Study programmes: Bachelor studies – Mathematics

Course name: Algebra 1

Lecturers: Milan Bozić, Aleksandar Lipkovski, Dragana Todorić, Zoran Petrović, Goran Đanković, Predrag Tanović, Zoran Petrić

Status: Compulsory

ECTS: 6

Attendance prerequisites: None

Course aims: Acquisition of general and specific knowledge of algebra.

Course outcome: Upon completion of the course, students have the basic knowledge of algebra and elementary number theory. They have acquired the following notions: algebraic structure, group, ring, field. Students are able to understand main algebraic constructions and basic theorems. They are qualified to solve problems from the mentioned areas and follow advanced courses of algebra and other mathematical disciplines in which algebra plays an important part.

Course content:

Elements of abstract algebra. Algebraic structures. Algebraic identities; algebraic theories and varieties, examples. Homomorphisms of algebras, subalgebras and generating sets; direct product of algebras. Kernel of homomorphism, fundamental theorem on homomorphisms.

Groups. Semigroups, groups, embedding of regular semigroup into a group, examples. Power of an element in a group, Lagrange's theorem, order of an element in a group. Cyclic groups and their subgroups, automorphisms of cyclic group; multiplicative group of integers modulo n, direct product of cyclic groups, multiplicativity of Euler's function. Inner automorphisms of a group, normal subgroups and quotient groups. Dihedral groups. Symmetric and alternating groups. Groups of order less than 10.

Finitely generated abelian groups. Free abelian groups. Finitely generated abelian groups. Primary decomposition, invariant factor decomposition. Generators and relations.

Rings and fields. Consequences of axioms. Characteristic of a ring. Field of fractions of an integral domain. Multiplicative group of a field. Polynomial ring. Euclidean division and Euclidean algorithm for polynomials. Roots of polynomial and polynomial factorization into irreducible factors. Kronecker's construction and splitting field of a polynomial. Viete's formulas and symmetric polynomials.

Introduction to number theory. Divisibility, GCD, LCM. Congruences. Euclidean division and Euclidean algorithm. Ring of integers modulo n, finite fields, Fermat's little theorem, Wilson's theorem, Euler's theorem, Chinese remainder theorem. Multiplicative arithmetic functions, Euler' function.

Literature:

1. G. Kalajdžić, Algebra, Matematički fakultet, Beograd, 1998.
2. Ž. Mijajlović, Algebra, Milgor, Beograd, 1998.
3. N. Božović, Ž. Mijajlović, Uvod u teoriju grupa, Naučna knjiga, Beograd, 1990.
4. A. Clark, Elements of Abstract algebra, Dover Publ. Co., New York, 1984.
5. A. Baker, A concise introduction to the theory of numbers, Cambridge Univ. Press., 1984.

Number of hours:	Lectures:	Tutorials:		
5	3	2		

Teaching and learning methods: Frontal / Interactive / Tutorials / Lectures / Exercises

Assessment (maximal 100 points)

Course assignments	points	Final exam	Points

Lectures	-	Written exam	30
Exercises / Tutorials	-	Oral exam	40
Colloquia	30	Written-oral exam	-
Essay / Project	-		