

Mathematical Institute SANU

Research Internship for Students 2020

During 2020, the Mathematical Institute SANU offers research internships for students interested in mathematics, computer sciences and mechanics and applications. The internship is open to all students from Serbia and abroad. The MISANU will provide working space and the mentors for the selected research topics, however in this moment it cannot offer any kind of financial support for the students.

In 2020, the following programs are opened. The interested students should send short CV and express the interest for the selected topic by sending an e-mail to studentski_seminar@mi.sanu.ac.rs or to contact mentors directly.

Research topic: *Formal systems for uncertain probabilistic reasoning, theory and applications*

Discipline: Mathematical Logic and Foundations, Mathematical Applications in Computer Science and Artificial Intelligence, Knowledge Management

Short description:

Mentors:

Zoran Ognjanović, zorano@mi.sanu.ac.rs

Aleksandar Perović, pera@sf.bg.ac.rs

Prerequisites: requested - familiarity with classical mathematical logic and probability theory; optional - familiarity with modal logics

Suggested material: Zoran Ognjanović, Miodrag Rašković, Zoran Marković, Probability Logics: Probability-Based Formalization of Uncertain Reasoning, Springer, 2016.

Research topic: *Wave propagation and heat conduction problems in hereditary and non-local media*

Discipline: Mechanics

Short description:

Mentors: Dušan Zorica, dusan_zorica@mi.sanu.ac.rs

Prerequisites:

Suggested material:

Research topic: *Contact mechanics at Finite Element Analysis– theory and applications*

Discipline: AppliedMechanics, Computational Mechanics

Short description: The summer internship on "*Contact mechanics at Finite Element Analysis– theory and applications*" will consist of two basic parts. In the first part students will be introduced to the theoretical basics of contact mechanics required for following this short course, while the second part will cover the basic knowledge of contact problem modeling in the Finite Element Analysis (FEA). The practical part of the course will be realized by specialized licensed software (ANSYS). Within this course basic theoretical examples in contact mechanics will be studied, as well as non-linear contact phenomena in real problems of Mechanics of Machines and Mechanisms.

During the internship, students will also be briefed on the scientific results published in recent years by leading authors in the field. Particular attention will be paid to the analysis of published papers and work on new original scientific contributions in this field, which, depending on student engagement, could result in the preparation of original work for submission to the appropriate scientific journal.

This summer internship course is intended for PhD and Master Students from Serbia and abroad who have basic backgrounds in mathematics and mechanics, necessary to follow the theory of the finite element method.

The duration of the internship is 2-3 weeks; the place of the internship is the Mathematical Institute SANU, Belgrade, Serbia.

Mentor: Ivana Atanasovska, iatanasovska@mi.sanu.ac.rs

Prerequisites: requested - basic engineering backgrounds in mathematics and mechanics; optional - familiarity with mechanics of continuum and theory of finite element method

Suggested material:

- Anthony C. Fischer-Cripps, *Introduction to ContactMechanics*, Springer, 2006
- J. R. Barber, *Contact Mechanics*, Springer, 2018
- Valentin L. Popov, Markus Heß, EmanuelWillert, *Handbook of ContactMechanics - Exact Solutions of AxisymmetricContact Problems*, Springer, 2019

- Alexander Konyukhov, Ridvan Izi, *Introduction to computational contact mechanics – a geometrical approach*, Wiley, 2015
- Peter Wriggers, *Computational ContactMechanics*, Springer, 2006
- *ANSYS Mechanical APDL Contact TechnologyGuide*, 2010

Research topic: *The Mathematical Spectra of Michael Petrovich*

Discipline: Foundations of mathematics

Short description: The spectral method of Michael Petrovich concerns a foundation of mathematics in terms of the real numbers. In opposition to the discrete language of formal logic, such a topic is based upon continuum which makes it connatural to the intuitionism of Brouwer. The aim is to elucidate its opportunities and achievements

Mentor: Miloš Milovanović, milosm@turing.mi.sanu.ac.rs

Prerequisites: required – real numbers; optional – p-adic numbers

Suggested material: M. Petrovitch. *Les spectres numeriques*. Gauthier-Villars, Paris, 1919.

M. Petrovitch. *Lecon sur les spectres mathematiques*. Gauthier-Villars, Paris, 1928.

Research topic: *Cosmology of the Church Fathers*

Discipline: Cosmology

Short description: Cosmology is a traditional science that has been restored by Albert Einstein in terms of the general relativity. The cosmological thought was initially presented by the Church fathers, such as Augustine of Hippo, Basil the Great, Maximus the Confessor and others. The *Divine Comedy* of Dante Alighieri has also based upon the ecclesial cosmology. The aim is to elucidate relationships to contemporary science.

Mentor: Miloš Milovanović, milosm@turing.mi.sanu.ac.rs

Prerequisites: required - reallivity theory; optional - fractal geometry

Suggested material: Aurelije Avgustin, *Ispovesti*, Dereta, Beograd 2009.

Bojan Tomić, *Fizika u šestodnevnu Vasilija Velikog*, Eparhijski upravni odbor Eparhije žicke, Kraljevo 2008.

Miloš Milovanović & Bojan M. Tomić, *Fractality and self-organization in the Orthodox iconography*, *Complexity* 21(S1), 2016, 55-68.

Vukašin Milićević, *Deseta nedoumica Svetog Maksima Ispovednika I bogoslovski problem vremena*, *Teološki pogledi* 48(2), 2015, 257-290.

Research topic: *The Calendar Issue Regarded through a Perspective of the Ecclesial Tradition*

Discipline: Cosmology

Short description: The topic is about an applied cosmology of the church fathers that permeates tradition, history, politics, art and other areas. The aim is to elucidate an interrelation between time and memory in that respect.

Mentor: Miloš Milovanović, milosm@turing.mi.sanu.ac.rs

Prerequisites: required - Julian calendar; optional - Passover rule

Suggested material: Miloš Milovanović, *Isihazam i kalendarsko pitanje*, Isihazam u životu Crkve srpskih i pomorskih zemalja, Zlatko Matić (ed.), Institut za sistematsko bogoslovlje Pravoslavnog bogoslovskog fakulteta, Beograd 2019, 157-173.

Miloš Milovanović, *Pitanje kalendara u svetlosti predanja Srpske pravoslavne crkve*, Arhipelag, Beograd 2020 (in the press).

Miloš Milovanović, *Spaljivanje Svetog Save Srpskog (prilog metodologiji istorijskog datovanja)*, Mitološki zbornik 42, 2019, 127-169.

Frances A. Yates, *The Art of Memory*, Ark Paperbacks, London 1966.

Research topic: *Harmonic quasiconformal mappings*

Discipline: Complex Analysis

Short description:

Mentors: Vesna Todorčević, vesna.todorcevic@fon.bg.ac.rs

Prerequisites:

Suggested material:

Research topic: *Hyperbolic type metrics*

Discipline: Complex Analysis

Short description:

Mentors: Vesna Todorčević, vesna.todorcevic@fon.bg.ac.rs

Prerequisites:

Suggested material:

Research topic: *Extremal Combinatorics*

Discipline: Combinatorics

Short description:

Mentor: Luka Milićević, luka.milicevic@mi.sanu.ac.rs

Prerequisites:

Suggested material:

Research topic: *New type of compandor to reduce the PAPR with maintaining the BER performance*

Discipline: Computer Science

Short description:

Mentor: Lazar Velimirović, velimirovic.lazar@gmail.com

Prerequisites:

Suggested material:

Research topic: *New hybrid techniques for reduce the PAPR*

Discipline: Computer Science

Short description:

Mentor: Lazar Velimirović, velimirovic.lazar@gmail.com

Prerequisites:

Suggested material:

Research topic: *Impact of companding techniques on bandwidth, noise, distortion and the ratio of power saving*

Discipline: Computer Science

Short description:

Mentor: Lazar Velimirović, velimirovic.lazar@gmail.com

Prerequisites:

Suggested material:

Research topic: Geometric representations in landscape gardens and their perception

Discipline: Multidisciplinary (geometry, perception), applied mathematics, computer sciences

Short description:

Landscape architectures all-over the world use geometry to design state-of-the-art gardens the beauty of which brings benefits to both the individual and the society. Geometry of a visual image conveys the affective meaning of a scene or an object. It is known that simple geometric forms convey emotions. For example downward-pointing V's are perceived as threatening and curvilinear forms are perceived as pleasant. Open gardens can be considered as complex geometrical structures with different levels of complexity. The aim of this research topic is to identify the most frequently used geometrical forms and their relationships in complex geometrical structures they build in space compositions in open gardens; to identify hierarchical models of composition of specific patterns. Does the dominance of certain geometric shapes in

composition and the level of complexity individually affect the perception of the garden-geometric pattern itself? The research should have 2 stages: 1. Analysis of existing gardens (3 formal and 3 informal gardens). 2. Generating a garden base scheme and testing their perception with real subjects. Perspectives: the data obtained could be used in the future to create virtual gardens.

Mentors:

Anđelka Hedrih, handjelka@gmail.com

Ivana Pedović, ivana.pedovic@gmail.com

Prerequisites: familiarity with visual data processing and fractals

Suggested material:

Christine L. Larson Joel Aronoff Elizabeth L. Steuer. Simple geometric shapes are implicitly associated with affective value. *Motiv Emot* (2012) 36:404–413 DOI 10.1007/s11031-011-9249-2

Patrick Spröte, Philipp Schmidt, Roland W. Fleming. Visual perception of shape altered by inferred causal history. *Scientific Reports* (2016) | 6:36245 | DOI: 10.1038/srep36245

Jay FriedenberG. The Perceived Beauty of Regular Polygon Tessellations. *Symmetry* 2019, 11, 984; doi:10.3390/sym11080984.

L. Dabbour, Geometric proportions: The underlying structure of design process for Islamic geometric patterns, *Frontiers of Architectural Research* (2012) 1(4): 380-391

I. Pedović, M. Stosić. A comparison of verbal and sensory presentation methods in measuring crossmodal correspondence within a semantic-based approach. *Československá psychologie* 2018 / ročník LXII / číslo 6

Research topic: Developing metaheuristic algorithms for optimization problems

Discipline: Operations research and management science

Short description: The main research topics are directed towards the development of mathematical models and (meta)heuristic optimization methods for various world-known optimization problems (optimization on graphs, scheduling, transportation, location, etc). Beside the application of different general purpose exact solution methods (CPLEX, Gurobi, LINGO, etc.), problem specific exact and heuristic algorithms will be developed. Although working with various metaheuristic methods, we particularly promote the ones developed by Serbian researchers: Variable Neighborhood Search (VNS) and Bee Colony Optimization (BCO). In addition, our current research project investigates parallelization, theoretical and empirical evaluation of metaheuristics. Our interest is

also directed towards the integration of Artificial Intelligence (AI) and optimization methods to deal with real-life optimization problems that occur in science and industry.

Mentor: Tatjana Davidović, tanjad@mi.sanu.ac.rs

Prerequisites: Good programming skills, C(C++), Java, Python.

Suggested material:

Talbi, El-Ghazali, *Metaheuristics: from design to implementation*, John Wiley & Sons, 2009.

Hansen, Pierre, et al., Variable neighborhood search: basics and variants, *EURO Journal on Computational Optimization* 5(3):423-454, 2017.

Davidović, T., Bee Colony Optimization: Recent Developments and Applications, (plenary talk), *Proc. Balkan Conference on Operational Research, BALCOR 2015*, Constanta, Romania, Sept. 9-12, 2015. *Mircea cel Batran Naval Academy Scientific Bulletin*, 18(2):225-235, 2015.

Research topic: *Algorithms to solving linear fractional programming problems in fuzzy environment*

Discipline: Operations Research

Short description: This research internship provides students an opportunity to participate in a scientific project on fuzzy fractional optimization. The recent results in the field will be briefly presented and possible directions for new researches will be analyzed. Particular attention will be paid to methodologies from fuzzy linear programming that can be extended to fuzzy linear fractional programming.

Mentor: Bogdana Stanojević, bgdnpop@gmail.com

Prerequisites: familiarity with basic mathematical programming and fuzzy sets theory

Suggested material: Frenk J.B.G., Schaible S. (2005) Fractional Programming. In: Hadjisavvas N., Komlósi S., Schaible S. (eds) *Handbook of Generalized Convexity and Generalized Monotonicity. Nonconvex Optimization and Its Applications*, vol 76. Springer, New York; Ghanbari, R., Ghorbani-Moghadam, K., Mahdavi-Amiri, N. *et al.* Fuzzy linear programming problems: models and solutions. *Soft Computing* (2019).

Research topic: *Algorithms to solving multiple objective non-linear programming problems*

Discipline: Operations Research

Short description: This research internship provides students an opportunity to participate in a scientific project on multiple objective optimization. Particular attention will be paid to non-linear problems and non-evolutionary solution algorithms. The recent results in the field will be briefly presented and possible directions for new researches will be analyzed.

Mentor: Bogdana Stanojević, bgdnpop@gmail.com

Prerequisites: familiarity with basic non-linear programming and optimization techniques

Suggested material: P. M. Pardalos, A. Zilinskas, and J. Zilinskas. Non-Convex Multi-Objective Optimization. Springer, (2017); El-Ghazali Talbi, Matthieu Basseur, Antonio J. Nebro, and Enrique Alba. Multi-objective optimization using metaheuristics: non-standard algorithms. International Transactions in Operational Research, 19(1-2):283-305, (2012).

Research topic: *Beyond Borsuk Ulam theorem: Equivariant methods in Discrete and Convex Geometry*

Discipline: Geometric Combinatorics, Algebraic Topology

Short description:

Mentor: Pavle Blagojević, pavle.v.m.blagojevic@gmail.com

Prerequisites:

Suggested material:

Research topic: *Positive Grassmannians and Amplituhedra*

Discipline: Geometric Combinatorics, Algebraic Topology

Short description:

Mentor: Pavle Blagojević, pavle.v.m.blagojevic@gmail.com

Prerequisites:

Suggested material:

Research topic: *Theory of arrangements of subspaces and their complements*

Discipline: Geometric Combinatorics, Algebraic Topology

Short description:

Mentor: Pavle Blagojević, pavle.v.m.blagojevic@gmail.com

Prerequisites:

Suggested material:

Research topic: *Topology, Combinatorics and Geometry of Configuration Spaces*

Discipline: Geometric Combinatorics, Algebraic Topology

Short description:

Mentor: Pavle Blagojević, pavle.v.m.blagojevic@gmail.com

Prerequisites:

Suggested material:

Research topic: *Using Spectral Sequences*

Discipline: Geometric Combinatorics, Algebraic Topology

Short description:

Mentor: Pavle Blagojević, pavle.v.m.blagojevic@gmail.com

Prerequisites:

Suggested material:

Research topic: *Rigidity Problems for Neighborly Polytopes in Toric Topology*

Discipline: Toric topology, Geometric Combinatorics

Short description: In this research project we are studying various types of rigidity in toric topology concerning toric spaces over neighborly polytopes such as the moment-angle manifolds, quasitoric manifolds and small covers. The principal questions we address are whether certain (cohomology) rings are isomorphic or not, and whether this implies certain rigidity property or not.

Mentor: Pavle Blagojević, djbaralic@mi.sanu.ac.rs

Prerequisites: required – familiarity with basic algebraic properties of rings, ideals and isomorphisms; optional – familiarity with homology and cohomology, as well as with basic notions in toric topology is welcomed, but not mandatory

Suggested material:

Suyoung Choi, Mikiya Masuda and Dong Youp Suh , *Rigidity problems in toric topology: A survey*, *Proceedings of the Steklov Institute of Mathematics* volume 275, 177–190(2011)

Victor Buchstaber and Taras Panov, Toric Topology, **AMS Mathematical Surveys and Monographs**

Volume: 204; 2015

Djordje Baralić and Lazar Milenković, *Small covers and quasitoric manifolds over neighborly polytopes*, preprint

Research topic: Elements of mathematical phenomenology and Phenomenological Mappings - Dynamics of hybrid systems with complex structures

Discipline: Mechanics, Mathematical physics, nonlinear sciences and applications

Short description: Introduction in research Ph.D. students and instruction for researchers into topics of multi-disciplinary nonlinear sciences; scientific methods and methodology in research in nonlinear dynamics; Theories of stability.

Elements of mathematical phenomenology and Phenomenological mappings: Theory and Applications

Chapter I. Mihailo Petrović (6 May 1868-8 June 1943)	5
Chapter II. Mihailo Petrović's Theory: Elements of Mathematical Phenomenology and Phenomenological Mappings.....	7
Chapter III. Graphical presentations of some elements of mathematical phenomenology and phenomenological mappings.....	9
Chapter IV. Linear and non-linear transformations.....	17

Chapter V. Central collision of two rolling balls: Theory and examples.....	57
Chapter VI. Trigger of the one side singular points in vibrations of the system with Amontons-Coulomb's type friction and with one degrees of freedom.....	101
Chapter VII. Chain system dynamics: Phenomenological mappings in vibrations, signals, resonances and dynamical absorptions in chain system dynamics by Katica R. (Stevanović) Hedrih and Andjelka N. Hedrih	161
Chapter VIII. Discrete fractional order system vibrations and fractional order signals by Katica R. (Stevanović) Hedrih and J. A. Tenreiro Machado.....	195
Chapter IX. Structural analogies on systems of deformable bodies coupled with non-linear layers by Katica R. (Stevanović) Hedrih and Julijana D. Simonović.....	235
Chapter X. Elements of mathematical phenomenology in dynamics of multi-body system with fractional order discrete continuum layers.....	271
Chapter XI. The mathematical analogies between vector models of stress state vector model, strain state vector model and mass inertia moment state vector model.....	297
Chapter XII. Logical, structural, qualitative and mathematical analogies	337

Seminar's work and the one day Mini-symposia with student lectures or poster presentations in the end of internship.

Mentor: *Katica (Stevanovica) Hedrih,*

http://www.mi.sanu.ac.rs/novi_sajt/research/conferences/ksh/default.htm

http://www.mi.sanu.ac.rs/novi_sajt/research/projects/174001a.php

e-mails: katicah@mi.sanu.ac.rs, katicah@mi.sanu.ac.rs, khedrih@sbb.rs

Prerequisites: *Basic foundations in Theoretical and Applied Mechanics and Mathematics (Bachelor degree in Engineering and Mechanics, BioMechanics)*

Suggested material: PDF files of each lecture and consultations

Research topic: THE THEORY OF BODY COLLISIONS IN ROLLING THROUGH GEOMETRY, KINEMATICS AND DYNAMICS OF BILLIARDS – Applications in methodology of research of vibro-impact system dynamics

Discipline: Mechanics, Nonlinear sciences and applications

Short description: Introduction in research Ph.D. students and instruction for researchers into topics of multi-disciplinary nonlinear sciences; scientific methods and methodology in research in nonlinear dynamics; Theories of stability.

Keywords: Theory of collision, Rolling balls, Billiards vibro-impact dynamics .

Abstract. The elements of geometry, kinematics and dynamics of rolling homogeneous balls along curvilinear lines are defined. The complete theory of the impact and collision of heavy rolling balls, through geometry, kinematics and dynamics of rolling balls, is defined. A new definition of the coefficient of restitution (collision) was introduced,

starting from the hypothesis of the conservation of the sum of angular momentum of the balls in rolling, for instant rolling axes, after the collision in relation to the before collision of the bodies. The expressions for the outgoing angular velocities of the ball rolling after the collision have been derived and their rolling paths after the impact or collision have been determined and various possible anchors have been shown. The difference between the content of the term billiards used in mathematical works of many mathematicians, as well as the research that remains in the field of geometry is pointed out. These results boil down to the task of inscribing open or closed polygonal lines in some restricted areas, and analogs are with tasks in optics, exploring the path of the light beam, which is reflected from mirrors at the boundaries defined by the regions. They are based on a series of Poncelet's theorems in geometry and do not reach the dynamics of real billiard systems. Our theory of ball rolling and collision is based on the examples of the abstraction of real rolling systems of heavy homogeneous billiards to a mechanical model.

Applications in methodology of research vibro-impact system dynamics

Seminar's work and the one day Mini-symposia with student lectures or poster presentations in the end of internship.

Mentor: *Katica (Stevanovica) Hedrih,*

http://www.mi.sanu.ac.rs/novi_sajt/research/conferences/ksh/default.htm

http://www.mi.sanu.ac.rs/novi_sajt/research/projects/174001a.php

e-mails: katicah@mi.sanu.ac.rs, katicah@mi.sanu.ac.rs, khedrih@sbb.rs

Prerequisites: *Basic foundations in Theoretical and Applied Mechanics and Mathematics (Bachelor degree in Engineering and Mechanics, BioMechanics)*

Suggested material: PDF files of each lecture and consultations

Research topic: Life and works of Serbian Scientists in area of Theoretical and Applied Mechanics

Discipline: Mechanics, Nonlinear sciences and applications, experimental mechanics

Short description: Introduction to research of Ph.D. students into topics of history of sciences in area of mechanics in Serbia

Abstract. Biobibliographies of Academicians and Professors of Mechanics in Serbia in Period (1870 – 1990). Ljubomir Klerić, Milutin Milanković, Anton Bilimović, Tatomir Andjelić, Ranilo Rašković, Vlatko Brčić, Jakov Hlitič, Djordje Zloković, Nikola Hajdin, i drugu. - Founder and Head of Department of Mechanics in Mathematical Institute of

SASA. Sroška škola nelinearnih oscilacija i naučni skupovi iz mehanike i nelinearnih nauka..

Seminar's work and the one day Mini-symposia with student lectures or poster presentations in the end of internship.

Mentor: *Katica (Stevanovica) Hedrih,*

http://www.mi.sanu.ac.rs/novi_sajt/research/conferences/ksh/default.htm

http://www.mi.sanu.ac.rs/novi_sajt/research/projects/174001a.php

e-mails: katicah@mi.sanu.ac.rs, katicah@mi.sanu.ac.rs, khedrih@sbb.rs

Prerequisites: *Basic foundations in Theoretical and Applied Mechanics and Mathematics (Bachelor degree in Engineering and Mechanics, BioMechanics)*

Suggested material: Edition of Serbian Academy of Sciences and Arts: "Life and work of Serbian Scientists".

