Financial and Actuarial Mathematics

Syllabus for a Master Course

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1. General goal of the course

The course provides students with an advanced academic education in Mathematical Finance and Actuarial Science. The students will be able to apply it in practice and will have a base for continuing their education in this field.

Students will be able to solve the basic problems in Financial and Actuarial fields. They will be able to integrate Actuarial, Statistical and Financial techniques in modeling and to take the right decisions in the area of insurance and financial practice.

The course gives a base for the students interested in a furhter PhD degree in Mathematical Modelling and Applications in Economics, Finance and Insurance.

2. Overview on the course modules

PART I: Mathematical Finance (30 units)

Module	No. of units	Contents
1. The Binomial Pricing Model.	2	One-period Binomial model. Multiperiod Binomial model.
2. Stochastic processes	4	Definitions. Examples. Martingales in discrete and continuous time. Local Martingales. The Optional Stopping Theorem. Martingale Convergence.
3. Stochastic processes with independent increments.	5	Random walk. Brownian Motion. First Passage Times. Properties. Stochastic counting processes. Poisson process. Mixed Poisson process.Polya process. Compound Poisson process. Polya-Aeppli process.
4. The Stochastic Integral.	4	Predictable processes. Square-integrable Martingales. Martingales of Bounded Variation. Integration with respect to local martingales.
5. Stochastic Calculus and Ito's formula.	5	Brownian Martingales. Exponential Processes. Change of measure and Girsanov's Theorem. Martingale Representation Theorem.
6. Risk-Neutral pricing.	3	Stock under risk-neutral measure. Black-Scholes-Merton formula. Hedging.
7. Fundamental theorems of asset pricing.	3	Existence of risk-neutral measure. Uniqueness of risk-neutral measure.

8. Stochastic differential equations.	4	Markov property. Feynmant – Kac theorem.

PART II: Insurance Risk Theory (30 units)

8. The risk process. Classical model of insurance risk.	4	Model description. Counting processes. Pure birth process. The Poisson process. The compound Poisson model. Cramer-Lundberg's theorem.
9. Ruin probability in the case of light tailed distributed claims.	5	Proof of Cramer – Lundberg's theorem. Ruin probability for exponential distributed claims.
10. Renewal Models.	4	Definition and properties. Renewal equation. Renewal arrivals. Probability of ruin. The Pollaczeck-Khinchine formula.
11. Random walk approach.	3	Random walk process. Probability of ruin and adjustment coefficient. Martingale approximation of the ruin probability.
12.Generalized risk models.	3	Compound Poisson process. Polya-Aeppli process. Ruin probability. Mixed Poisson process. Polya process. Models with dependent input. Approximations of the ruin probability.
13. Ruin probability in the presence of heavy tails.	3	Subexponential distributions. Ruin when claim sizes are heavy tailed. Ruin when both claim sizes and inter-arrival times are heavy tailed.
14. Approximations of the risk process.	4	Weak convergence of stochastic processes. Diffusion approximation. Ruin probability. Approximation with α – stable Levy motion.
15. Reinsurance.	4	Reinsurance policies. Investment in risky assets. On minimizing the ruin probability by investment and reinsurance. Approximation of ruin probability under optimal investment and reinsurance.
Total no. of units	60	

3. References:

1. Asmussen S. (2000). Ruin Probabilities, World Scientific Publishing Co.
2. Grandell J. (1991) Aspects of Risk Theory, Springer.
3. Grandell J. (1997) Mixed Poisson Processes. Chapman & Hall.
4. Kaas R., Goovaerts M., Dhaene J. and Denuit M. ((2001) Modern Actuarial Risk Theory, Kluwer Academic Publishers.
5. Mikosch Th. (2004) Non–Life Insurance Mathematics, Springer.
6. Elliott R.J. and Kopp P.E. (1999) Mathematics of Financial Markets, Springer.
7. Etheridge A. (2002) A Course in Financial Calculus, Cambridge University Press.
8. Karatzas I. and Shreve S. (1998) Brownian Motion and Stochastic Calculus, Springer.
9. Musiela M. and Rutkowski M. (1997) Martingale Methods in Financial Modelling, Springer.
10 Oksendal B. (1998) Stochastic Differential Equations, 5th edition, Springer.
11. Protter Ph.E. (2004) Stochastic Integration and Differential Equations, 2-nd edition, Springer.
12. Shreve S.E. (2004) Stochastic Calculus for Finance I, The Binomial Asset Pricing Model, Springer.
13. Shreve S.E.(2004) Stochastic Calculus in Finance II, Continuous – Time Models, Springer.
14. Minkova L.D. Lecture notes.

4. Teaching

The course should be accompanied by homework exercises. The students can work on their completion during at most 2 of the afternoon sessions. The major part of the afternoon sessions should be spent by working independently in teams. The results also should be presented in the afternoon sessions. During the afternoon sessions the lecturer should be available for questions and be present in order to get an impression on the performance of the students.

The course is planned to last 4 weeks, with lectures from Monday to Friday. This implies that there will be 3 teaching units (45 minutes each) per day. The following schedule is proposed for each day:

8:00 till 11:00: three units with breaks in between;
11:30 till 12:30: discussion with the lecturer;
15:00 till 17:30 work on homework exercises,
work in teams on problems posed by the lecturer,
presentation of results.

5. Grading

The basis for grading is provided by the performance of the students in the following items:

a) Homework exercises will be regularly given in order to achieve a better understanding of the lectures.

b) During each week of the course a project should be performed by the students. They should work in teams of 4 – 5 persons. The results obtained have to be presented by the teams.

c) An oral examination is planned to take place. It can consist of several parts taken during the course.

In order to obtain the grade for the course the following weights will be used for the items a) – c) from above:

Homework exercises	20%
Projects	50%
Oral examination	30%

The European Credit Transfer System (ECTS) is used for the grading of all performance assessments.