(\frac{1}{2}\rho u^2\right)} + \frac{\partial}{\partial t}\left(\frac{3}{2}p\right) + \boxed{\nabla\cdot\left(\frac{1}{2}\rho u^2\vec{u}\right)} + \nabla\cdot\left(\frac{3}{2}p\vec{u}\right) +$$

$$+ \nabla\cdot(\vec{u}\cdot\hat{P}) + \nabla\cdot\vec{q} = \vec{j}\cdot\vec{E}$$

$\Rightarrow$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho u^2\right) + \nabla\cdot\left(\frac{1}{2}\rho u^2 \vec{u}\right) = \frac{1}{2}u^2\frac{\partial \rho}{\partial t} + \frac{1}{2}\rho\frac{\partial}{\partial t}(\vec{u}\cdot\vec{u}) +$$

$$+ \rho\vec{u}\cdot\boxed{\nabla\left(\frac{1}{2}u^2\right)} + \frac{1}{2}u^2\,\nabla\cdot(\rho\vec{u}) \ominus$$

$$\nabla\left(\frac{1}{2}u^2\right) = \vec{u}\times(\nabla\times\vec{u}) + (\vec{u}\cdot\nabla)\vec{u}$$

$$\rightarrow \vec{u}\cdot\nabla\left(\frac{1}{2}u^2\right) = \underbrace{\vec{u}\cdot\left(\vec{u}\times(\nabla\times\vec{u})\right)}_{\Rightarrow 0} + \vec{u}\cdot(\vec{u}\cdot\nabla)\vec{u}$$

$$\ominus \frac{1}{2}u^2\frac{\partial\rho}{\partial t} + \cancel{\frac{1}{2}}\rho\,\cancel{2}\,\frac{\partial\vec{u}}{\partial t}\cdot\vec{u} + \rho\vec{u}\cdot(\vec{u}\cdot\nabla)\vec{u} +$$

$$+ \frac{1}{2}u^2\,\nabla\cdot(\rho\vec{u}) =$$

$$= \frac{1}{2}u^2\underbrace{\left(\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{u})\right)}_{\text{jdna kont. } \sum_\alpha C_{\alpha} = 0} + \rho\vec{u}\cdot\underbrace{\left(\frac{\partial\vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u}\right)}_{\frac{1}{\rho}(-\nabla\cdot\hat{P} + \vec{j}\times\vec{B})} =$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho u^2\right) + \nabla\cdot\left(\frac{1}{2}\rho u^2\vec{u}\right) = \vec{u}\cdot(-\nabla\cdot\hat{P}) + \vec{u}\cdot(\vec{j}\times\vec{B})$$

$$\vec{u}\cdot\left(-\nabla\cdot\hat{P}\right)+\vec{u}\cdot\left(\vec{j}\times\vec{B}\right)+\frac{\partial}{\partial t}\left(\frac{3}{2}p\right)+\nabla\cdot\left(\frac{3}{2}p\vec{u}\right)+$$
$$+\nabla\cdot\left(\vec{u}\cdot\hat{P}\right)+\nabla\cdot\vec{q}=\vec{j}\cdot\vec{E}$$

$\hat{P}\to$ simetrican tenzor $\Rightarrow\ \vec{u}\cdot\hat{P}=\hat{P}\cdot\vec{u}$

$$\nabla\cdot\left(\vec{u}\cdot\hat{P}\right)=\nabla\cdot\left(\hat{P}\cdot\vec{u}\right)=\vec{u}\cdot\left(\nabla\cdot\hat{P}\right)+\left(\hat{P}\cdot\nabla\right)\cdot\vec{u}$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2}p\right)\cancel{-\vec{u}\cdot\left(\nabla\cdot\hat{P}\right)}+\vec{u}\cdot\left(\vec{j}\times\vec{B}\right)+\nabla\cdot\left(\frac{3}{2}p\vec{u}\right)+$$
$$\cancel{+\vec{u}\cdot\left(\nabla\cdot\hat{P}\right)}+\left(\hat{P}\cdot\nabla\right)\cdot\vec{u}+\nabla\cdot\vec{q}=\vec{j}\cdot\vec{E}$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2}p\right)+\nabla\cdot\left(\frac{3}{2}p\vec{u}\right)+\nabla\cdot\vec{q}+\left(\hat{P}\cdot\nabla\right)\cdot\vec{u}=\vec{j}\cdot\vec{E}-\vec{u}\cdot\left(\vec{j}\times\vec{B}\right)$$

$$\vec{j}=\sum_{\alpha}{}'\,\vec{j}_\alpha=\sum_{\alpha}{}'\,\rho^{el}_\alpha\vec{u}_\alpha=\sum_{\alpha}{}'\,n_\alpha q_\alpha\vec{u}_\alpha=$$

$\vec{u}_{diff\,\alpha}=\vec{u}_\alpha-\vec{u}$

$$=\sum_{\alpha}{}'\,n_\alpha q_\alpha\vec{u}+\sum_{\alpha}{}'\,n_\alpha q_\alpha\vec{u}_{diff\,\alpha}=$$

$$=\rho^{el}\vec{u}+\sum_{\alpha}{}'\,\rho^{el}_\alpha\vec{u}_{diff,\alpha}$$

$\Rightarrow$ član $\vec{u}\cdot(\vec{j}\times\vec{B})$ opstaje, $\vec{u}_\alpha\cdot(\vec{j}_\alpha\times\vec{B})=0$

$\downarrow$ fgcniše $\vec{u}_\alpha$

$$\frac{\partial}{\partial t}\left(\frac{3}{2}p\right)+\vec{u}\cdot\nabla\left(\frac{3}{2}p\right)+\frac{3}{2}p(\nabla\cdot\vec{u})+(\hat{P}\cdot\nabla)\cdot\vec{u}+\nabla\cdot\vec{q}=$$
$$=\vec{j}\cdot(\vec{E}+\vec{u}\times\vec{B})$$

uvodi se:

$$\tilde{\vec{E}}=\vec{E}+\vec{u}\times\vec{B}$$

$\circledast$ $\rho^{el}\approx 0$ (makroskopski elektroneutralna

$\circledast$ $\frac{\partial p}{\partial t}+\vec{u}\cdot(\nabla p)=\frac{dp}{dt}$

$$\frac{3}{2}\frac{dp}{dt}+\frac{3}{2}p(\nabla\cdot\vec{u})+(\hat{P}\cdot\nabla)\cdot\vec{u}+\nabla\cdot\vec{q}=\vec{j}\cdot\tilde{\vec{E}}$$

$$\frac{3}{2}p=\rho e$$

$$\frac{d}{dt}\left(\frac{3}{2}p\right)=\frac{d}{dt}(\rho e)=\frac{d\rho}{dt}e+\rho\frac{de}{dt}=$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$

$$\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \vec{u} + \vec{u} \cdot \nabla \rho = 0$$

$$\frac{d\rho}{dt} = -\rho \nabla \cdot \vec{u}$$

$$\ominus \quad -\rho e (\nabla \cdot \vec{u}) + \rho \frac{de}{dt}$$

$$\rho \frac{de}{dt} - \cancel{\rho e (\nabla \cdot \vec{u})} + \cancel{\rho e (\nabla \cdot \vec{u})} + (\hat{P} \cdot \nabla) \cdot \vec{u} + \nabla \cdot \vec{q} = \vec{j} \cdot \tilde{\vec{E}}$$

$$\boxed{\rho \frac{de}{dt} + (\hat{P} \cdot \nabla) \cdot \vec{u} + \nabla \cdot \vec{q} = \vec{j} \cdot \tilde{\vec{E}}}$$

kod jedne kretanja smo pps. $\rho^{el} \approx 0$

(da nismo → ostaje član $\rho^{el} \vec{E}$ →

$$\to \tilde{\vec{j}} \cdot \vec{E} \quad , \quad \tilde{\vec{j}} = \vec{j} - \rho^{el} \vec{u}$$

① Polazeći od:

$$\frac{3}{2}\frac{dp_\alpha}{dt} + \frac{3}{2}p_\alpha \nabla\cdot\vec{u}_\alpha + (\hat{P}_\alpha\cdot\nabla)\cdot\vec{u}_\alpha + \nabla\cdot\vec{q}_\alpha =$$

$$= C_{3,\alpha} - \vec{u}_\alpha\cdot\vec{C}_{2\alpha} + \frac{1}{2}u_\alpha^2 C_{1\alpha}$$

pokazati da važi:

$$\frac{3}{2}n_\alpha k\frac{dT_\alpha}{dt} + (\hat{P}_\alpha\cdot\nabla)\cdot\vec{u}_\alpha + \nabla\cdot\vec{q}_\alpha = C_{3\alpha} - \vec{u}_\alpha\cdot\vec{C}_{2\alpha} +$$

$$+ \left(\frac{1}{2}u_\alpha^2 - \frac{3}{2}\frac{kT_\alpha}{m_\alpha}\right)C_{1\alpha}$$

$$p_\alpha = n_\alpha k T_\alpha$$

$$\frac{3}{2}\frac{dp_\alpha}{dt} = \frac{3}{2}\frac{d}{dt}(n_\alpha k T_\alpha) = \frac{3}{2}n_\alpha k\frac{dT_\alpha}{dt} +$$

$$+ \frac{3}{2}kT_\alpha \frac{dn_\alpha}{dt} =$$

jdna kont. $\frac{\partial \rho_\alpha}{\partial t} + \nabla\cdot(\rho_\alpha\vec{u}_\alpha) = C_{1\alpha}$

$$\rho_\alpha = n_\alpha m_\alpha$$

$$\frac{\partial}{\partial t}(n_\alpha m_\alpha) + m_\alpha n_\alpha \nabla\cdot\vec{u}_\alpha + m_\alpha \vec{u}_\alpha \cdot \nabla n_\alpha = C_{1\alpha}$$

$$\frac{d\rho_\alpha}{dt} = C_{1\alpha} - \rho_\alpha \nabla\cdot\vec{u}_\alpha \quad /:m_\alpha$$

$$\frac{dn_\alpha}{dt} = \frac{C_{1\alpha}}{m_\alpha} - n_\alpha \nabla\cdot\vec{u}_\alpha$$

$$\ominus\ \frac{3}{2} n_\alpha k \frac{dT_\alpha}{dt} + \frac{3}{2} kT_\alpha(-n_\alpha)\nabla\cdot\vec{u}_\alpha + \frac{3}{2}kT_\alpha \frac{C_{1\alpha}}{m_\alpha}$$

p_α

$$\frac{3}{2}\frac{dp_\alpha}{dt} = \frac{3}{2} n_\alpha k \frac{dT_\alpha}{dt} - \frac{3}{2} p_\alpha \nabla\cdot\vec{u}_\alpha + \frac{3}{2} kT_\alpha \frac{C_{1\alpha}}{m_\alpha}$$

$$\frac{3}{2} n_\alpha k \frac{dT_\alpha}{dt} - \cancel{\frac{3}{2} p_\alpha \nabla\cdot\vec{u}_\alpha} + \frac{3}{2} kT_\alpha \frac{C_{1\alpha}}{m_\alpha} + \cancel{\frac{3}{2} p_\alpha \nabla\cdot\vec{u}_\alpha} +$$

$$+ (\hat{P}_\alpha \cdot \nabla)\cdot\vec{u}_\alpha + \nabla\cdot\vec{q}_\alpha = C_{3,\alpha} - \vec{u}_\alpha\cdot\vec{C}_{2\alpha} + \frac{1}{2}u_\alpha^2 C_{1\alpha}$$

$$\frac{3}{2} n_\alpha k \frac{dT_\alpha}{dt} + (\hat{P}_\alpha\cdot\nabla)\cdot\vec{u}_\alpha + \nabla\cdot\vec{q}_\alpha =$$

$$= C_{3\alpha} - \vec{u}_\alpha\cdot\vec{C}_{2\alpha} + \left(\frac{1}{2}u_\alpha^2 - \frac{3}{2}\frac{kT_\alpha}{m_\alpha}\right)C_{1\alpha} \quad \blacksquare$$

(2) Neka je $T = \frac{p}{nk}$. Pokazati da važi:

$$T = \frac{1}{n}\sum_\alpha{}' n_\alpha \left(T_\alpha + \frac{1}{3k} m_\alpha u^2_{diff,\alpha} \right)$$

Kod jdne prenosa energije smo izveli:

$$p = \sum_\alpha{}' p_\alpha + \frac{1}{3}\sum_\alpha{}' \rho_\alpha u^2_{diff\alpha}$$

$$p_\alpha = n_\alpha k T_\alpha \qquad \rho_\alpha = m_\alpha n_\alpha$$

$$T = \frac{1}{nk}\left(\sum_\alpha{}' p_\alpha + \frac{1}{3}\sum_\alpha{}' \rho_\alpha u^2_{diff\alpha}\right) =$$

$$= \frac{1}{nk}\sum_\alpha{}' \left(n_\alpha k T_\alpha + \frac{1}{3} m_\alpha n_\alpha u^2_{diff\alpha} \right) =$$

$$= \frac{1}{n}\sum_\alpha{}' n_\alpha \left(T_\alpha + \frac{1}{3k} m_\alpha u^2_{diff\alpha} \right) \blacksquare$$

③ Neka je $\chi(\vec{v})$ neka fizička karakteristika čestica plazme. Polazeći od O.K.j.u.f.p. odrediti jdn prenosa $\chi(\vec{v})$ tj. $\langle \chi(\vec{v}) \rangle$ u vremenu i prostoru.

$$\frac{\partial f_\alpha}{\partial t} + \vec{v}\cdot\nabla f_\alpha + \vec{a}\cdot\nabla_{\vec{v}} f_\alpha = \mathcal{I}_\alpha \quad \Big/ \int_{V_{\vec{v}}} \chi(\vec{v})\, d^3\vec{v}$$

$$I_1 = \int_{V_{\vec{v}}} \frac{\partial f_\alpha}{\partial t} \underbrace{\chi(\vec{v})}_{\text{nije ekspl. fja } t}\, d^3\vec{v} = \frac{\partial}{\partial t}\int_{V_{\vec{v}}} f_\alpha \chi(\vec{v})\, d^3\vec{v} =$$

$$= \frac{\partial}{\partial t}\left(n_\alpha \langle \chi(\vec{v})\rangle_\alpha\right)$$

$$I_2 = \int_{V_{\vec{v}}} \chi(\vec{v})\, \vec{v}\cdot \nabla f_\alpha\, d^3\vec{v} =$$

$\nabla = \nabla_{\vec{r}}$

$$\left\{ \nabla\cdot\left(\chi(\vec{v})\vec{v} f_\alpha\right) = \vec{v} f_\alpha \cdot \nabla\chi(\vec{v}) + \right.$$

$\nabla_{\vec{r}}\, \chi(\vec{v}) = 0$

$$+\chi(\vec{v}) f_\alpha \nabla \cdot \vec{v} + \chi(\vec{v})\vec{v} \cdot \nabla f_\alpha$$

$$\nabla_{\vec{r}} \cdot \vec{v} = 0$$

$$\ominus \int_{V_{\vec{v}}} \nabla_{\vec{r}} \cdot (\chi(\vec{v})\vec{v} f_\alpha)\, d^3\vec{v} = \nabla \cdot \int_{V_{\vec{v}}} \chi(\vec{v})\vec{v} f_\alpha\, d^3\vec{v} =$$

$$= \nabla \cdot \left(n_\alpha \langle \chi(\vec{v})\vec{v} \rangle_\alpha\right)$$

$$I_3 = \int_{V_{\vec{v}}} \chi(\vec{v})\, \vec{a} \cdot \nabla_{\vec{v}} f_\alpha\, d^3\vec{v} \ominus$$

$$\nabla_{\vec{v}} \cdot (\chi(\vec{v})\vec{a} f_\alpha) =$$

$$\vec{a} f_\alpha \cdot \nabla_{\vec{v}} \chi(\vec{v}) + \chi(\vec{v}) f_\alpha \nabla_{\vec{v}} \cdot \vec{a} + \chi(\vec{v})\vec{a} \cdot \nabla_{\vec{v}} f_\alpha$$

$$0$$

$$\ominus \underbrace{\int_{V_{\vec{v}}} \nabla_{\vec{v}} \cdot (\chi(\vec{v})\vec{a} f_\alpha)\, d^3\vec{v}}_{} - \int_{V_{\vec{v}}} \vec{a} f_\alpha \cdot \nabla_{\vec{v}} \chi(\vec{v})\, d^3\vec{v} =$$

Gauss: $f_\alpha \to 0$ за $\vec{v} \to \infty \Rightarrow 0$

$$= -n_\alpha \langle \vec{a} \cdot \nabla_{\vec{v}} \chi(\vec{v}) \rangle_\alpha$$

Konačno:

$$\frac{\partial}{\partial t}\left(n_\alpha \langle \chi(\vec{v}) \rangle_\alpha\right) + \nabla \cdot \left(n_\alpha \langle \chi(\vec{v})\vec{v} \rangle_\alpha\right) - n_\alpha \langle \vec{a} \cdot \nabla_{\vec{v}} \chi(\vec{v}) \rangle_\alpha =$$

$$= \int_{V_{\vec{v}}} \chi(\vec{v}) I_\alpha \, d^3\vec{v}$$

④ Isto kao u ③, samo $\chi = \chi(\vec{r}, \vec{v}, t)$

$$\frac{\partial f_\alpha}{\partial t} + \vec{v} \cdot \nabla f_\alpha + \vec{a} \cdot \nabla_{\vec{v}} f_\alpha = I_\alpha \quad \Big/ \int_{V_{\vec{v}}} \chi(\vec{r}, \vec{v}, t) \, d^3\vec{v}$$

! pišemo dalje samo χ podrazumevajući da je $\chi = \chi(\vec{r}, \vec{v}, t)$! $f_\alpha = f_\alpha(\vec{r}, \vec{v}, t)$

$$I_1 = \int_{V_{\vec{v}}} \chi \frac{\partial f_\alpha}{\partial t} \, d^3\vec{v} =$$

$$\frac{\partial}{\partial t}(\chi f_\alpha) = \chi \frac{\partial f_\alpha}{\partial t} + f_\alpha \frac{\partial \chi}{\partial t}$$

$$\overset{(=)}{} \int_{V_{\vec v}} \frac{\partial}{\partial t}(\chi f_\alpha)\, d^3\vec v - \int_{V_{\vec v}} f_\alpha \frac{\partial \chi}{\partial t}\, d^3\vec v =$$

$$= \frac{\partial}{\partial t}\left(n_\alpha \langle \chi \rangle_\alpha\right) - n_\alpha \left\langle \frac{\partial \chi}{\partial t} \right\rangle_\alpha$$

$$I_2 = \int_{V_{\vec v}} \chi\, \vec v \cdot \nabla f_\alpha\, d^3\vec v \;\overset{(=)}{}$$

$\nabla \equiv \nabla_{\vec r}$

$$\boxed{\nabla \cdot (\chi \vec v f_\alpha) = \vec v f_\alpha \cdot \nabla \chi + \chi f_\alpha \underbrace{\nabla \cdot \vec v}_{0} + \chi \vec v \cdot \nabla f_\alpha}$$

$$\overset{(=)}{} \int_{V_{\vec r}} \nabla_r \cdot (\chi \vec v f_\alpha)\, d^3\vec v - \int_{V_{\vec v}} \vec v f_\alpha \cdot \nabla_r \chi\, d^3\vec v =$$

$$= \nabla \cdot \left(n_\alpha \langle \chi \vec v \rangle_\alpha\right) - n_\alpha \langle \vec v \cdot \nabla \chi \rangle_\alpha$$

$$I_3 = \int_{V_{\vec v}} \chi\, \vec a \cdot \nabla_{\vec v} f_\alpha\, d^3\vec v \;\overset{(=)}{}$$

$$\boxed{\nabla_{\vec v} \cdot (\chi \vec a f_\alpha) = \vec a f_\alpha \cdot \nabla_{\vec v} \chi + \chi f_\alpha \underbrace{\nabla_{\vec v} \cdot \vec a}_{\to 0} + \chi \vec a \cdot \nabla_{\vec v} f_\alpha}$$

$$\ominus \int_{V_{\vec{v}}} \nabla_{\vec{v}} \cdot (\chi \vec{a} f_\alpha)\, d^3\vec{v} - \int_{V_{\vec{v}}} \vec{a} f_\alpha \cdot \nabla_{\vec{v}} \chi \, d^3 \vec{v} =$$

Gay $\to 0$

$$= -n_\alpha \langle \vec{a} \cdot \nabla_{\vec{v}} \chi \rangle_\alpha$$

Konačno: $I_1 + I_2 + I_3$! $\chi = \chi(\vec{r}, \vec{v}, t)$

$$\frac{\partial}{\partial t}\left(n_\alpha \langle \chi \rangle_\alpha\right) - n_\alpha \left\langle \frac{\partial \chi}{\partial t} \right\rangle_\alpha + \nabla \cdot \left(n_\alpha \langle \chi \vec{v} \rangle\right) -$$

$$- n_\alpha \langle \vec{v} \cdot \nabla \chi \rangle_\alpha - n_\alpha \langle \vec{a} \cdot \nabla_{\vec{v}} \chi \rangle_\alpha =$$

$$= \int_{V_{\vec{v}}} \chi I_\alpha \, d^3 \vec{v}$$