Number Theory

Syllabus for the TEMPUS-SEE PhD Course

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Course goals

Number theory has always exhibited a unique feature that some appealing and easily stated problems tend to resist the attempts for solution over very long periods of time. It has influenced and has been influenced by developments in many mathematical disciplines. Several breaktroughs that took place during last decades on one hand and unprecented range of applications on the other, have significantly enlarged the interested mathematical community. The course is designed to provide insights into some areas of modern research in analytic, algebraic and computational/algorithmic number theory. The extent of exposure to advanced themes will depend on the mathematical background of paricipants.

Prerequisites

Prerequisites vary from one part of the course to another and range from elementary number theory, complex analysis, some Fourier analysis, standard course in algebra (basics of finite group theory commutative rings, ideals, basic Galois theory of fields), to data structures and programming skills.

Course modules (20 units each)

I. Analytic Number Theory
Lecturer: Prof. Dr. Muharem Avdispahić, University of Sarajevo
II. Algebraic Number Theory
Lecturer: Asso. Prof. Dr. Ivan Chipchakov, IMI, Bulgarian Academy of Sciences
III. Computational Number Theory
Lecturer: Dr. habil. Wolfgang A. Schmid, University of Graz/Ecole Polytechnique Paris

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Analytic Number Theory

Euler's proof of infinitude of primes
Dirichlet theorem on primes in arithmetic progressions
Functional equation for the Riemann zeta function
Prime number theorem
The Selberg class of functions
Poisson summation formula as a trace formula
Weil’s functional
Hyperbolic geometry
Hyperbolic Laplacian
The Selberg trace formula
Selberg's zeta and prime geodesic theorems
Explicit formulas in the fundamental class

Algebraic Number Theory

Number fields and algebraic integers
Unique factorization of ideals
Ideal class group
Dirichlet theorem on units
p-adic fields and local to global principle
Dedekind zeta and Hecke L-function
Elliptic curves over number fields
Zeta function of an elliptic curve
Birch and Swinnerton-Dyer conjecture
Shimura-Taniyama and Fermat's last theorem

Computational Number Theory

Basic algorithms and some algorithms of elementary number theory
Algorithmic linear algebra for number theory
Main tasks of computational algebraic number theory
Applications in cryptography
Pime-testing and factorization
Computational problems of non-unique factorization theory and zero-sum theory
Recent developments/events

Literature

A prospective course participant may wish to have a look at number theory related informative articles in T. Gowers (ed.), The Princeton Companion to Mathematics, Princeton University Press, Princeton 2008, in particular: B. Mazur, Algebraic numbers (pp. 315-332); A. Granville, Analytic Number Theory (332-348); C. Pomerance, Computational Number Theory (348-362).
Sample of surveys closer to research frontiers is given by

E. Bombieri,The Rosetta Stone of L-functions. *Perspectives in analysis,* 1--15, Math. Phys. Stud., 27, *Springer, Berlin* 2005
K. Soundararajan, Small gaps between prime numbers; the work of Goldston-Pintz-Yildrim. *Bulletin of the American Mathematical Society* **44** (2007), no. 1, 1-18

He/she may be interested to see a recent research article achieving an important result without as heavy machinery as might have been expected

M. Agrawal, N. Kayal, N. Saxena, PRIMES is in P. *Ann. of Math. (2)* **160** (2004), no. 2, 781--793.

One is often attracted to ellegance of brief expositions like

H.P.F. Swinnerton-Dyer, A brief guide to algebraic number theory. London Mathematical Society Student Texts, 50. *Cambridge University Press, Cambridge,* 2001. x+146 pp.
G. Tenenbaum, M. Mendès France, The prime numbers and their distribution. Student Mathematical Library, 6. *American Mathematical Society, Providence, RI,* 2000. xx+115 pp
D. J. Newman, Analytic number theory. Graduate Texts in Mathematics, 177. *Springer-Verlag, New York,* 1998. viii+76 pp.

However, eventually one has to reach for comprehensive accounts. The latter include

H. Cohen, A course in computational algebraic number theory. Graduate Texts in Mathematics, 138. *Springer-Verlag, Berlin,* 1993
H. Iwaniec, E. Kowalski, Analytic number theory. American Mathematical Society Colloquium Publications, 53. *American Mathematical Society, Providence, RI,* 2004
Yu. I. Manin, A. A. Panchishkin, Introduction to modern number theory. Fundamental problems, ideas and theories. Encyclopaedia of Mathematical Sciences, 49. *Springer-Verlag, Berlin,* 2005
H. L. Montgomery, R. C. Vaughan, Multiplicative number theory I. Classical theory, Cambridge Studies in Advanced Mathematics, 97. *Cambridge University Press, Cambridge* 2006
W. Narkiewicz, Elementary and analytic theory of algebraic numbers. Third edition. Springer Monographs in Mathematics. *Springer-Verlag, Berlin,* 2004
J. Neukirch, Algebraic number theory. Grundlehren der Mathematischen Wissenschaften, 322. *Springer-Verlag, Berlin,* 1999

Grading

Homework 20%
Project 40%
Final exam 40%