Hausdorff Geometry of Polynomials

Bl. Sendov

Abstract

Let $D(c;r)$ be the smallest disk, with center c and radius r, containing all zeros of the polynomial $p(z) = (z - z_1)(z - z_2)\cdots(z - z_n)$. Half a century ago, we conjectured that for every zero z_k of $p(z)$, the disk $D(z_k;r)$ contains at least one zero of the derivative $p'(z)$. More than 100 papers are devoted to this conjecture, in which it is proved for different special cases. But in general, this conjecture is proved only for the polynomials of degree $n \leq 8$.

In this lecture a stronger conjecture is discussed and proved for polynomials of degree $n = 3$. A number of other conjectures are presented, including a variation of the Smale's mean value conjecture.

References

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