Regular Variation of order n and applications

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A positive and measurable function $f(x)$ is regularly varying with index α if for all $x > 0$ we have $\lim_{t\longrightarrow\infty} f(tx)/f(t) = x^{\alpha}$. Notation $f \in RV(\alpha)$. To study the rate of convergence, we can study second order relations of the form

$$\lim_{t\longrightarrow\infty} \frac{f(tx) - x^{\alpha} f(t)}{g_1(t)} = h_1(x),$$

where $g_1(t)$ is measurable, is ultimately of one sign and satisfies $g_1(t) = o(1)f(t)$. In this paper we study higher order relations of the form

$$\lim_{t\longrightarrow\infty} \frac{f(tx) - x^{\alpha} f(t) - h_1(x)g_1(t) - ... - h_n(x)g_n(t)}{g_{n+1}(t)} = h_{n+1}(x), \qquad (1)$$

where the rate functions $g_i(t)$ are measurable, ultimately of one sign and satisfy $g_{i+1}(t) = o(1)g_i(t)$, $i = 1, 2, ..., n$.

Among others we show that the above relation holds locally uniformly in $x > 0$, we prove a representation theorem for such functions and completely characterize the functions $h_i(x)$ that appear in the defintion. Relations of this type are useful in studying asymptotic properties in extreme value theory. More precisely, we show that many distribution functions alow an expansion of the form (1). We also discuss new asymptotic expansions for the difference between the product and the convolution product of distribution functions.

References

[1] Omey, E. and Segers, J. Generalised regular variation of arbitrary order. Submitted (2009).

[2] Omey, E. and Willekens, E. (1988). Π-variation with remainder. J. London Math. Soc. *J. London Math. Soc.2* **37 (1)** (1988), 105–118.