TITLE: Distance in the absolute plane and Cauchy functional equations

Abstract: Let $A$ denote the absolute plane and $d_a$ the distance function on it. Using a constructive approach which leads to the functional equations, we study which conditions on a “measure” of segments on a given half-line $l$ in the absolute plane are essential to be the restriction of $d_a$ on $l$.

Description: We consider two well-known models of the absolute plane which we call E-model and H-model. The E-model is actually the standard model of the Euclidean plane and the H-model is the Poincaré disk model of the hyperbolic plane. We used same proposition that states that there is a distance $d_a$ (which is in accordance with the relations between and congruence) on the absolute plane and that it is unique up to a multiplicative constant. Let $d_e$ and $d_h$ be the interpretations of $d_a$ in the E-model and in the H-model, respectively. We consider a fixed half-line $l$ ($l = [0, 1)$ in the H-model and $l = [0, +\infty)$ in the E-model) and a function $f : l \to [0, +\infty)$ with some additive properties.

Scientific field: Mathematics

Scientific subfield: Geometry and Complex analysis