

Study programmes: Bachelor studies – Mathematics				
Course name: Selected Topics of Algebra and Mathematical Logic				
Lecturers: Milan Božić, Aleksandar Lipkovski, Dragana Todorčić, Zoran Petrović, Goran Danković				
Status: Optional				
ECTS: 6				
Attendance prerequisites: Introduction to Mathematical Logic, Linear algebra, Algebra 1, Algebra 2				
Course aims: Acquisition of general and specific knowledge of algebra and mathematical logic.				
Course outcome: Upon completion of the course, the students have advanced knowledge of selected topics from lattice theory and Boolean algebras, model theoretic algebra, theory of free algebras and category theory. The students understand the following concepts: free algebra, presentation of algebras, the word problem, functor, universal object. The students know the theorems of Stone, Birkhoff and Sikorski. They are able to solve problems in these fields, and to attend advanced courses in algebra and other fields of mathematics in which algebraic methods are applied.				
Course content:				
Lattices. Partially ordered sets. Axioms for lattices. Modular and distributive lattices. Complete lattices, Tarsky lemma and Cantor-Bernstein theorem. Ideals of lattices; principle, prime and maximal ideals. Boolean algebras and rings. Representation of finite Boolean algebras. Filters, ultrafilters and ultraproducts. Stone's representation theorem for Boolean algebras.				
Model theoretical algebra. Satisfaction relation, compactness theorem, applications in algebra. Quantifier elimination in fields.				
Congruences of algebras. The lattice of congruences. Congruences and quotient algebras; examples.				
Free algebras. Definition and examples of free algebras. First order logic with equality. Theorem of the existence of free algebras in algebraic varieties. Birkhoff's theorem. Free Boolean algebras (Sikorski's theorem). Free groups. Free abelian groups. Generators of algebras and relators. Presentations of algebras. The word problem.				
Categories. Definition and examples. Functors and natural transformations. Universal objects. Examples: products, coproducts and free objects.				
Literature:				
Ž. Mijajlović, <i>An Introduction to Model Theory</i> , PMF Novi Sad, 1987; G. Kalajdžić, <i>Algebra</i> , Matematički fakultet, Beograd, 1998; Ž. Mijajlović, <i>Algebra</i> , Milgor, Beograd, 1998; N. Božović, Ž. Mijajlović, <i>Uvod u teoriju grupa</i> , Naučna knjiga, Beograd, 1990; P.M. Cohn, <i>Universal algebra</i> , D. Reidel Publ. Co. Dodrecht, Netherlands, 1981; S. MacLane, <i>Categories for the working Mathematician</i> , 2 nd edition, Springer, New York, 1998.				
Number of hours: 5	Lectures: 3	Tutorials: 2	Laboratory: -	Research: -
Teaching and learning methods: Lectures/ Tutorials				
Assessment (maximal 100 points)				
Course assignments	points	Final exam	points	
Lectures	-	Written exam	30	
Exercises / Tutorials	-	Oral exam	40	
Colloquia	20	Written-oral exam	-	
Essay / Project	10			