Study programmes: Bachelor studies – Mathematics
Course name: Linear Algebra
Lecturers: Milan Bozić, Aleksandar Lipkovski, Dragana Todorić, Zoran Petrović, Goran Đanković, Predrag Tanović, Zoran Petrić
Status: Compulsory
ECTS: 13
Attendance prerequisites: no prerequisites.
Course aims: Acquisition of advanced general and specific knowledge of linear algebra.
Course outcome: Upon completion of the course, students have the basic knowledge of linear algebra. They can solve systems of linear equations and they become familiar with the solutions set structure. Students are able to understand fundamental concepts, main constructions and basic theorems of the theory of vector spaces. They have acquired the following notions: linear map, minimal polynomial and the determinant of a matrix, as well as their essential features. They are qualified to solve problems from the mentioned areas and follow other courses in which the areas are applied. Also, after the second semester, students have advanced knowledge of linear algebra and get familiar with diagonalization theorems for linear and bilinear maps. They know the theory of inner product spaces and theorems of important classes of their operators. They are prepared to follow advanced courses in which linear algebra plays an important part.
Course content: **Basic algebraic structures.** Semigroup, group. Ring, field; polynomials and matrices. **Vector spaces and linear maps.** Modules, vector spaces and linear algebras. Basic examples and first consequences of vector space axioms; Cartesian product of vector spaces. Subspaces, intersection of subspaces, sum of subspaces; affine subspaces and quotient spaces. Linear maps, kernel and image of a linear map. Rank-nullity theorem. **Basis and dimension.** Linear combination, linear span, generating set. Linear independence. Basis and dimension of a vector space. Grassman formula. The rank and the nullity of a linear map. **Coordinates of a vector and matrix representations of a linear map.** Coordinates of a vector in the given basis; change of basis and coordinates. Matrix representations of a linear map; equivalent and similar matrices. **Algebra of square matrices.** The isomorphism beetween the algebra of linear operators and the algebra of square matrices. Minimal polynomial of a matrix and linear operator; nth power of matrix. **Rank of a matrix.** Elementary row (column) operations. Echelon form of a matrix; rank of a matrix. Applications (the inverse of a matrix). **Determinants.** Determinants-definition, basic properties and Cauchy-Binet formula. Laplace expansion of the determinant; inverse of a matrix; the determinant rank of a matrix. **Systems of linear equations.** Solutions set structure; Gaussian elimination. Kronecker-Capelli theorem. Cramer's rule. **Reductions of linear operators and matrices**. Invariant subspaces, eigenvalues and eigenvectors. Characteristic polynomial, Cayley-Hamilton theorem. Triangular and diagonal form of a linear operator, Jordan canonical form of a matrix. Linear difference equations. **Linear and multilinear forms.** Linear forms, the dual space, the transpose of a linear map. Alternating multilinear forms, determinants. **Bilinear and quadratic forms.** Matrix representation of a bilinear form. Quadratic forms, diagonalization, equivalence of quadratic forms. Classification of real and complex quadratic forms. Positive definite quadratic forms. **Inner product spaces.** Inner product, Euclidean vector spaces. Norm of a vector, Cauchy-

Schwarz inequality, distance and angle between vectors. Orthogonality, orthonormal basis, orthogonal matrices. Orthogonal projection of a vector onto a subspace, applications. The mixed product and the vector product in a Euclidian space.
Linear maps of Euclidean spaces. Symmetric operators, diagonalization and canonical forms in Euclidean spaces. Orthogonal operators and canonical forms. The polar decomposition.
Isometries of Euclidean space. Isometries, translations. Canonical form of an isometric transformation.
Hermitian spaces. Hermitian inner product, unitary and Hermitian operators. Normal operators, canonical form.
Applications in geometry. Second-order curves and surfaces. Canonical equationes of second-order curves and surfaces.

Literature:

1. G. Kalajdžić, Linearna algebra, 5th edition, Matematički fakultet, Beograd, 2007.
2. A. Lipkovski, Linearna algebra i analitička geometrija, 2nd edition, Zavod za udžbenike i nastavna sredstva, Beograd, 2007.
3. A. Lipschutz, Schaum's Outline of Theory and Problems of Linear Algebra, 2nd edition, Mc Graw-Hill, New York, 1991.

Number of hours: 11=5+6	**Lectures:** 5=2+3	**Tutorials:** 6=3+3		

Teaching and learning methods: Frontal / Interactive / Tutorials / Lectures / Exercises

Assessment (maximal 100 points)

Course assignments	points	Final exam	Points
Lectures	-	Written exam	30
Exercises / Tutorials	-	Oral exam	40
Colloquia	30	Written-oral exam	-
Essay / Project	-		